Topic: Initial value problems

Question: Solve the initial value problem.

$$\frac{dy}{dx} = 2x + 3$$

$y = 5$ when $x = 0$

Answer choices:

A $\quad y = x^2 + 3x + 5$

B $\quad y = 5$

C $\quad y = 4x^2 + 3x + 5$

D $\quad y = x^2 + 3x - 40$

Solution: A

In order to find y, we multiply both sides of the equation by dx and then integrate both sides.

$$dy = (2x + 3)\ dx$$

$$\int dy = \int 2x + 3\ dx$$

$$y = x^2 + 3x + C$$

Now, in order to find the specific equation that passes through $y = 5$ when $x = 0$, we substitute these values into the general equation we found and solve for C.

$$5 = 0^2 + 3(0) + C$$

$$5 = C$$

Therefore, the specific equation we are looking for it

$$y = x^2 + 3x + 5$$

Topic: Initial value problems

Question: Solve the initial value problem.

$$f''(x) = \cos x$$

$f'(0) = 1$ and $f(0) = 3$

Answer choices:

A $f(x) = \sin x + 1$

B $f(x) = -\cos x + x + 4$

C $f(x) = -\sin x + 1$

D $f(x) = \cos x + x + 2$

Solution: B

Before we can find the equation for $f(x)$, we must first find the equation for $f'(x)$, which we do by integrating $f''(x)$.

$$f'(x) = \int \cos x \ dx$$

$$f'(x) = \sin x + C$$

Now we find the specific equation for $f'(x)$ by solving for C with the initial condition given.

$$f'(0) = \sin 0 + C = 1$$

$$C = 1$$

$$f'(x) = \sin x + 1$$

In order to find $f(x)$, we integrate $f'(x)$ and find C by using the initial condition for $f(x)$.

$$f(x) = \int (\sin x + 1) \ dx$$

$$f(x) = -\cos x + x + C$$

$$f(0) = -\cos 0 + 0 + C = 3$$

$$-1 + C = 3$$

$$C = 4$$

Therefore,

$$f(x) = -\cos x + x + 4$$