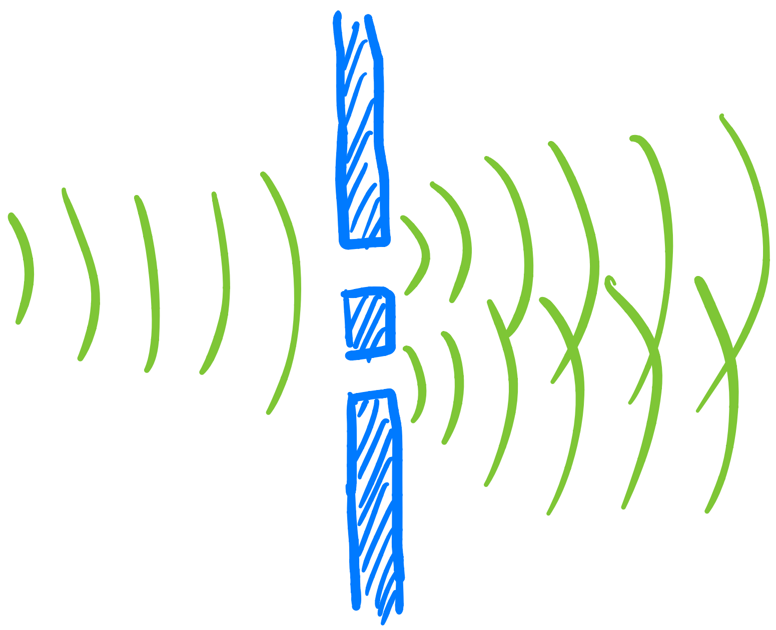
The quantum eraser

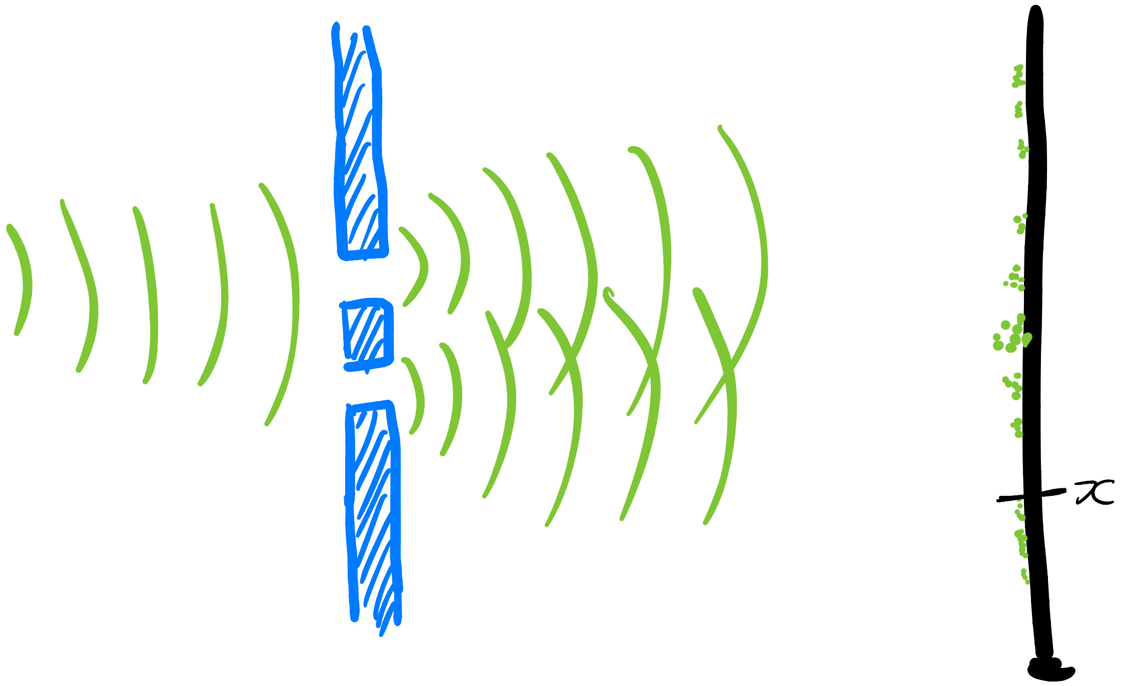
The quantum eraser experiment is an excellent example of "decoherence"; where a quantum state appears to collapse. We are then able to recover the original state by "erasing" the effect.

We start with light that is in a superposition of going left and right through a double slit:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\text{left}\rangle + \frac{1}{\sqrt{2}}|\text{right}\rangle$$



This light will interfere with itself and cause a double slit pattern. We can see why mathematically by considering, what happens when we measure the light in the position basis at the back wall?



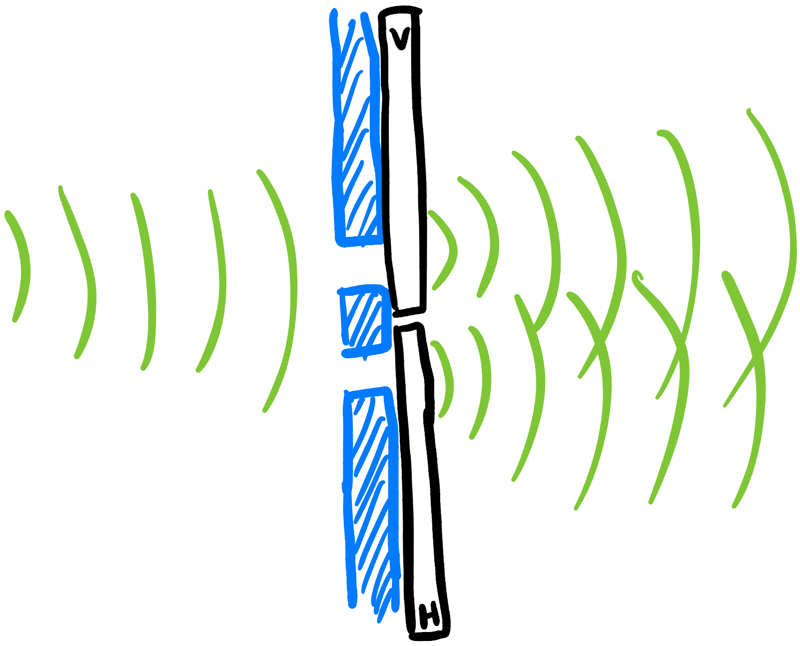
To calculate the probability the light ends up around x, we use the Born Rule.

$$\text{probability light lands near } x = \left(\langle x|\psi\rangle\right)^2$$
$$= \left(\langle x|\left(\tfrac{1}{\sqrt{2}}|\text{left}\rangle + \tfrac{1}{\sqrt{2}}|\text{right}\rangle\right)\right)^2$$
$$= \left(\tfrac{1}{\sqrt{2}}\langle x|\text{left}\rangle + \tfrac{1}{\sqrt{2}}\langle x|\text{right}\rangle\right)^2$$

Suppose $\langle x|\text{left}\rangle$ is positive, but $\langle x|\text{right}\rangle$ is negative. Then these 2 terms (coming from the 2 different slits) would cancel, and this probability would be low. This is destructive interference. On the other hand, if $\langle x|\text{left}\rangle$ and $\langle x|\text{right}\rangle$ are both positive, and this probability is large. This is constructive interference. So the light from the 2 slits will reenforce each other or cancel eachother at different places on the wall. That's what leads to the double slit pattern.

Now we want to change the experiment to get rid of interference.

We will mark which way the light goes with two polarization filters. These filters will mean that only vertical light can pass through the left slit, and only horizontal through the right.



The state of the light that goes through the left slit is:

$$|V\rangle|\text{left}\rangle$$

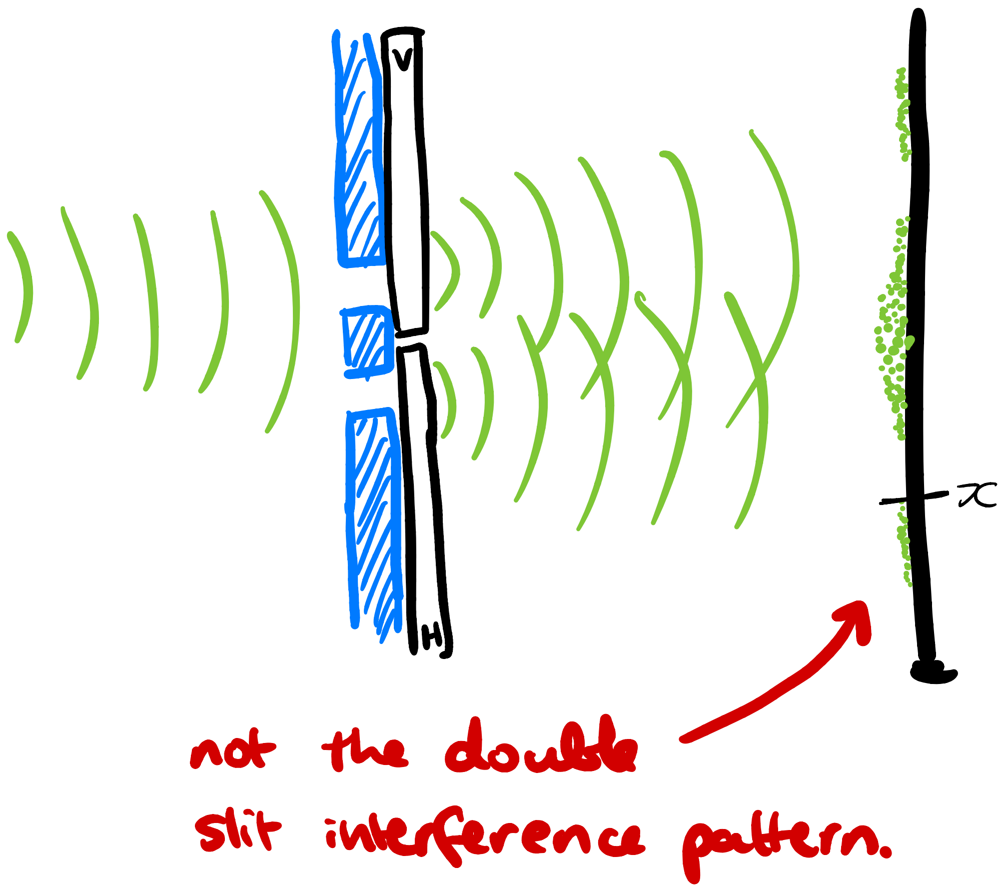
And the state of any light that goes to the right is:

$$|H\rangle|\text{right}\rangle$$

So the combined state is this superposition:

$$|\Psi_{\text{marked}}\rangle = \frac{1}{\sqrt{2}}|V\rangle|\text{left}\rangle + \frac{1}{\sqrt{2}}|H\rangle|\text{right}\rangle$$

This state will <u>not</u> have interference:



Why? Well let's do the same calculation as before and see if there is any interference.

$$\text{probability light lands near } x = |\langle x | \psi_{\text{marked}} \rangle|^2$$

$$= \left|\langle x | \left(\tfrac{1}{\sqrt{2}} |V\rangle |\text{left}\rangle + \tfrac{1}{\sqrt{2}} |H\rangle |\text{right}\rangle \right)\right|^2$$

$$= \left| \tfrac{1}{\sqrt{2}} |V\rangle \langle x | \text{left}\rangle + \tfrac{1}{\sqrt{2}} |H\rangle \langle x | \text{right}\rangle \right|^2$$

To actually calculate this probability we need to take the modulus of this vector, which is not so straightforward. But we don't need to do that.

All we want to do is convince ourselves that interference between $|\text{left}\rangle$ and $|\text{right}\rangle$ can't happen anymore. To do that, let's consider the case we did before.

- If $\langle x|\text{left}\rangle$ is negative but $\langle x|\text{right}\rangle$ is positive.
  - Last time we saw these terms cancel, reducing the probability that the light lands at that x_0
  - Here, no cancelation can happen.

$$|V\rangle \underbrace{\langle x|\text{left}\rangle}_{-} + |H\rangle \underbrace{\langle x|\text{right}\rangle}_{+}$$

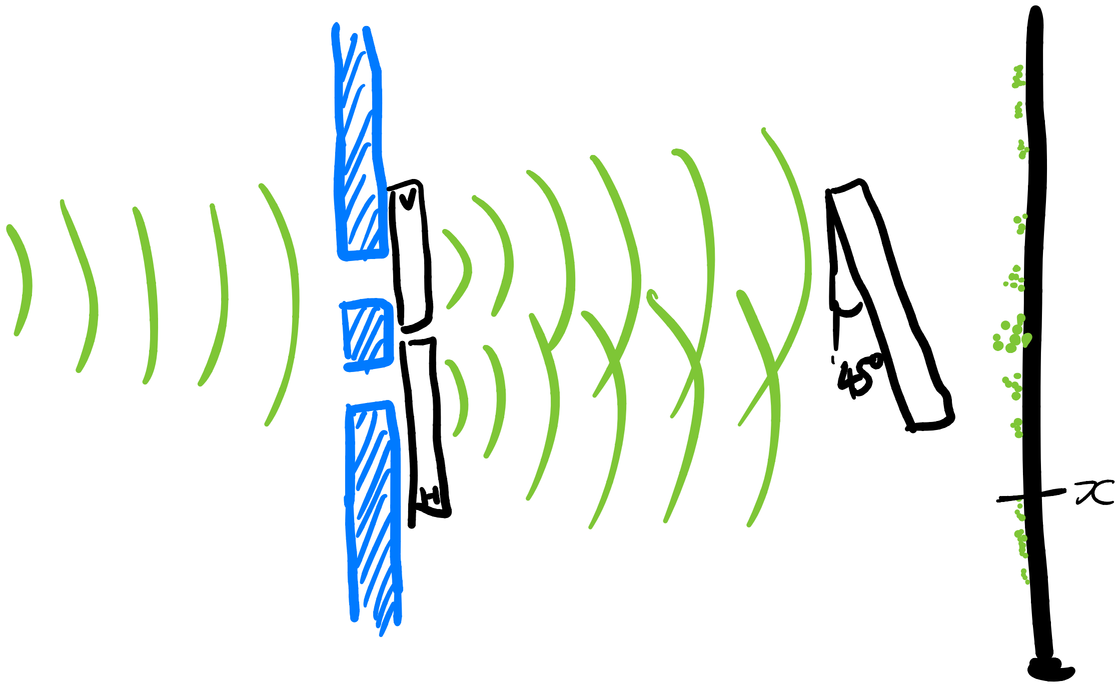
These terms want to cancel, but can't, because of the $|V\rangle$ & $|H\rangle$ in the way. That's because

$$|V\rangle \times (-1) + |H\rangle \times (1)$$

doesn't cancel to zero; $-|V\rangle + |H\rangle \neq 0$.

  - Essentially, these terms are incompatible, so they can't cancel or reenforce.

The quantum eraser experiment:



The quantum eraser involves adding another filter to the set up, which only allows through light oriented in the $|\nearrow\rangle$ direction. Our light however is either $|H\rangle$ or $|V\rangle$. What happens to it? All light that makes it through the filter will be $|\nearrow\rangle$ light. So the filter changes the state like so:

$$|\Psi_{marked}\rangle = \frac{1}{\sqrt{2}} |V\rangle|left\rangle + \frac{1}{\sqrt{2}} |H\rangle|right\rangle$$

$$|\Psi_{erased}\rangle = \frac{1}{\sqrt{2}} |\nearrow\rangle|left\rangle + \frac{1}{\sqrt{2}} |\nearrow\rangle|right\rangle$$

Now there can be interference between the light from the two slits again:

probability light lands near x = $\left(\langle x | \psi_{\text{erased}} \rangle\right)^2$

$$= \left(\langle x | \left(\tfrac{1}{\sqrt{2}} |\nearrow\rangle |\text{left}\rangle + \tfrac{1}{\sqrt{2}} |\nearrow\rangle |\text{right}\rangle\right)\right)^2$$

$$= \left(\tfrac{1}{\sqrt{2}} |\nearrow\rangle \langle x | \text{left}\rangle + \tfrac{1}{\sqrt{2}} |\nearrow\rangle \langle x | \text{right}\rangle\right)^2$$

Suppose these two terms are negative and positive. They can actually cancel now:

$$|\nearrow\rangle \times (-1) + |\nearrow\rangle \times (1)$$

$$= -|\nearrow\rangle + |\nearrow\rangle = 0$$

This is destructive interference again! The probability of light in this spot is low because the light from the left and right were able to cancel each other.

This is why we recover the double slit pattern. The 45° filter "erases" the path information from the polarization. This is what makes it possible for the two bits of light to interfere again.