Scholastic Video Book Series

Artificial Neural Networks

Part 1

(with English Narrations)

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Artificial Neural Networks - #1

Classification using Single Layer Perceptron Model

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(ANN-001)

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Classification Example

- The following table shows sample data obtained from two different fruits.

	Weight (grams)	Length (cm)
Fruit 1 (Class C1)	121	16.8
	114	15.2
Fruit 2 (Class C2)	210	9.4
	195	8.1

- Train a single layer perceptron model using the above parameters to classify the two fruits.
- Using the model parameters you have obtained classify the fruit with weight 140gm and length 17.9cm.

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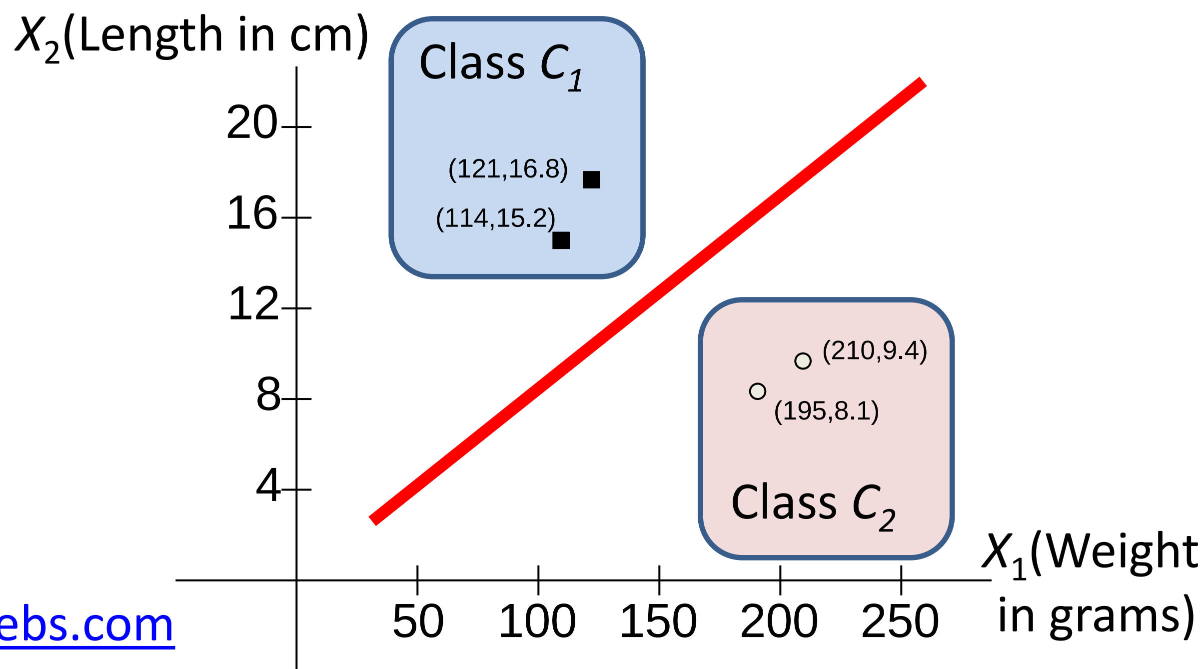
Classification Example

	Weight (grams)	Length (cm)
Fruit 1 (Class C1)	121	16.8
	114	15.2
Fruit 2 (Class C2)	210	9.4
	195	8.1



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Artificial Neural Networks Theory

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Artificial Neural Networks Theory

- **Artificial Neural Networks (ANN)** are computers whose architecture is modeled after the brain.
- They typically consist of many hundreds of simple processing units which are wired together in a complex communication network.
- Each unit or **node** is a simplified model of a real neuron which **fires** (sends off a new signal) if it receives a sufficiently strong input signal from the other nodes to which it is connected.

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Intelligent Machine

- The strength of these connections may be varied in order for the network to perform different tasks corresponding to different patterns of node firing activity.
- This structure is very different from traditional computers and hence make way to Intelligent Machines.

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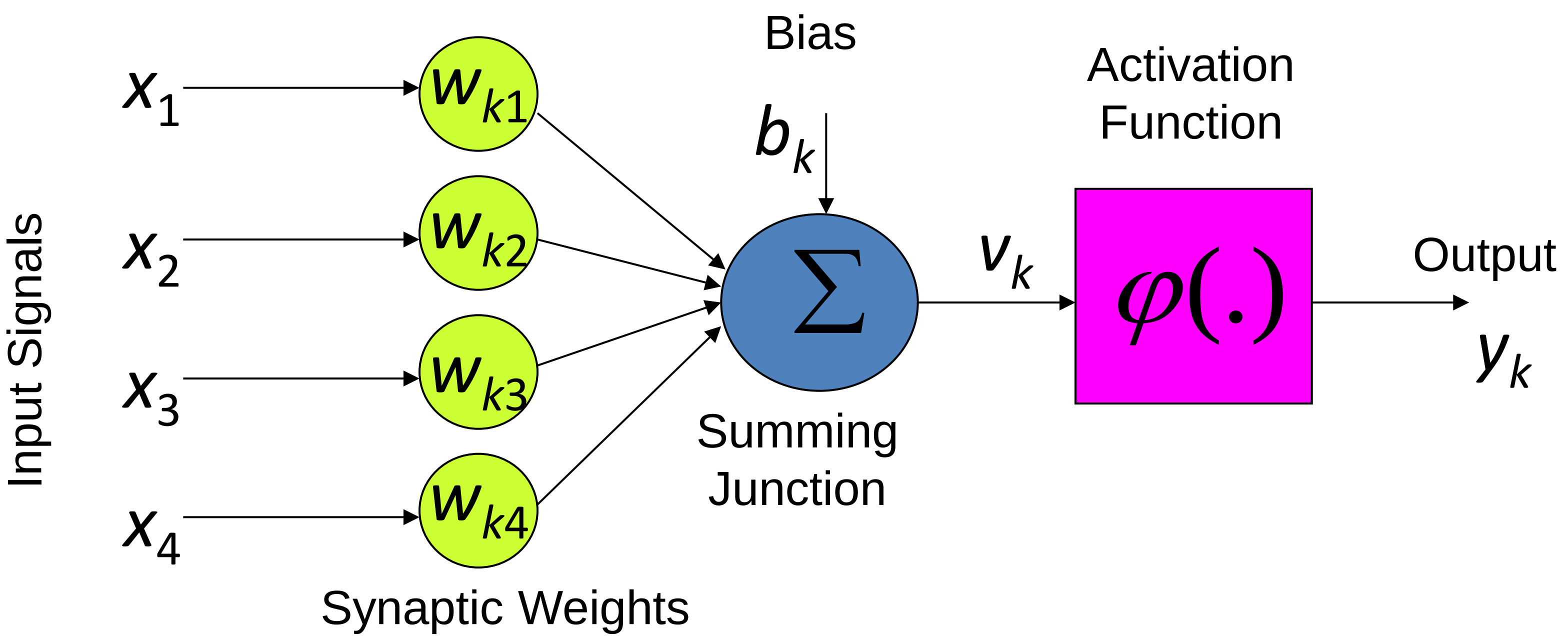
Model of a Neuron

- The model consists of a set of synapses each of which is characterized by a weight or strength of its own.
- An adder, an activation function and a bias.



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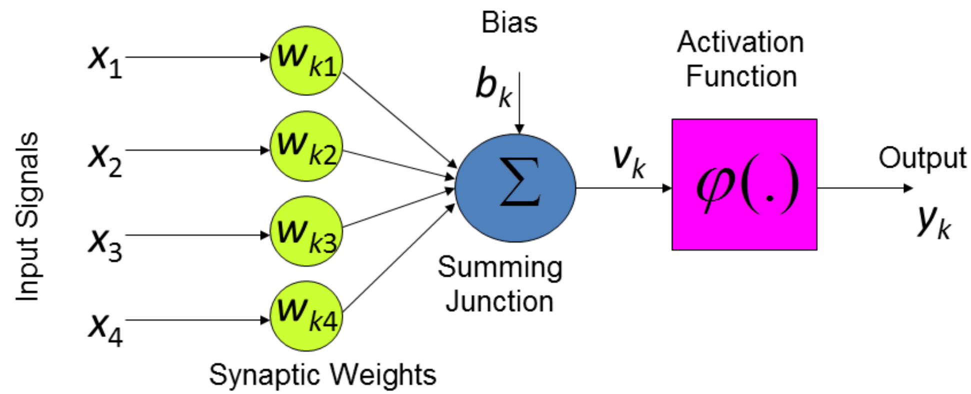


- In mathematical terms, a neuron k can be described by:

$$u_k = \sum_{j=1}^{m} w_{kj} x_j$$

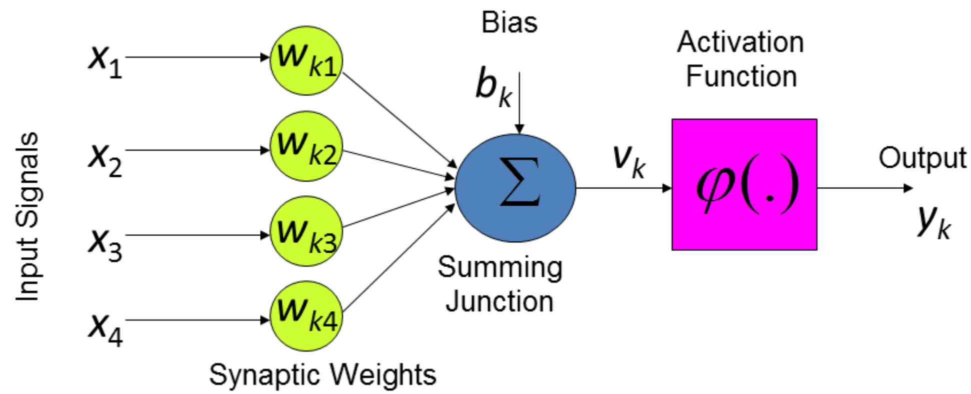
and

$$y_k = \varphi\left(u_k + b_k\right)$$

where u_k is the linear combiner output due to input signals.

- Also

$$v_k = u_k + b_k$$



- The bias is an external parameter of artificial neuron and can be included into the equations as follows:

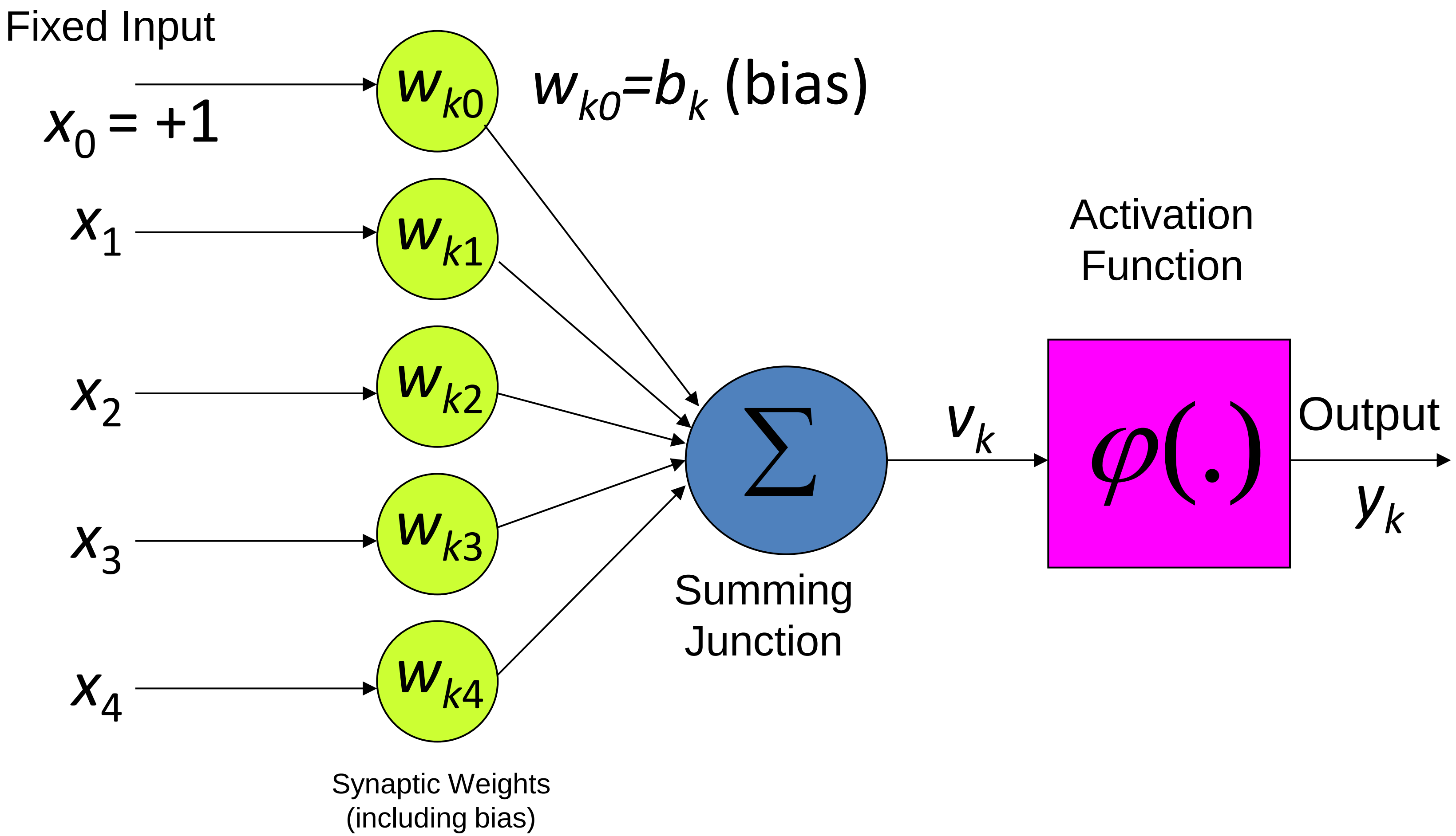
$$v_k = \sum_{j=0}^{m} w_{kj} x_j$$

- and

$$y_k = \varphi(v_k)$$

- Note the change of limits of *j* from 1 to 0.

Modified Neuron Structure



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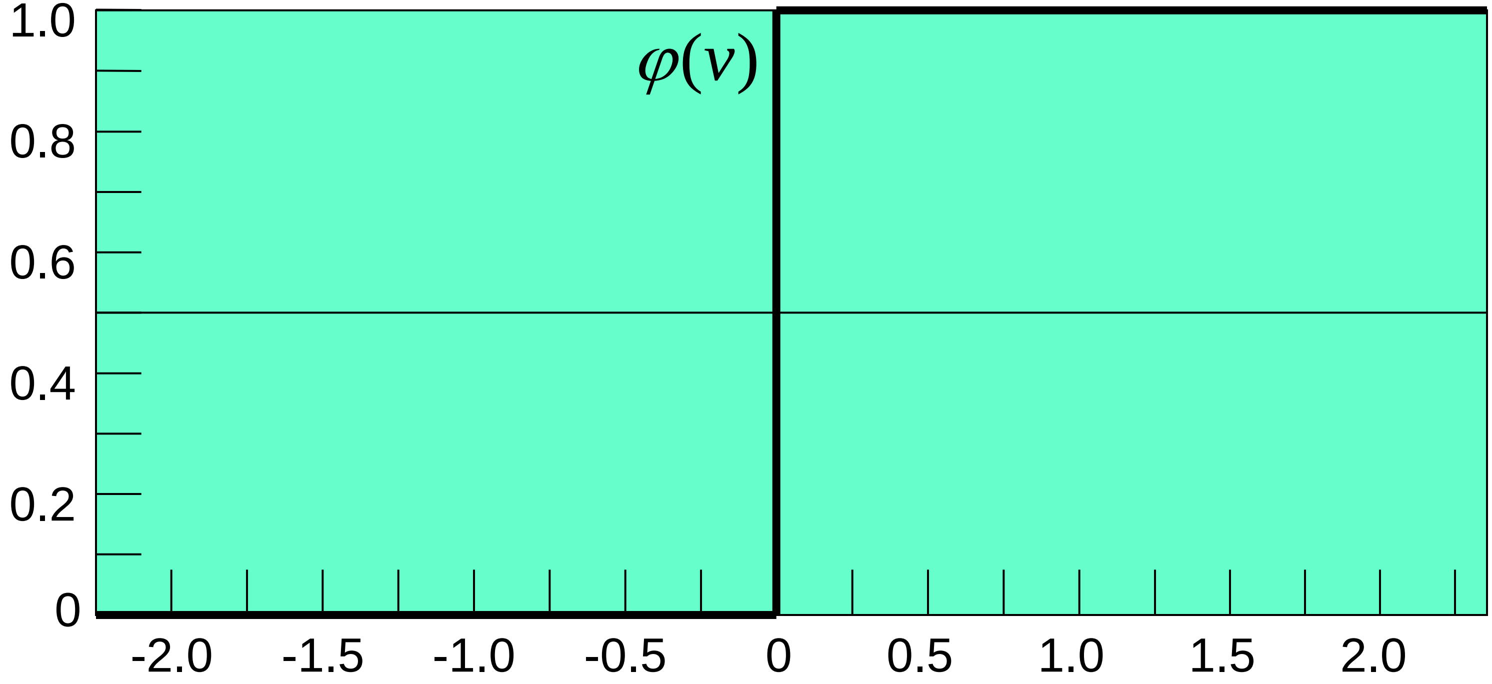
Types of Activation Functions

- Threshold Function or Heaviside Function:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases}$$



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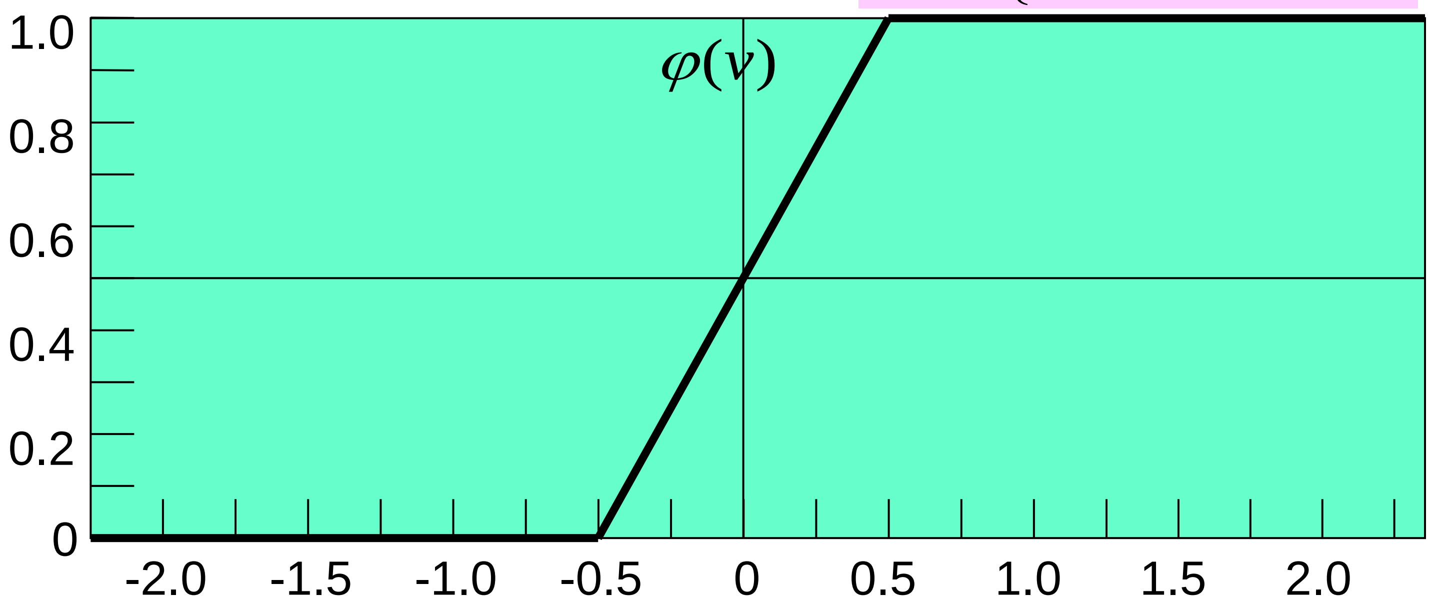
Types of Activation Functions (cont…)

- Piecewise-Linear Function:

$$\varphi(v)=\begin{cases}1, & v \geq +\frac{1}{2} \\ v, & +\frac{1}{2} > v > -\frac{1}{2} \\ 0, & v \leq -\frac{1}{2}\end{cases}$$



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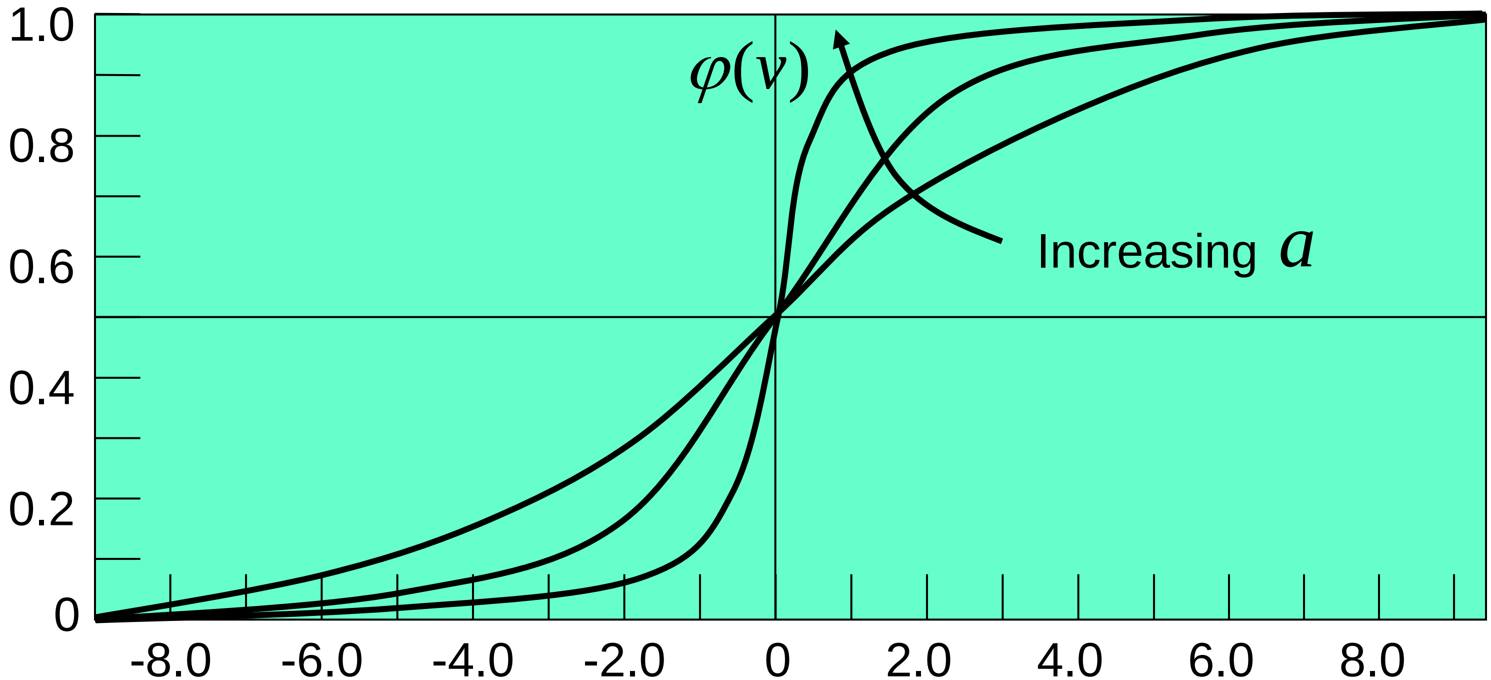
Types of Activation Functions (cont...)

- Sigmoid Function: $$\varphi(v) = \frac{1}{1+\exp(-av)}$$

where a is the slope parameter of the sigmoid function.



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Single Layer Perceptron

- The neuronal model we have just discussed is also known as a perceptron.
- The perceptron is the simplest form of a neural network used for the classification of patterns said to be linearly separable.
- Basically, it consists of a single neuron with adjustable synaptic weights and bias.
- Now we will look at a method of achieving **learning** in our model we have formulated.

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Perceptron Convergence (Learning) Algorithm

- Variables and Parameters

$$\mathbf{x}(n) = (m+1)\times 1 \quad \text{input vector}$$
$$= \left[+1, x_1(n), x_2(n), \ldots, x_m(n)\right]^T$$
$$\mathbf{w}(n) = (m+1)\times 1 \quad \text{weight vector}$$
$$= \left[b(n), w_1(n), w_2(n), \ldots, w_m(n)\right]^T$$
$$b(n) = \text{bias}$$
$$y(n) = \text{actual response}$$
$$d(n) = \text{desired response}$$
$$\eta = \text{learning-rate parameter, a postive constant less than unity}$$

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Perceptron Convergence (Learning) Algorithm (cont…)

1. ***Initialization***. Set $\mathbf{w}(0) = \mathbf{0}$. Then perform the following computations for time step $n = 1, 2, \ldots$
2. ***Activation***. At time step n, activate the perceptron by applying input vector $\mathbf{x}(n)$ and desired response $d(n)$.
3. ***Computation of Actual Response***. Compute the actual response of the perceptron:

$$y(n) = \operatorname{sgn}[\mathbf{w}^T(n)\mathbf{x}(n)]$$

where $\operatorname{sgn}(.)$ is the signum function.

$$\operatorname{sgn}(x) = \begin{cases} +1, & \text{if} \quad x \geq 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$$

Perceptron Convergence (Learning) Algorithm (cont…)

4. ***Adaptation of Weight Vector***. Update the weight vector of the perceptron:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \eta\left[d(n) - y(n)\right]\mathbf{x}(n)$$

where

$$d(n) = \begin{cases} +1 & \text{if } \mathbf{x}(n) \text{ belongs to class } C_1 \\ -1 & \text{if } \mathbf{x}(n) \text{ belongs to class } C_2 \end{cases}$$

5. ***Continuation***. Increment time step n by one and go back to step 2.

Decision Boundary

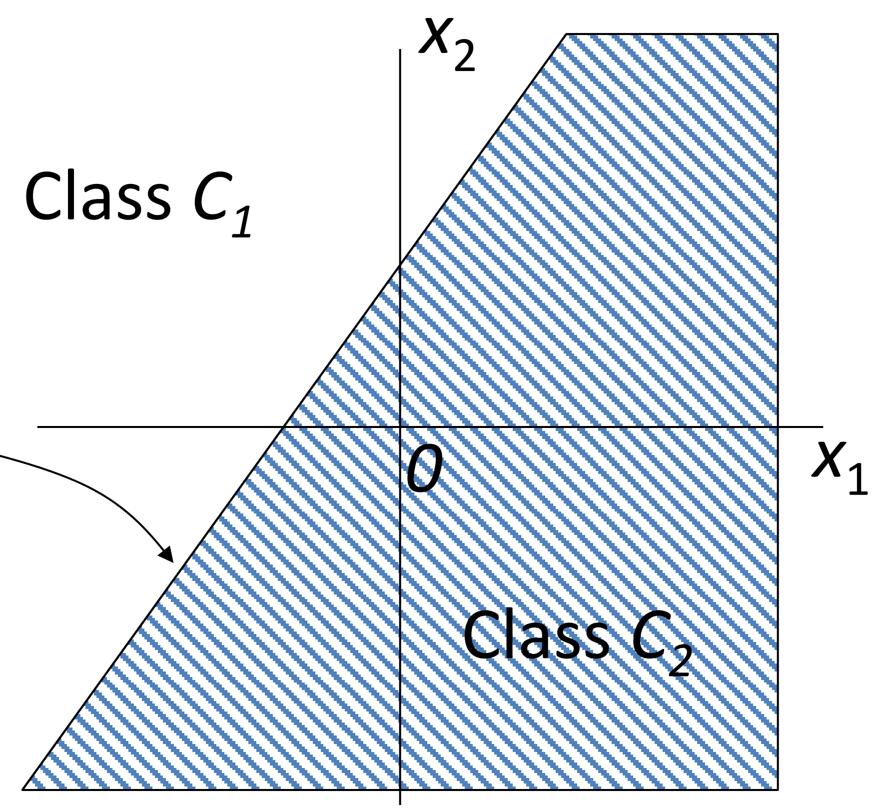
- The hyper-plane

$$\sum_{i=1}^{m} w_i x_i + b = 0$$

or

$$w_1 x_1 + w_2 x_2 + b = 0$$

is the decision boundary for a two class classification problem.



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Solution to the Example Question
(with correct initial weights and bias)

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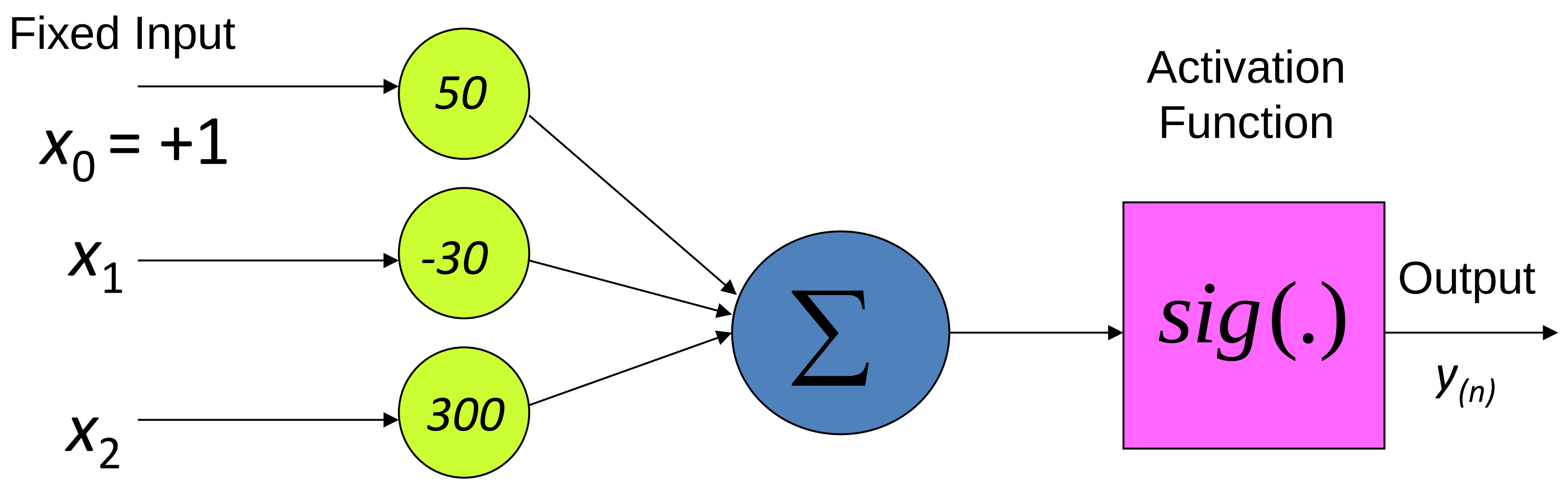
Solution to the Example
(with correct initial weights and bias)

With correct initial weights and bias

$$\left.\begin{array}{l} w_1(0) = -30,\ w_2(0) = 300, \\ b(0) = 50,\ \eta = 0.01 \end{array}\right\} given$$



$$\mathrm{sgn}(x) = \begin{cases} +1, & \text{if} \quad x \geq 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$$

Solution to the Example

(with correct initial weights and bias) (cont…)

$$\left.\begin{aligned} &w_1(0) = -30,\ w_2(0) = 300, \\ &b(0) = 50,\ \eta = 0.01 \end{aligned}\right\} given$$

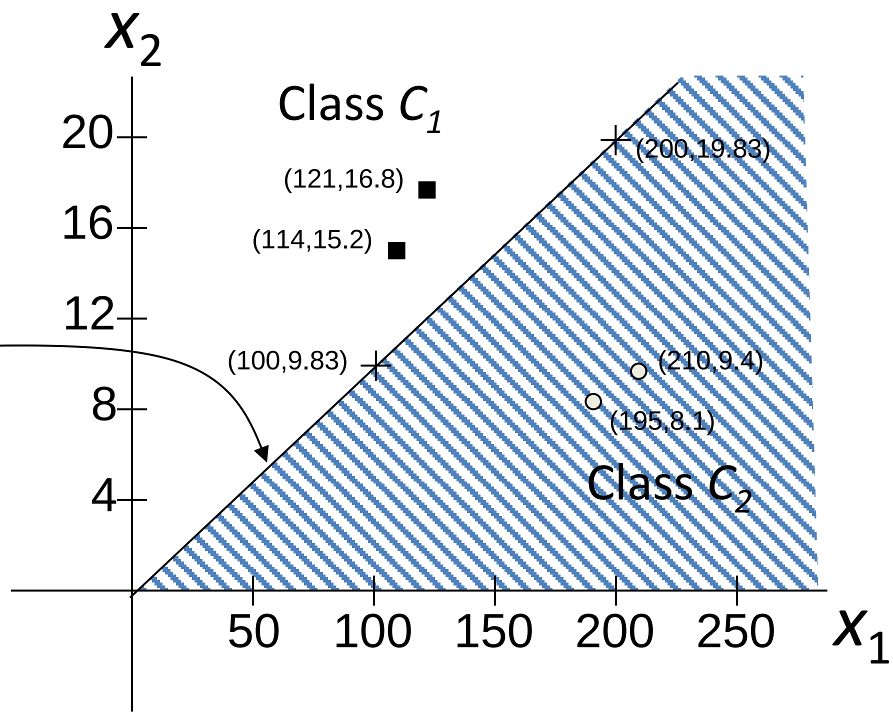
Therefore the Initial Decision Boundary for this example is:

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$-30x_1 + 300x_2 + 50 = 0$$

$$x_1 = 100,\ x_2 = \frac{30 \times 100 - 50}{300} = 9.83$$

$$x_1 = 200,\ x_2 = \frac{30 \times 200 - 50}{300} = 19.83$$



Initial hyper-plane does separate the two classes.

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Classification of the Unknown Fruit

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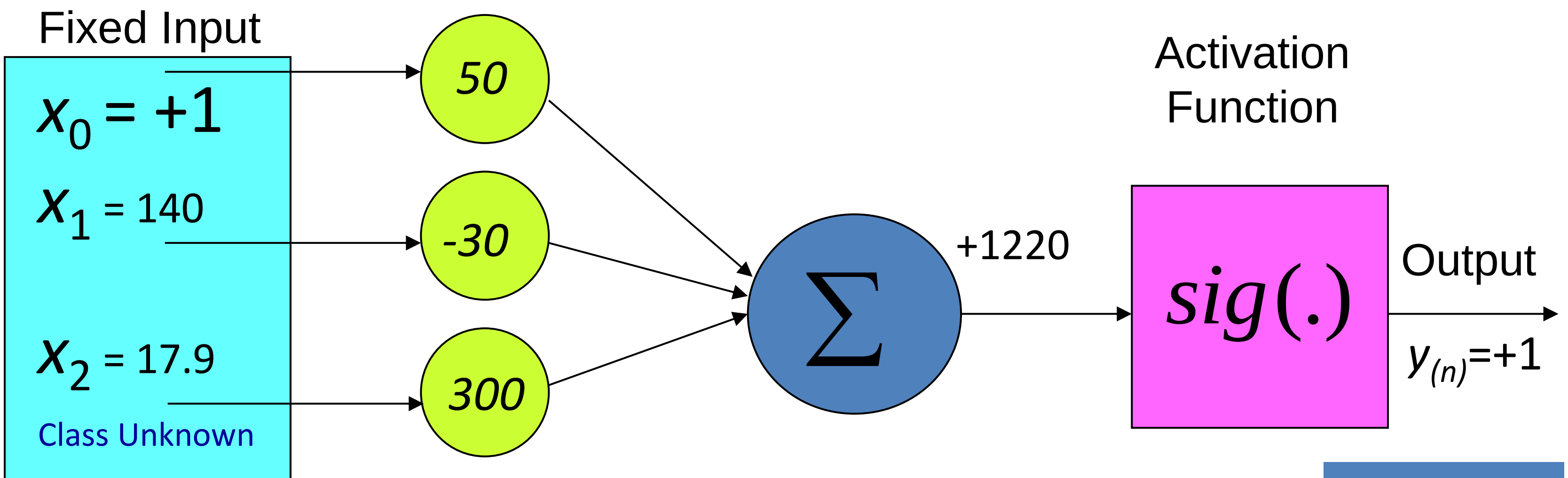

For Class C1,
Output = +1

Now use the above model to classify the unknown fruit.

$$\mathbf{x}(\text{unknown}) = [+1, 140, 17.9]^T$$

$$\mathbf{w}(3) = [50, -30, 300]^T$$

$$y(\text{unknown}) = \text{sgn}\left(\mathbf{w}^T(3)\mathbf{x}(\text{unknown})\right) = \text{sgn}(50\times 1 - 30\times 140 + 300\times 17.9)$$

$$= \text{sgn}(1220) = +1$$

$\therefore$ this unknown fruit belongs to the class C_1.

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Solution to the Example Question
(with unknown initial weights and bias)

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Solution to the Example

(with unknown initial weights and bias)

With unknown initial weights and bias

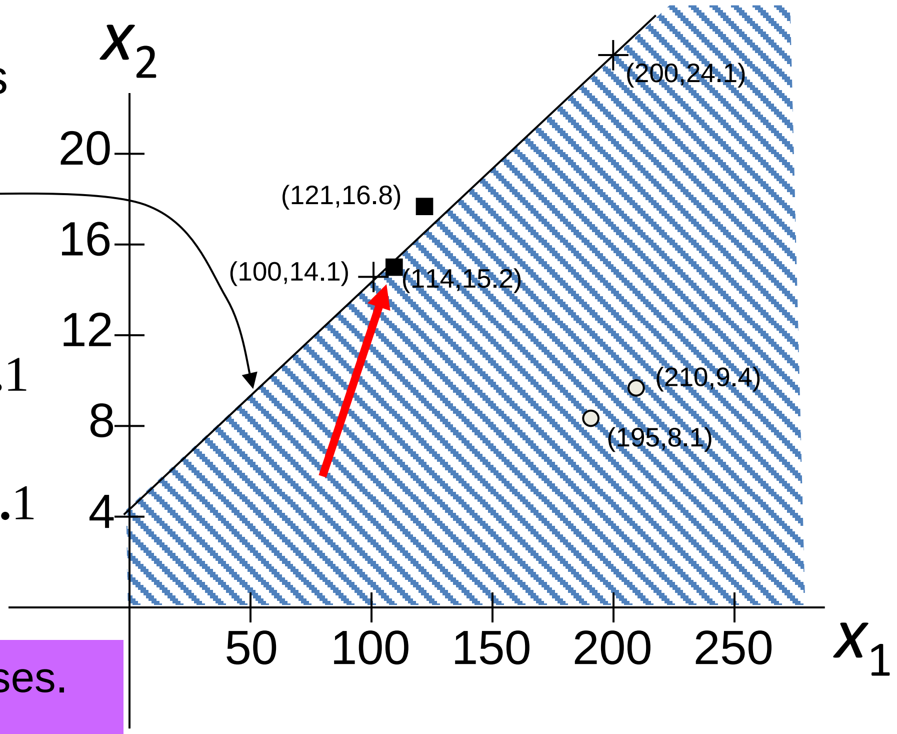
$$\left.\begin{array}{l} w_1(0) = -30,\ w_2(0) = 300, \\ b(0) = -1230,\ \eta = 0.01 \end{array}\right\} given$$

Therefore the Decision Boundary for this case:

$$-30x_1 + 300x_2 - 1230 = 0$$

$$x_1 = 100,\ x_2 = \frac{30 \times 100 + 1230}{300} = 14.1$$

$$x_1 = 200,\ x_2 = \frac{30 \times 200 + 1230}{300} = 24.1$$



Initial hyperplane does not separate the two classes.
Therefore we need to **Train** the Neural Network

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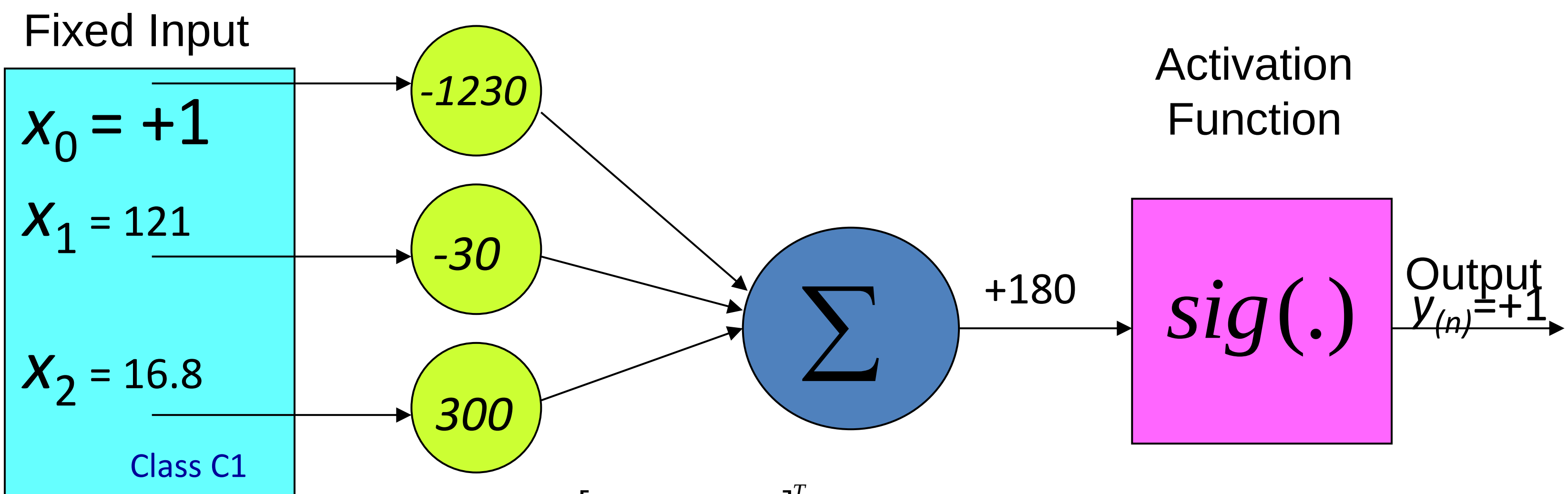
Training with known Fruit (121,16.8)



$$\mathbf{x}(n) = \mathbf{x}(0) = [+1, 121, 16.8]^T \quad \text{and} \quad d(0) = +1$$

$$\mathbf{w}(n) = \mathbf{w}(0) = [-1230, -30, 300]^T$$

$$y(n) = y(0) = \operatorname{sgn}\left(\mathbf{w}^T(0)\mathbf{x}(0)\right)$$

$$= \operatorname{sgn}\left(-1230 \times 1 - 30 \times 121 + 300 \times 16.8\right)$$

$$= \operatorname{sgn}(180) = +1 = d(0)$$

Hence no need to recalculate the weights.

$$\therefore \mathbf{w}(n+1) = \mathbf{w}(1) = [-1230, -30, 300]^T$$

For Class C1, Output = +1

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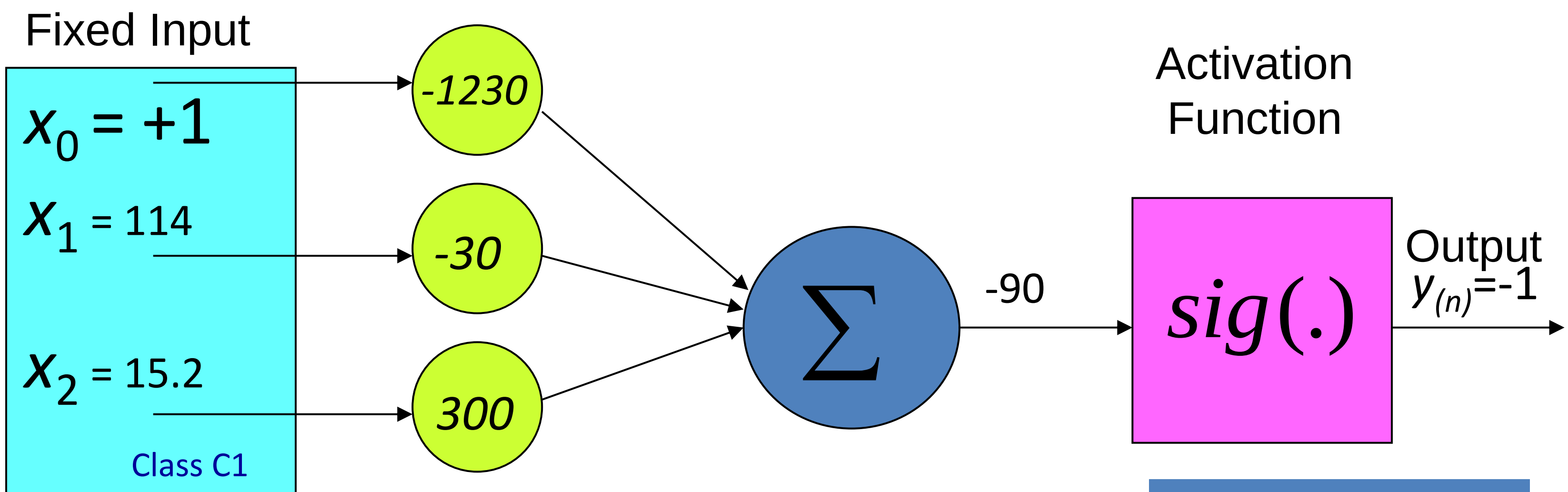
Training with known Fruit (114,15.2)



For Class C1, the output should be +1, but what we get is -1. Hence we have to recalculate the weights.

$$\mathbf{x}(1) = [+1, 114, 15.2]^T \quad \text{and} \quad d(1) = +1$$

$$\mathbf{w}(1) = [-1230, -30, 300]^T$$

$$y(1) = \operatorname{sgn}\left(\mathbf{w}^T(1)\mathbf{x}(1)\right) = \operatorname{sgn}\left(-1230\times 1 - 30\times 114 + 300\times 15.2\right)$$

$$= \operatorname{sgn}(-90) = -1 \neq d(1)$$

Hence **we have to** recalculate the weights.

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Adaptation of Weight Vector

Here we use ***Adaptation of Weight Vector*** (Step 4) to update the weight vector of the perceptron.

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \eta\left[d(n) - y(n)\right]\mathbf{x}(n)$$

$$\mathbf{w}(1) = \left[-1230, -30, 300\right]^T$$

$$\mathbf{x}(1) = \left[+1, 114, 15.2\right]^T$$

$$d(1) = +1, y(1) = -1, \eta = 0.01$$

$$\mathbf{w}(1+1) = \mathbf{w}(2) = \left[-1230, -30, 300\right]^T + 0.01\left[+1-(-1)\right]\left[+1, 114, 15.2\right]^T$$

$$= \left[-1230, -30, 300\right]^T + \left[+0.02, 2.28, 0.304\right]^T$$

$$\therefore \mathbf{w}(2) = \left[-1229.08, -27.72, 300.304\right]^T$$

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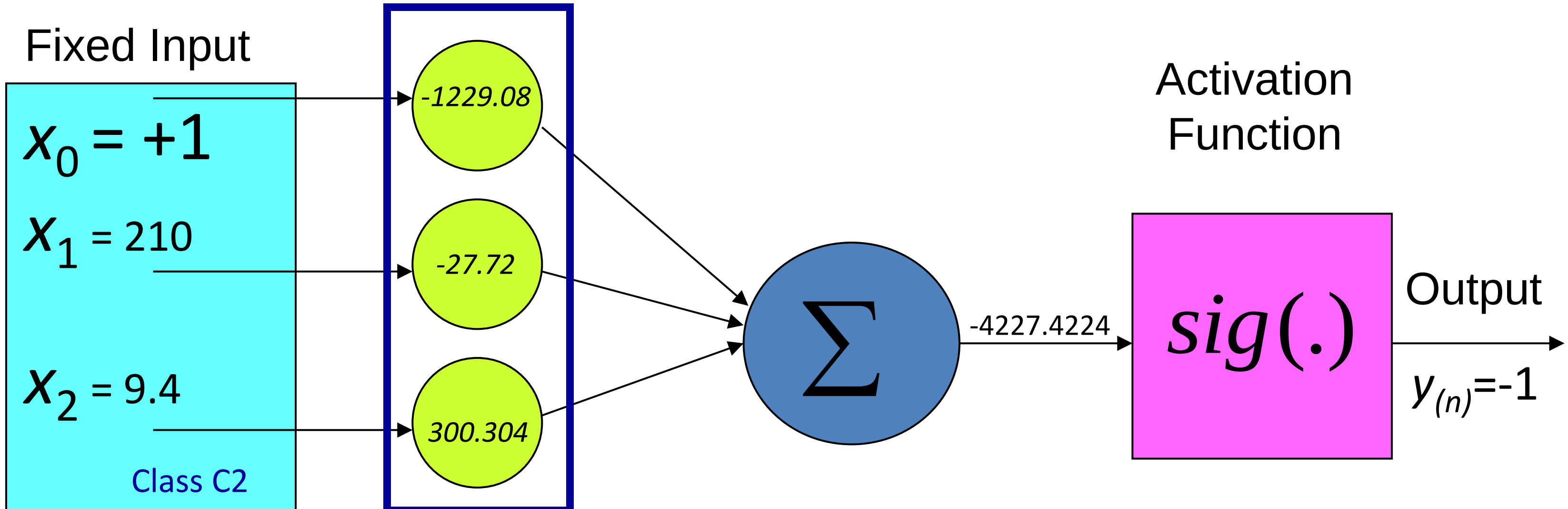
Training with known Fruit (210,9.4)



For Class C2, Output = -1

$$\mathbf{w}(2) = [-1229.08, -27.72, 300.304]^T$$

$$\mathbf{x}(2) = [+1, 210, 9.4]^T \quad \text{and} \quad d(2) = -1$$

$$\mathbf{y}(2) = \text{sgn}\left(\mathbf{w}^T(2)\mathbf{x}(2)\right) = \text{sgn}\left(-1229.08\times1 - 27.72\times210 + 300.304\times9.4\right)$$

$$= \text{sgn}(-4227.4224) = -1 = d(2)$$

Hence no need to recalculate the weights.

$$\therefore \mathbf{w}(n+1) = \mathbf{w}(3) = [-1229.08, -27.72, 300.304]^T$$

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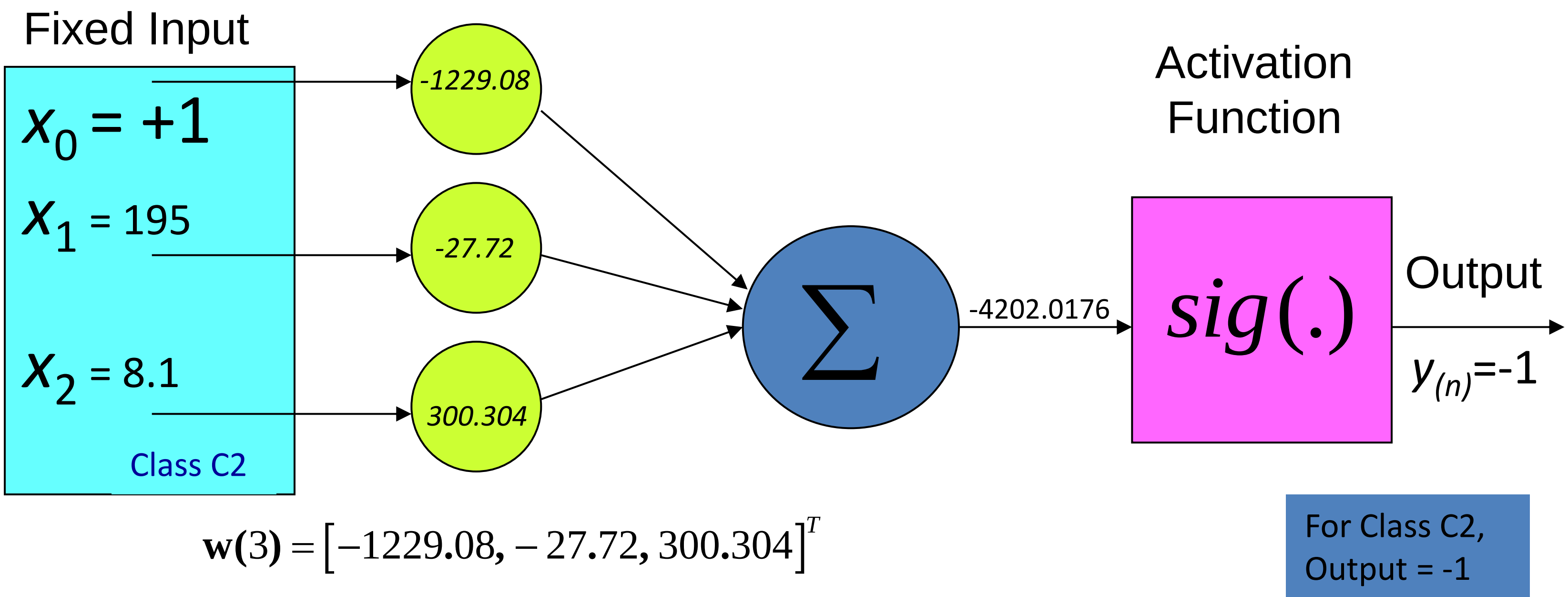
Training with known Fruit (195,8.1)



$$\mathbf{w}(3) = \left[-1229.08, -27.72, 300.304\right]^T$$

$$\mathbf{x}(3) = \left[+1, 195, 8.1\right]^T \quad \text{and} \quad d(3) = -1$$

$$\mathbf{y}(3) = \operatorname{sgn}\left(\mathbf{w}^T(3)\mathbf{x}(3)\right) = \operatorname{sgn}\left(-1229.08 \times 1 - 27.72 \times 195 + 300.304 \times 8.1\right)$$

$$= \operatorname{sgn}(-4202.0176) = -1 = d(3)$$

Hence no need to recalculate the weights.

$$\therefore \mathbf{w}(n+1) = \mathbf{w}(4) = \left[-1229.08, -27.72, 300.304\right]^T$$

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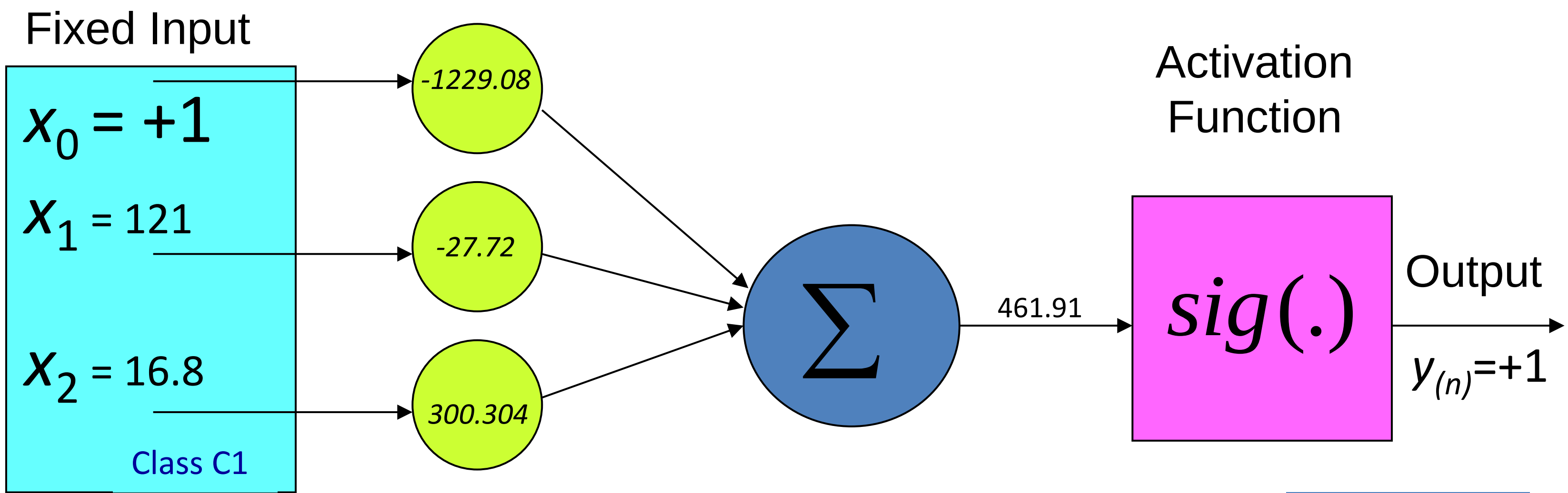
Training with known Fruit (121,16.8)



For Class C1, Output = +1

$$\mathbf{w}(4) = [-1229.08, -27.72, 300.304]^T$$

$$\mathbf{x}(4) = [+1, 121, 16.8]^T \quad \text{and} \quad d(4) = +1$$

$$\mathbf{y}(4) = \text{sgn}\left(\mathbf{w}^T(4)\mathbf{x}(4)\right) = \text{sgn}\left(-1229.08 \times 1 - 27.72 \times 121 + 300.304 \times 16.8\right)$$

$$= \text{sgn}(461.91) = +1 = d(4)$$

Hence no need to recalculate the weights.

$$\therefore \mathbf{w}(n+1) = \mathbf{w}(5) = [-1229.08, -27.72, 300.304]^T$$

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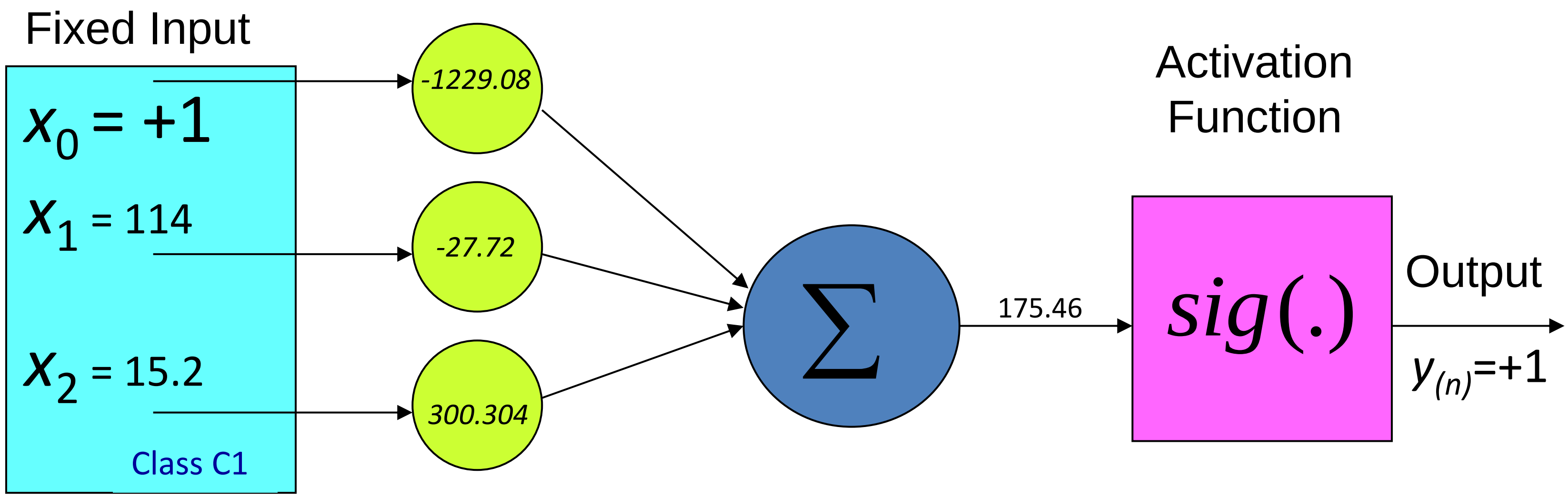
Training with known Fruit (114,15.2)



$$\mathbf{w}(5) = [-1229.08, -27.72, 300.304]^T$$

$$\mathbf{x}(5) = [+1, 114, 15.2]^T \quad \text{and} \quad d(5) = +1$$

$$\mathbf{y}(5) = \text{sgn}\left(\mathbf{w}^T(5)\mathbf{x}(5)\right) = \text{sgn}\left(-1229.08 \times 1 - 27.72 \times 114 + 300.304 \times 15.2\right)$$

$$= \text{sgn}(175.46) = +1 = d(5)$$

Hence no need to recalculate the weights.

$$\therefore \mathbf{w}(n+1) = \mathbf{w}(6) = [-1229.08, -27.72, 300.304]^T$$

For Class C1, Output = +1

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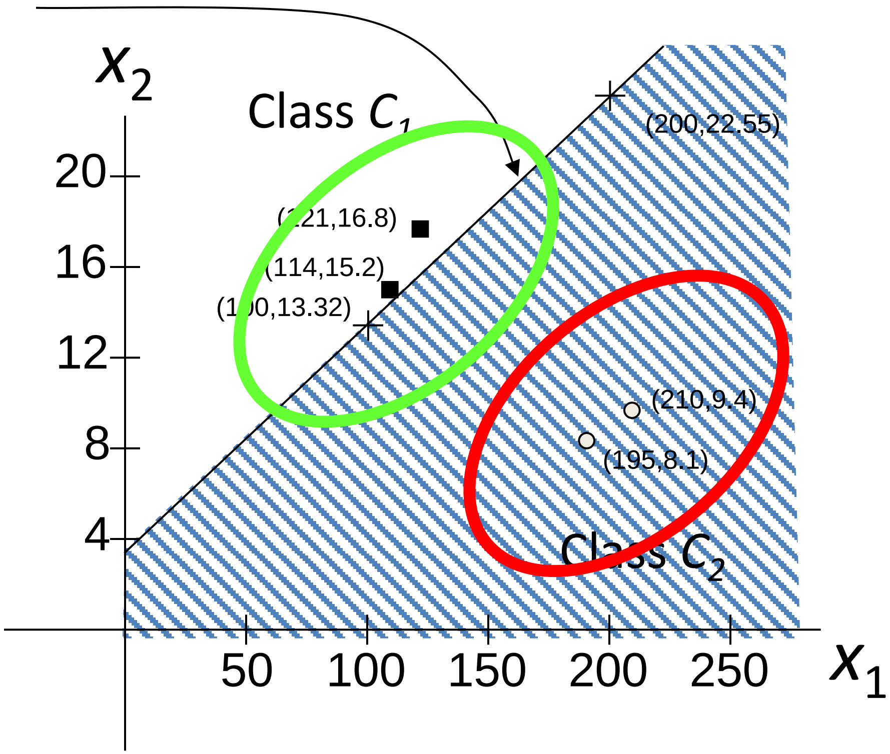
Decision Boundary After Training

Therefore the decision boundary obtained after Neural Network training is:

$$-27.72x_1 + 300.304x_2 - 1229.08 = 0$$

$$x_1 = 100,\ x_2 = \frac{27.72 \times 100 + 1229.08}{300.30} = 13.32$$

$$x_1 = 200,\ x_2 = \frac{27.72 \times 200 + 1229.08}{300.30} = 22.55$$



The new decision boundary can now be used to classify any unknown item.

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Classification of the Unknown Fruit using the New Decision Boundary

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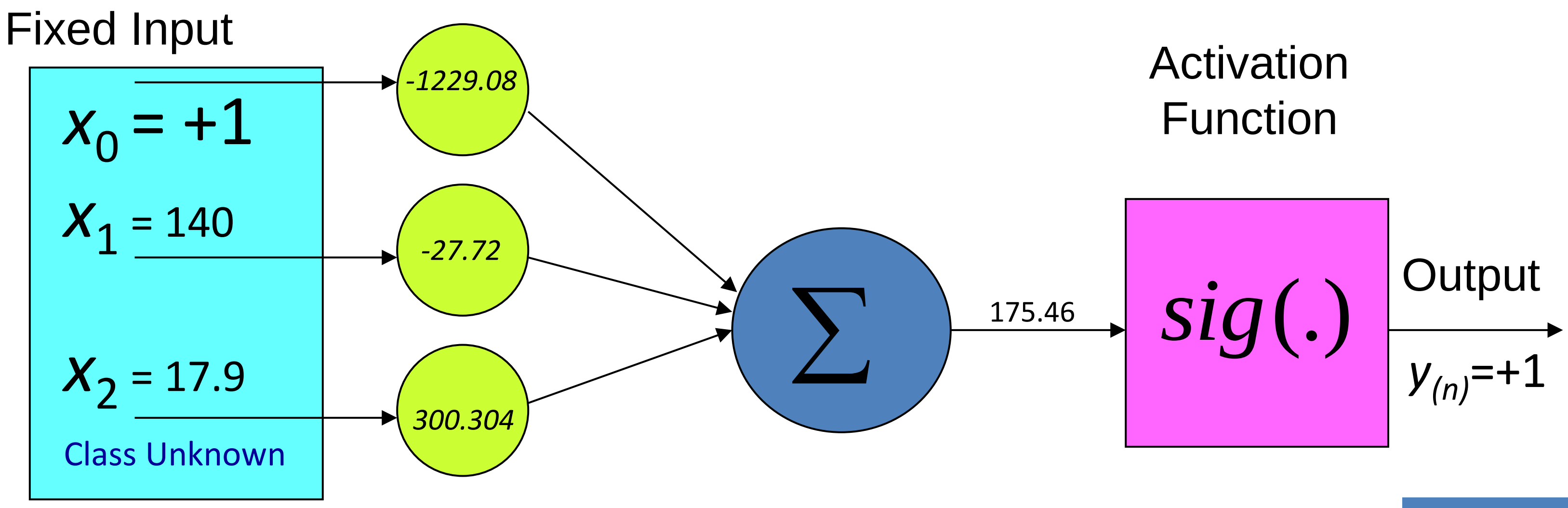
Classification of the Unknown Fruit using the New Decision Boundary



For Class C1, Output = +1

Now use the above model to classify the unknown fruit.

$$\mathbf{x}(\text{unknown}) = [+1,140,17.9]^T$$
$$\mathbf{w}(5) = [-1229.08, -27.72, 300.304]^T$$
$$\text{y}(\text{unkown}) = \text{sgn}(\mathbf{w}^T(5)\mathbf{x}(\text{unknown}))$$
$$= \text{sgn}(-1229.08 \times 1 - 27.72 \times 140 + 300.304 \times 17.9)$$
$$= \text{sgn}(265.56) = +1$$

$\therefore$ this unknown fruit belongs to the class C_1

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Artificial Neural Networks - #1

Classification using Single Layer Perceptron Model

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END of the Book

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Scholastic Video Book Series

Artificial Neural Networks

Part 1

(with English Narrations)

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