ROTATIONAL MOTION

Angular Position and Displacement

1. Which of the following are examples of objects in rotational motion? (Select all that apply)
   - A A CD spinning in a CD player (the CD)
   - B The handle on a door which is swinging open (the handle)
   - C A screw being tightened with a screwdriver (the screw)
   - D A person riding on a Ferris wheel (the person)

2. What variable do we use to represent angular position?
   - A x
   - B α
   - C ω
   - D θ

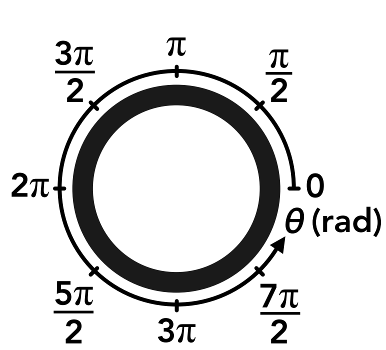
3. What is the SI unit for angular position?
   - A m
   - B deg
   - C rad
   - D rev

4. What is the SI unit for angular displacement?
   - A rev
   - B rad
   - C m
   - D deg

5. A wrench is used to tighten a bolt clockwise 3 revolutions. The angular displacement of the bolt is
   - A positive
   - B negative

6. Which of these are valid axes for rotational motion? (Select all that apply)

A



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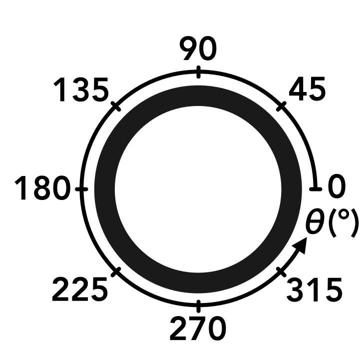

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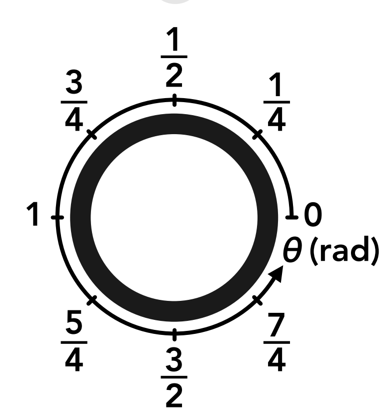

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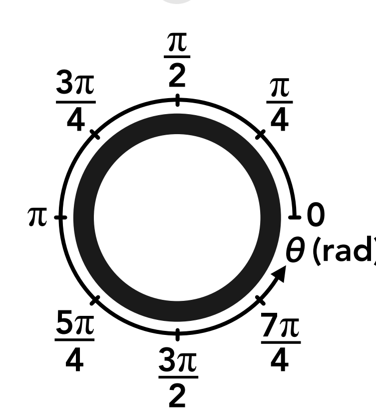


7. If a bike wheel rotates 270° over a short period, how many radians does the wheel rotate?

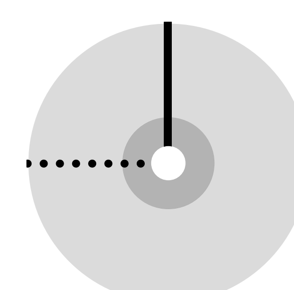
8. If a door starts at an angular position of -15° and rotates clockwise 45°, what is the final angular position?

9. The drum in a washing machine starts at an angular position of $3\pi/2$ rad and rotates counterclockwise 2.5 revolutions. What is the final angular position of the drum in radians?

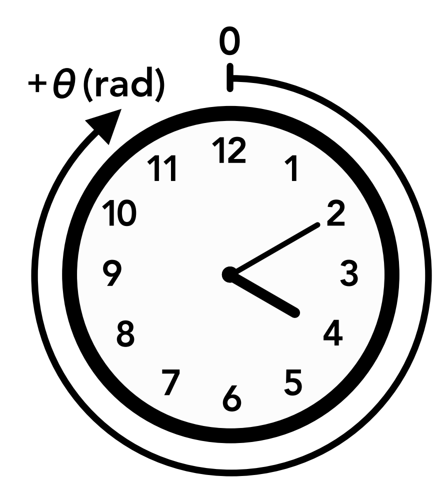
10. An olympic diver jumps and begins flipping through the air. During a short period of time, an observer sees the diver rotate from an angular position of 115° to a position of -80°. What was the angular displacement of the diver based on that point of view?

11. A carousel ride starts at an angular position of 270° and rotates clockwise 5.5π radians. What is the final angular position of the carousel in degrees?

12. The solid line and dotted line on the CD shown on the right form a 90° angle between them. If the solid line on the spinning CD rotates 270°, how many degrees does the dotted line rotate in the same amount of time?



13. If we overlay an axis on a clock as shown on the right, what would be the angular positions of the hour hand at the following times, in radians: 3:00, 6:30, 10:45 ?



Angular Velocity

14. What variable do we use to represent angular velocity?

 A θ
 B v
 C ω
 D α

15. What is the SI unit for angular velocity?

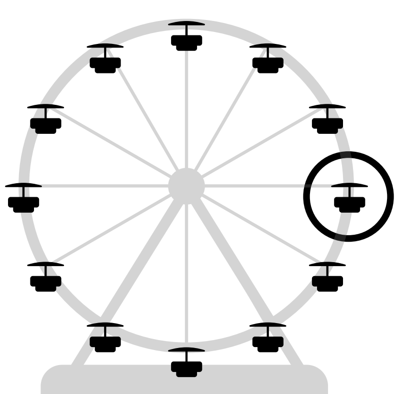
 A m/s
 B rad/s
 C deg/s
 D rpm

16. A football is passed down the field with some spin added to it. From the receiver's perspective, if the ball is spinning counterclockwise, is the angular velocity of the ball positive or negative by convention?

 A Positive
 B Negative

17. Your friend is on a Ferris wheel which is rotating clockwise at a constant 1.5 rpm. When you look up you see your friend is at the top of the Ferris wheel. Where is your friend 30 seconds later?

A



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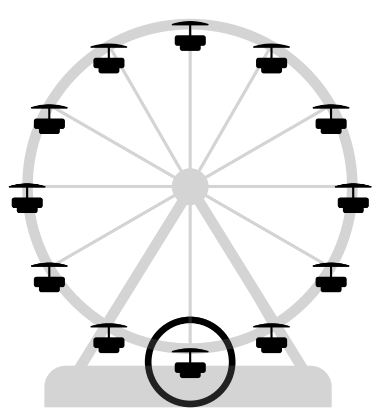

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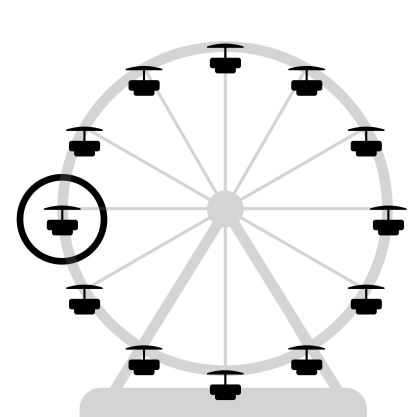

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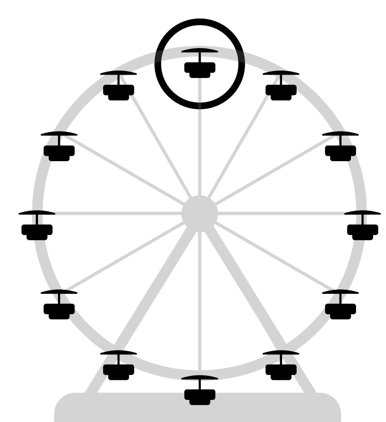
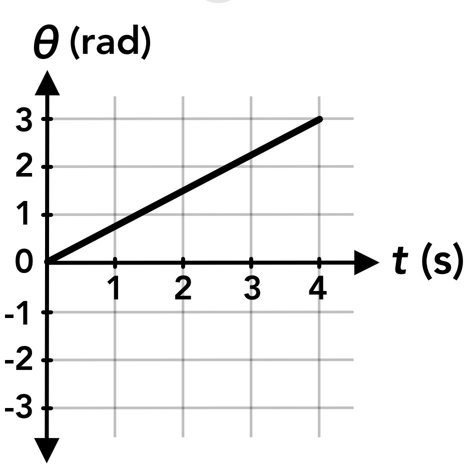
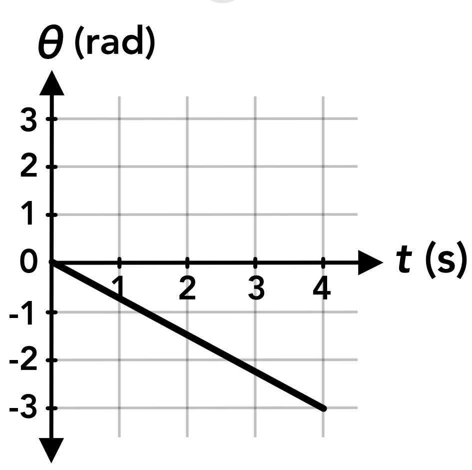


18. A vinyl record is spinning at a constant 45 rpm clockwise. Which of these graphs would represent its motion?

A



B



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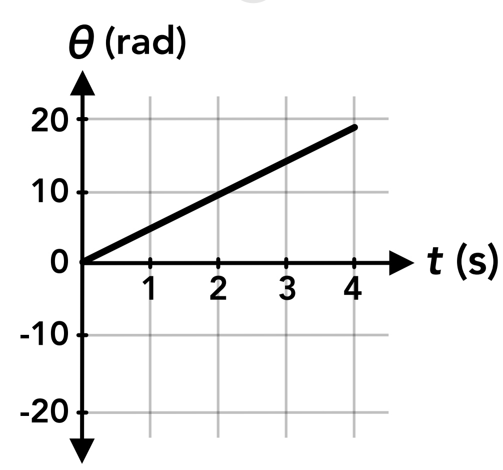

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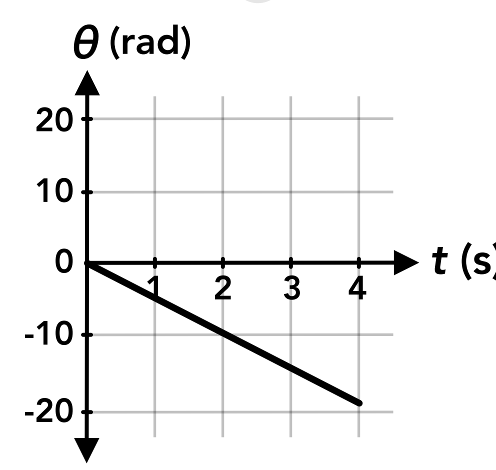


19. A carousel at a local fair rotates at an angular velocity of 18 rad/s. What is the angular velocity in rpm?

20. If the Earth rotates once per day counterclockwise as seen from above the North Pole, what is the angular velocity of the Earth in rad/s?

21. What is the angular speed of the hour, minute, and second hands on a clock (a 12-hour clock) in rpm?

22. If a vinyl record rotates 700° in the clockwise direction in 3.5 s, what is its angular velocity in deg/s?

23. An art student is using a potter's wheel to make a vase. Over a period of 0.5 s, the wheel rotates from an angular position of $-\pi/2$ rad to a position of $3\pi/2$ rad. What is the angular velocity of the wheel in rad/s?

24. If a Ferris wheel is currently at an angular position of $\pi/8$ rad and is rotating at a constant angular velocity of $-\pi/60$ rad/s, how long does it take for the Ferris wheel to reach an angular position of $-4\pi/3$ rad?

25. If a wheel rotates 500π rad in 4 min, what is the average angular velocity of the wheel in deg/s?

26. A vinyl record is spinning at 33 1/3 rpm. How many seconds does it take for the record to rotate 3π rad?

Angular Acceleration

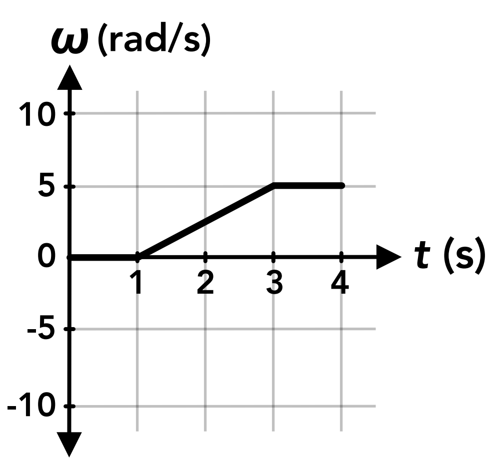
27. What variable do we use to represent angular acceleration?
   - A θ
   - B a
   - C ω
   - D α

28. What is the SI unit for angular acceleration?
   - A m/s^2
   - B rev/s^2
   - C deg/s^2
   - D rad/s^2

29. True or false: if an object is rotating and experiences an angular acceleration, its angular speed must increase.

30. A vinyl record starts from rest and accelerates clockwise at 2.5 rad/s^2 for 2 s, then maintains its speed. Which of these graphs would represent its motion?

A



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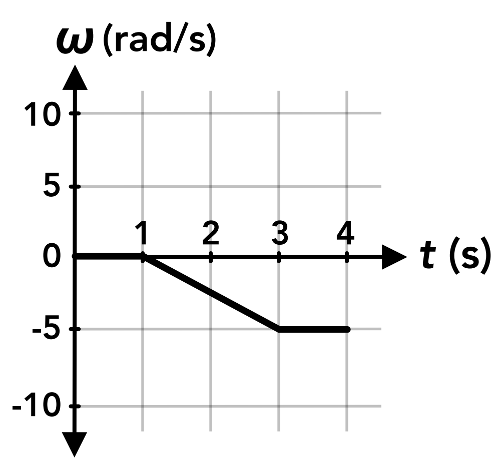

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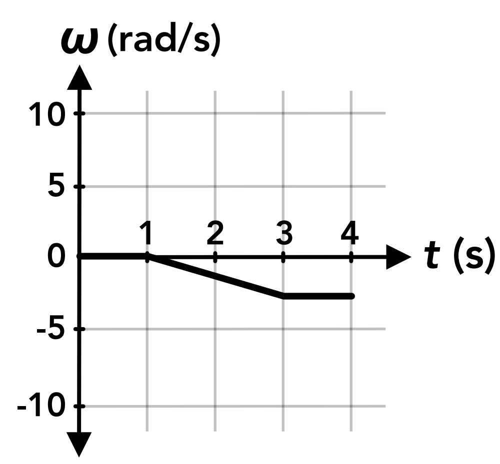

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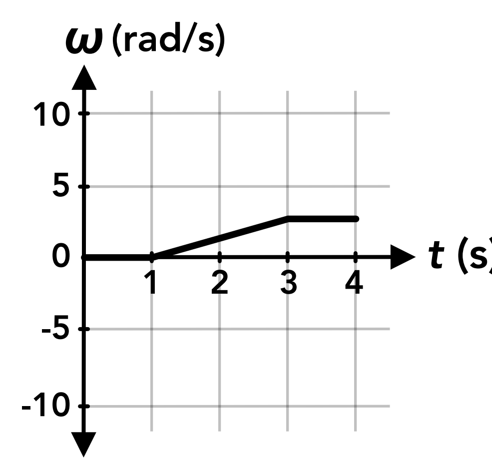
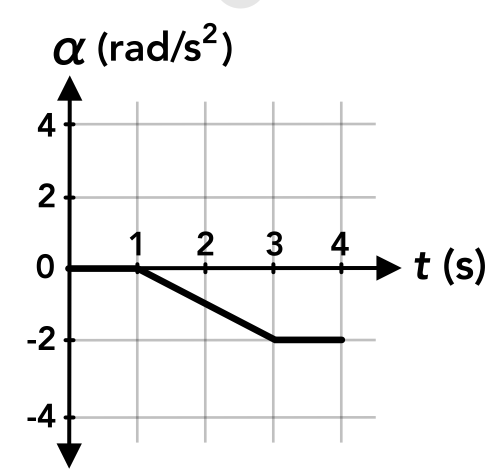


31. A vinyl record starts from rest and then speeds up with a constant clockwise acceleration for 2 s, reaching a final angular velocity of -4 rad/s. Which of these graphs would represent its motion?

A



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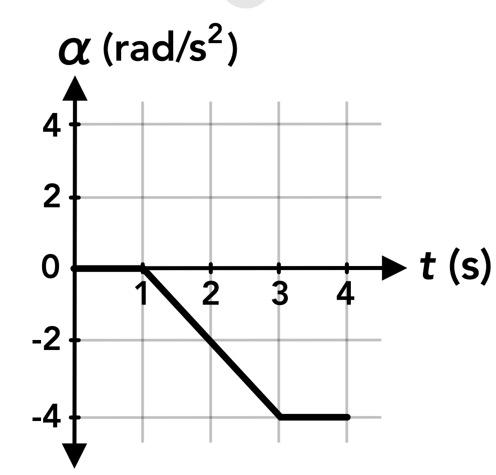

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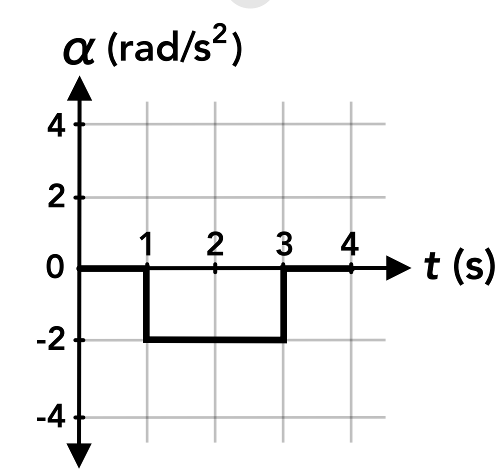

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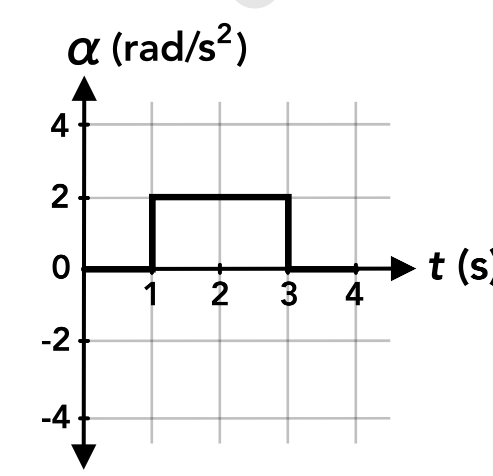

32. When a blender is turned on, the blades inside experience an angular acceleration of 20 rev/s^2. What is the angular acceleration in rad/s^2?

33. A lab centrifuge spinning counterclockwise at 100 rad/s has its speed turned up. If it takes 8 s to reach an angular speed of 500 rad/s, what was the angular acceleration of the centrifuge?

34. A cyclist is stopped on a hill. The moment they release the brakes, they accelerate down the hill and the wheels experience an angular acceleration of 1.5 rad/s^2. After 4 s, what is the angular speed of the wheels in rad/s?

35. You're in the mood to listen to some music. You pick an album, and when you turn on your record player the record (starting from rest) takes 2 s to speed up to an angular velocity of 45 rpm clockwise. What was the angular acceleration of the record during that period in rad/s^2 ?

36. After a few seconds of listening to the record you start to wonder why the singer's voice is a higher pitch than you remember. You suddenly realize the record is supposed to be played at 33 1/3 rpm, so you change the speed of the record. If it takes 0.6 s for the record to change from 45 rpm to 33 1/3 rpm (both clockwise), what is the angular accleration of the record during that time in rad/s^2 ?

37. When the first side of the record is finished playing you hit "Stop". The record that was spinning at 33 1/3 rpm clockwise comes to a stop in 1.5 s. What was the angular acceleration of the record during that time in rad/s^2 ?

38. If a lab centrifuge starts from rest at an angular position of 0 deg and accelerates counterclockwise at 200 deg/s^2, what is the final angular position after 3.5 s, in deg?

39. The operator of a carousel ride that is spinning counterclockwise decides to increase the speed. Over a period of 5 s the ride accelerates at 1 rad/s^2 and rotates 20 rad during that time. What was the initial angular velocity of the ride?

40. The axle connected to the front wheels of a car is spinning at a constant 420 rpm as the car drives forwards. The car then accelerates and the axle experiences a constant angular acceleration. After a period of 4 s the axle has turned 60 revolutions since the car began accelerating. What was the angular acceleration of the axle in rev/s^2 during that time?

41. A vinyl record starts from rest and then accelerates, reaching an angular velocity of -3.5 rad/s while rotating through an angular displacement of -10 rad. What was the angular acceleration of the record?

42. A CD is spinning at 20 rad/s and then the CD accelerates at a constant 8 rad/s^2. From the moment the CD begins to accelerate, how many radians does the CD rotate by the time it reaches a velocity of 50 rad/s?

43. The wheels on a bike are spinning at 6.3 rad/s. When the rider hits the brakes, the wheels experience a constant angular acceleration and rotate 25 rad as the bike and wheels come to a full stop. What was the angular acceleration of the wheels?

44. When a ceiling fan is turned on, it accelerates (from rest) at a constant 3 rad/s^2. How long does it take the fan to rotate 10 revolutions?

45. A ceiling fan is spinning counterclockwise at a constant speed. The fan is turned off and experiences an angular acceleration of -2 rad/s^2. During the time it takes the fan to come to a stop, the fan turns 7 revolutions. What was the initial angular speed of the fan, in rpm?

46. A wheel on a car is spinning at 840 rpm. The car accelerates, and the wheel experiences an angular acceleration for 3 s. If the wheel rotates 50 revolutions during that period of time, what is the angular velocity of the wheel after those 3 s, in rpm?

Answers

1. A, C
2. D
3. C
4. B
5. B
6. B, D
7. $3\pi/2$ rad (4.7 rad)
8. -60°
9. $13\pi/2$ rad (20.4 rad)
10. -195°
11. -720°
12. 270°
13. 3:00 = $\pi/2$ rad
    6:30 = $13\pi/12$ rad
    10:45 = $43\pi/24$ rad
14. C
15. B
16. A
17. C
18. D
19. 171.9 rpm
20. **7.3×10^{-5} rad/s**
21. Hour = 1/720 rpm
    Minute = 1/60 rpm
    Second = 1 rpm
22. -200 deg/s
23. 4π rad/s (12.6 rad/s)
24. 87.5 s
25. 375 deg/s
26. 2.7 s
27. D
28. D
29. False
30. B
31. C
32. 40π rad/s² (125.7 rad/s²)
33. 50 rad/s²
34. 6 rad/s
35. $-3\pi/4$ rad/s² (2.4 rad/s²)
36. 2.0 rad/s²
37. 2.3 rad/s²
38. 1225°
39. 1.5 rad/s
40. 4 rev/s²
41. -0.6 rad/s²
42. 131.3 rad
43. -0.8 rad/s²
44. 6.5 s
45. 126.6 rpm
46. 1160.4 rpm

Answers - Angular Position and Displacement

1. **Answer: A, C**
   The spinning CD and the screw are both rotating about their own center so they are in rotational motion. The door handle is in circular motion because it follows a circular path as the door swings open (but the door is in rotational motion). A person on a Ferris wheel is in circular motion because they follow a circular path.

2. **Answer: D**
   We use the variable θ (the Greek letter "theta") to represent angular position and angles in geometry.

3. **Answer: C**
   The SI unit for angular position is radians (rad). Meters (m) is the SI unit for linear or tangential position, degrees (deg) is a valid unit for angular position but is not the SI unit. Revolutions (rev) is also a valid unit for angular position but is not the SI unit.

4. **Answer: B**
   The SI unit for angular displacement is radians (rad), the same as for angular position. Revolutions (rev) is a valid unit for angular displacement but is not the SI unit. Meters (m) is the SI unit for linear or tangential position. Degrees (deg) is a valid unit for angular displacement but is not the SI unit.

5. **Answer: B**
   By convention, clockwise is the negative direction.

6. **Answer: B, D**
   Axes for rotational motion can use the unit of radians (rad) which is the SI unit, or degrees (deg). If the axis uses radians then one revolution must be equal to 2π radians. If the axis uses degrees then one revolution must be equal to 360°.

7. **Answer: $3\pi/2$ rad or 4.7 rad**
   We can convert from degrees to radians using the relationship of 2π rad = 360°:

$$\frac{270°}{} \times \frac{2\pi \text{ rad}}{360°} = 3\pi/2 \text{ rad}$$

8. **Answer: -60°**
The final angular position can be found using the equation for angular displacement:
$\Delta\theta = \theta_f - \theta_i$ (-45°) $= \theta_f -$ (-15°) $\theta_f =$ -60°

9. **Answer: 13π/2 rad or 20.4 rad**
First we can convert the angular displacement from rev to rad:
$$\frac{2.5 \text{ rev}}{} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 5\pi \text{ rad}$$
Then we can find the final angular position using the equation for angular displacement:
$\Delta\theta = \theta_f - \theta_i$ $(5\pi \text{ rad}) = \theta_f - (3\pi/2 \text{ rad})$ $\theta_f = 13\pi/2 \text{ rad}$

10. **Answer: -195°**
The angular displacement is the final angular position minus the initial angular position:
$\Delta\theta = \theta_f - \theta_i = (-80°) - (115°) = -195°$

11. **Answer: -720°**
First we can convert the angular displacement from rad to deg:
$$\frac{-5.5\pi \text{ rad}}{} \times \frac{360°}{2\pi \text{ rad}} = -990°$$
Then we can find the final angular position using the equation for angular displacement:
$\Delta\theta = \theta_f - \theta_i$ $(-990°) = \theta_f - (270°)$ $\theta_f = -720°$

12. **Answer: 270°**
All points or lines on the same, solid rotating object will rotate together the same amount.

13. **Answer: 3:00 = π/2 rad, 6:30 = 13π/12 rad, 10:45 = 43π/24 rad**
We can multiply the fraction of a revolution for each time by 2π rad to get the angular position in rad. The hour hand rotates 1/12 of a revolution for each hour and 1/720 of a revolution for each minute.

3:00 $$\frac{3}{12} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \pi/2 \text{ rad}$$

6:30 $$\left(\frac{6}{12} + \frac{30}{720}\right) \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{780\pi \text{ rad}}{720} = 13\pi/12 \text{ rad}$$

10:45 $$\left(\frac{10}{12} + \frac{45}{720}\right) \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{1290\pi \text{ rad}}{720} = 43\pi/24 \text{ rad}$$

Answers - Angular Velocity

14. **Answer: C**
We use the variable ***ω*** (the Greek letter "omega") to represent angular velocity.

15. **Answer: B**
The SI unit for angular velocity is radians per second (rad/s). Meters per second (m/s) is the SI unit for linear or tangential velocity. Degrees per second (deg/s) is a valid unit for angular velocity but is not the SI unit. Revolutions per minute (rpm) is a valid unit for angular velocity but is not the SI unit.

16. **Answer: A**
By convention, counterclockwise is the positive direction.

17. **Answer: C**

First we can convert the angular velocity from rpm to rev/s:

$$\frac{1.5\text{ rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 0.025\text{ rev/s clockwise}$$

Then we can find the angular displacement in rev during the 30 second period:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (0.025\text{ rev/s}) = \frac{\Delta\theta}{(30\text{ s})} \qquad \Delta\theta = 0.75\text{ rev clockwise}$$

The friend started at the top of the Ferris wheel and it rotated 0.75 or 3/4 of a revolution clockwise.

18. **Answer: D**

These are all graphs of the angular positive vs time. The slope of the angular position vs time graph is equal to the angular velocity. A slope of -45 rpm (clockwise is negative) is equal to -4.7 rad/s.

$$\frac{45\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = -4.7\text{ rad/s}$$

After 4 seconds the angular position would be -18.8 rad, using the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (-4.7\text{ rad/s}) = \frac{\Delta\theta}{(4\text{ s})} \qquad \Delta\theta = -18.8\text{ rad}$$

19. **Answer: 171.9 rpm**

We need to convert from rad/s to rev/min (rpm) using the relationships between rad, rev, s and min:

$$\frac{18\text{ rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 171.9\text{ rev/min (rpm)}$$

20. **Answer: 7.3×10^{-5} rad/s**

We need to convert from rev/day to rad/s and use scientific notation since the answer is small:

$$\frac{1\text{ rev}}{\text{day}} \times \frac{1\text{ day}}{24\text{ h}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 7.3 \times 10^{-5}\text{ rad/s}$$

21. **Answer: Hour hand = 1/720 rpm, minute hand = 1/60 rpm, second hand = 1 rpm**

We can start with the angular speed of each hand based on how long it takes to complete 1 revolution:

Hour hand: $\frac{1\text{ rev}}{12\text{ h}} \times \frac{1\text{ h}}{60\text{ min}} = 1/720\text{ rev/min (rpm)}$

Minute hand: $\frac{1\text{ rev}}{1\text{ h}} \times \frac{1\text{ h}}{60\text{ min}} = 1/60\text{ rev/min (rpm)}$

Second hand: $\frac{1\text{ rev}}{1\text{ min}} = 1\text{ rev/min (rpm)}$

22. **Answer: -200 deg/s**

We can use the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(-700°)}{(3.5\text{ s})} = -200\text{ deg/s}$$

23. **Answer: 4π rad/s or 12.6 rad/s**

We can use the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t} = \frac{(3\pi/2\text{ rad}) - (-\pi/2\text{ rad})}{(0.5\text{ s})} = 4\pi\text{ rad/s}$$

24. **Answer: 87.5 s**
We can find the period of time using the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (-\pi/60 \text{ rad/s}) = \frac{(-4\pi/3 \text{ rad}) - (\pi/8 \text{ rad})}{\Delta t} \qquad \Delta t = 87.5 \text{ s}$$

25. **Answer: 375 deg/s**
First we can find the average angular velocity in rad/min:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(500\pi \text{ rad})}{(4 \text{ min})} = 125\pi \text{ rad/min}$$

Then we can convert from rad/min to deg/s:

$$\frac{125\pi \text{ rad}}{\text{min}} \times \frac{360°}{2\pi \text{ rad}} \times \frac{1 \text{ min}}{60 \text{ s}} = 375 \text{ deg/s}$$

26. **Answer: 2.7 s**
First we can convert the angular velocity from rpm to rad/s:

$$\frac{33\ 1/3 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 10\pi/9 \text{ rad/s}$$

Then we can find the period of time using the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad (10\pi/9 \text{ rad/s}) = \frac{(3\pi \text{ rad})}{\Delta t} \qquad \Delta t = 2.7 \text{ s}$$

Answers - Angular Acceleration

27. **Answer: D**
We use the variable α (the Greek letter "alpha") to represent angular acceleration.

28. **Answer: D**
The SI unit for angular acceleration is radians per second squared (rad/s²). Meters per second squared (m/s²) is the SI unit for linear or tangential acceleration. Revolutions per minute squared (rev/min²) is a valid unit for angular acceleration but is not the SI unit. Degrees per second squared (deg/s²) is a valid unit for angular acceleration but is not the SI unit.

29. **Answer: False**
Angular acceleration is the change in angular speed, but the speed may increase or decrease.

30. **Answer: B**
These are all graphs of the angular velocity vs time. The slope of the angular velocity vs time graph is equal to the angular acceleration. The slope should be -2.5 rad/s² because clockwise is the negative direction. After 2 seconds (3 s - 1 s) the angular velocity would be -5 rad/s, using the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad (-2.5 \text{ rad/s}^2) = \frac{\Delta\omega}{(2 \text{ s})} \qquad \Delta\omega = -5 \text{ rad/s}$$

31. **Answer: C**
These are all graphs of the angular acceleration vs time. The acceleration should be constant (a flat line) for 2 s and negative because clockwise is the negative direction by convention.

32. **Answer: 40π rad/s² or 125.7 rad/s²**
We can convert from rev/s² to rad/s²:

$$\frac{20\text{ rev}}{\text{s}^2} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 40\pi\text{ rad/s}^2$$

33. **Answer: 50 rad/s²**
We can use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(500\text{ rad/s}) - (100\text{ rad/s})}{(8\text{ s})} = 50\text{ rad/s}^2$$

34. **Answer: 6 rad/s**
We can find the final angular velocity using the equation for angular acceleration. The initial angular velocity is 0.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} \qquad (1.5\text{ rad/s}^2) = \frac{\omega_f - (0\text{ rad/s})}{(4\text{ s})} \qquad \omega_f = 6\text{ rad/s}$$

35. **Answer: -3π/4 rad/s² or -2.4 rad/s²**
First we can convert the final angular velocity from rpm to rad/s:

$$\frac{-45\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = -3\pi/2\text{ rad/s}$$

Then we can find the angular acceleration. The initial angular velocity is 0 (the record starts from rest).

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(-3\pi/2\text{ rad/s}) - (0\text{ rad/s})}{(2\text{ s})} = -3\pi/4\text{ rad/s}^2$$

36. **Answer: 2.0 rad/s²**
First we can convert the initial and final angular velocities from rpm to rad/s:

$$\frac{-45\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = -3\pi/2\text{ rad/s}$$

$$\frac{-33\ 1/3\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = -10\pi/9\text{ rad/s}$$

Then we can find the angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(-10\pi/9\text{ rad/s}) - (-3\pi/2\text{ rad/s})}{(0.6\text{ s})} = 2.0\text{ rad/s}^2$$

37. **Answer: 2.3 rad/s²**
First we can convert the initial angular velocity from rpm to rad/s:

$$\frac{-33\ 1/3\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = -10\pi/9\text{ rad/s}$$

Then we can find the angular acceleration. The final angular velocity is 0 (the record comes to a stop).

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(0\text{ rad/s}) - (-10\pi/9\text{ rad/s})}{(1.5\text{ s})} = 2.3\text{ rad/s}^2$$

38. **Answer: 1225°**
We can find the final angular position using the kinematic equation below, using the unit of degrees:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (0\text{ deg}) + (0\text{ deg/s})(3.5\text{ s}) + \frac{1}{2}(200\text{ deg/s}^2)(3.5\text{ s})^2 = 1{,}225\text{ deg}$$

39. **Answer: 1.5 rad/s**
We can find the initial angular velocity using the kinematic equation below, using 0 for the initial angular position:
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$ $\quad$ $(20\text{ rad}) = (0\text{ rad}) + \omega_i(5\text{ s}) + \frac{1}{2}(1\text{ rad/s}^2)(5\text{ s})^2$ $\quad$ $\omega_i = 1.5\text{ rad/s}$

40. **Answer: 4 rev/s²**
First we can convert the initial angular velocity from rpm to rev/s:
$$\frac{420\text{ rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 7\text{ rev/s}$$
Then we can find the angular acceleration, using the unit of revolutions and using 0 for the initial position:
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$ $\quad$ $(60\text{ rev}) = (0\text{ rev}) + (7\text{ rev/s})(4\text{ s}) + \frac{1}{2}\alpha(4\text{ s})^2$ $\quad$ $\alpha = 4\text{ rev/s}^2$

41. **Answer: -0.6 rad/s²**
We can find the angular acceleration using this kinematic equation, using 0 for the initial angular velocity:
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$ $\quad$ $(-3.5\text{ rad/s})^2 = (0\text{ rad/s})^2 + 2\alpha(-10\text{ rad})$ $\quad$ $\alpha = -0.6\text{ rad/s}^2$

42. **Answer: 131.3 rad**
We can find the angular displacement using this kinematic equation:
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$ $\quad$ $(50\text{ rad/s})^2 = (20\text{ rad/s})^2 + 2(8\text{ rad/s}^2)\Delta\theta$ $\quad$ $\Delta\theta = 131.3\text{ rad}$

43. **Answer: -0.8 rad/s²**
We can find the angular acceleration using this equation, using 0 for the initial position and final velocity:
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$ $\quad$ $(0\text{ rad/s})^2 = (6.3\text{ rad/s})^2 + 2\alpha(25\text{ rad} - 0\text{ rad})$ $\quad$ $\alpha = -0.8\text{ rad/s}^2$

44. **Answer: 6.5 s**
First we can convert the angular displacement from rev to rad:
$$\frac{10\text{ rev}}{} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 20\pi\text{ rad}$$
Then we can find the period of time using this kinematic equation, using 0 for the initial position and velocity:
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$ $\quad$ $(20\pi\text{ rad}) = (0\text{ rad}) + (0\text{ rad/s})t + \frac{1}{2}(3\text{ rad/s}^2)t^2$ $\quad$ $t = 6.5\text{ s}$

45. **Answer: 126.6 rpm**
First we can convert the angular displacement from rev to rad:
$$\frac{7\text{ rev}}{} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 14\pi\text{ rad}$$
Then we can find the initial angular velocity using this equation, using 0 for final angular velocity:
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$ $\quad$ $(0\text{ rad/s})^2 = \omega_i^2 + 2(-2\text{ rad/s}^2)(14\pi\text{ rad})$ $\quad$ $\omega_i = 13.26\text{ rad/s}$
Then we can convert the initial angular velocity from rad/s to rpm:
$$\frac{13.26\text{ rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 126.6\text{ rev/min (rpm)}$$

46. **Answer: 1160.4 rpm**

First we can convert the initial angular velocity from rpm to rev/s:

$$\frac{840\text{ rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 14\text{ rev/s}$$

Then we can find the angular acceleration, using the unit of revolutions and 0 for the initial position:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \qquad (50\text{ rev}) = (0\text{ rev}) + (14\text{ rev/s})(3\text{ s}) + \frac{1}{2}\alpha(3\text{ s})^2 \qquad \alpha = 1.78\text{ rev/s}^2$$

Then we can find the final angular velocity, using the unit of revolutions:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} \qquad (1.78\text{ rev/s}^2) = \frac{\omega_f - (14\text{ rev/s})}{(3\text{ s})} \qquad \omega_f = 19.34\text{ rev/s}$$

Then we can convert the final angular velocity from rev/s to rpm:

$$\frac{19.34\text{ rev}}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} = 1160.4\text{ rev/min (rpm)}$$