

Sequences Mixed Exam Questions



Attempt these exam questions independently showing full and clear solutions. Check each answer as you go against the exam board mark scheme.

Arithmetic Sequences

1.

The first term of an arithmetic series is 1. The common difference of the series is 6.

- (a) Find the tenth term of the series. (2 marks)
- (b) The sum of the first n terms of the series is 7400.
- (i) Show that $3n^2 - 2n - 7400 = 0$. (3 marks)
- (ii) Find the value of n . (2 marks)
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2.

The first term of an arithmetic series is a and the common difference is d .

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

- (a) Use this information to write down two equations for a and d . (2)
- (b) Show that $a = -17.5$ and find the value of d . (2)

The sum of the first n terms of the series is 2750.

- (c) Show that n is given by $n^2 - 15n = 55 \times 40$. (4)
- (d) Hence find the value of n . (3)



3.

A sequence u_1, u_2, u_3, \dots is defined by

$$u_1 = 8 \quad \text{and} \quad u_{n+1} = u_n + 3.$$

- (i) Show that $u_5 = 20$. [2]
- (ii) The n th term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q . [2]
- (iii) State what type of sequence it is. [1]
- (iv) Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^N u_n = 1256$. [5]

Geometric Sequences

4.

The second term of a geometric series is 48 and the fourth term is 3.

- (a) Show that one possible value for the common ratio, r , of the series is $-\frac{1}{4}$ and state the other value. (4 marks)
- (b) In the case when $r = -\frac{1}{4}$, find:
- (i) the first term; (1 mark)
- (ii) the sum to infinity of the series. (2 marks)

5.

A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

- (a) Find the common ratio of the series. (1 mark)
- (b) Find the sum to infinity of the series. (2 marks)
- (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. (2 marks)
- (d) Prove that the n th term of the series is 25×0.8^n . (2 marks)



6. [Hint: if you can't do (a), the rest of the question is still doable]

- (a) A geometric series has first term a and common ratio r . Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r} \quad (4)$$

Mr. King will be paid a salary of £35 000 in the year 2005. Mr. King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

- (b) Find, to the nearest £100, Mr. King's salary in the year 2008. (2)

Mr. King will receive a salary each year from 2005 until he retires at the end of 2024.

- (c) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024. (4)

Inductive Sequences

7.

A sequence u_1, u_2, u_3, \dots is defined by

$$u_1 = 4 \quad \text{and} \quad u_{n+1} = \frac{2}{u_n} \quad \text{for } n \geq 1.$$

- (i) Write down the values of u_2 and u_3 . [2]
- (ii) Describe the behaviour of the sequence. [1]

8.

You are given that

$$u_1 = 1,$$
$$u_{n+1} = \frac{u_n}{1 + u_n}.$$

Find the values of u_2, u_3 and u_4 . Give your answers as fractions. [2]

Mixed Questions



9.

- (i) In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.

(A) Haroon answers six questions correctly. Show that he receives a total of £210. [1]

(B) State, in a simple form, a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant who receives £10 350 from this game. [4]

- (ii) In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.

(A) Gary received £75 from the game. How many questions did he get right? [1]

(B) Bethan answered 9 questions correctly. How much did she receive from the game? [2]

(C) State a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant in this game who receives £2 621 435. [4]

10.

- (a) An arithmetic progression has first term $\log_2 27$ and common difference $\log_2 x$.

(i) Show that the fourth term can be written as $\log_2 (27x^3)$. [3]

(ii) Given that the fourth term is 6, find the exact value of x . [2]

- (b) A geometric progression has first term $\log_2 27$ and common ratio $\log_2 y$.

(i) Find the set of values of y for which the geometric progression has a sum to infinity. [2]

(ii) Find the exact value of y for which the sum to infinity of the geometric progression is 3. [5]

11.

- (i) The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x . [3]

- (ii) The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

(a) Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case. [4]

(b) Find the other possible value of x that also gives a geometric progression. [4]