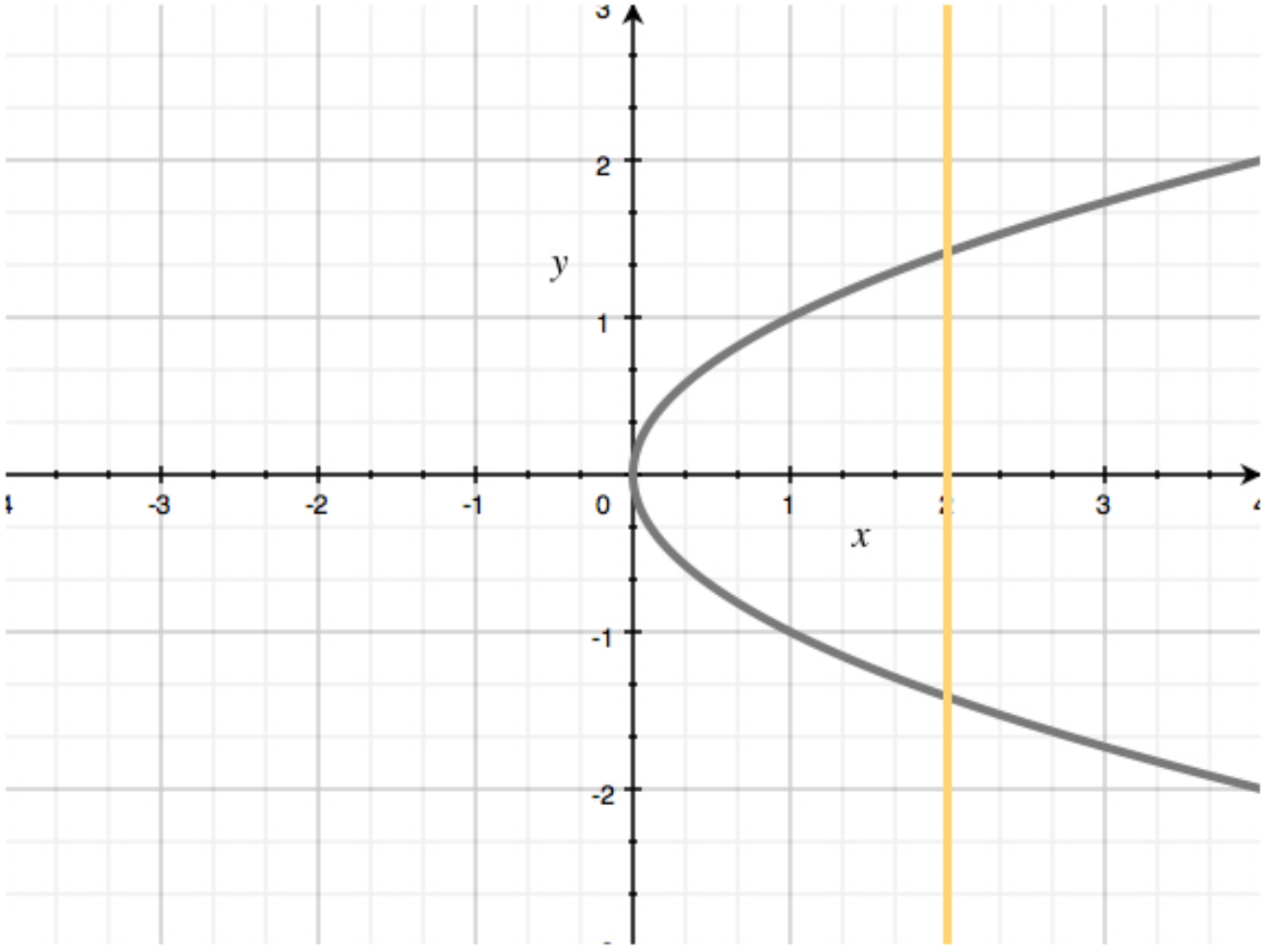
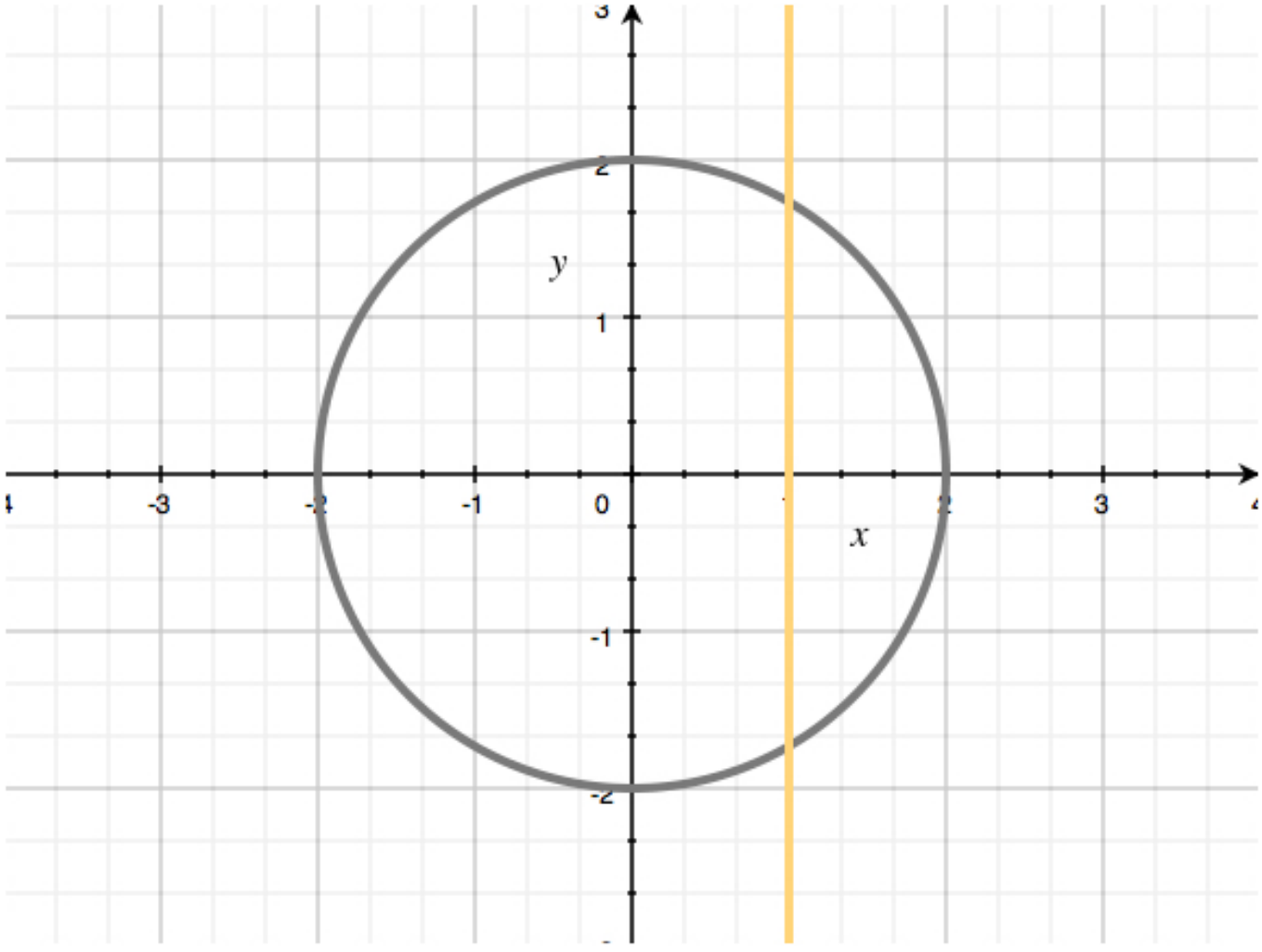
Vertical line test

Most of the equations you'll encounter in calculus are functions. Since not all equations are functions, it's important to understand that only functions can pass the Vertical Line Test. In other words, in order for a graph to be a function, no completely vertical line can cross its graph more than once. The graph below doesn't pass the Vertical Line Test because a vertical line intersects it more than once.



Passing the Vertical Line Test also implies that the graph has only one output value for y for any input value of x. You know that an equation is not a function if y can be two different values at a single x value.

You know that the circle below is not a function because any vertical line you draw between $x = -2$ and $x = 2$ will cross the graph twice, which causes the graph to fail the Vertical Line Test. In fact, circles can never be called functions because they'll never pass the Vertical Line Test.



You can also test this algebraically by plugging in a point between -2 and 2 for x, such as $x = 1$.

Example

Determine algebraically whether or not $x^2 + y^2 = 1$ is a function.

Plug in 0 for x and simplify.

$$(0)^2 + y^2 = 1$$

$$y^2 = 1$$

$$\sqrt{y^2} = \sqrt{1}$$

$$y = \pm 1$$

Looking at it another way, at $x = 0$, y can be both 1 and -1 . Since a function can only have one unique output value for y for any input value of x, the function fails the Vertical Line Test and is therefore not a function. We've now proven with both the graph and with algebra that this circle is not a function.
