Polynomial Functions & Equations

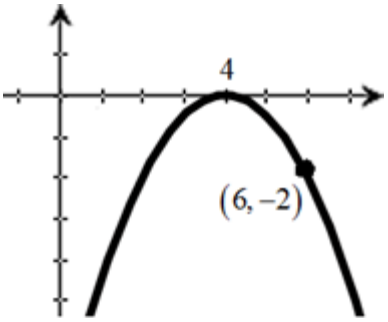
syllabus content: polynomial functions & their graphs, factor & remainder theorems, fundamental theorem of algebra, solving polynomial equations & inequalities both graphically & algebraically, sum & product of the roots of a polynomial equation

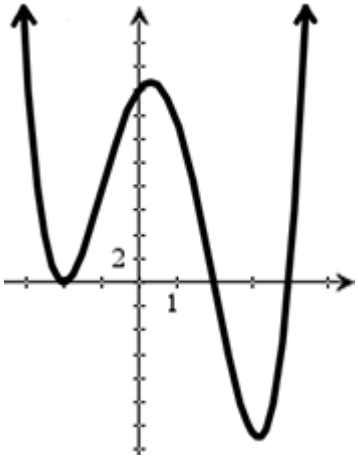
[answers on last page]

Exercises

Part I – questions 1-11, **no** calculator allowed

1. Consider a parabola (shown in diagram) that has a vertex at $(4, 0)$ and passes through the point $(6, -2)$. Find the equation of this parabola and write it in the form $y = ax^2 + bx + c$.





2. The graph of a quartic function (shown in diagram) has exactly three x-intercepts at $x = -2$, $x = 2$ and $x = 4$, and has a y-intercept at $y = 16$. Express the equation for this function in the form $y = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.

3. Given that $x = 3$ and $x = -1$ are zeros of the polynomial function $g(x) = 3x^3 - 8x^2 - 5x + 6$ find the remaining zero of g.

4. Given that $x^3 + Ax^2 + x + B$ is exactly divisible by both $x - 1$ and $x + 3$, find the value of A and the value of B.

5. Find a polynomial with integer coefficients of lowest degree having zeros of $x = -\frac{1}{2}$ and $x = 2 + \mathrm{i}$.

6. $x - 1$ is a factor of the polynomial $P(x) = x^3 + mx^2 + nx - 4$.

(a) Show that $m + n = 3$.

(b) Show that P can be written in factorised form as $P(x) = (x-1)\left[x^2 + (p+1)x + 4\right]$.

7. The polynomial $ax^3 + 4x^2 + cx - 36$ is divisible by $x^2 - 9$. Find the value of a and the value of c.

8. When the function $g(x) = x^3 + bx^2 - 5x + 2$ is divided by $(x-1)$ it leaves the same remainder as when it is divided by $(x+2)$. Find the value of b.

Polynomial Functions & Equations

Part I continued - **no** calculator allowed

9. If α and β are the roots of the quadratic equation $x^2+3x-5=0$, find a quadratic equation with integer coefficients that has roots of:

(a) $\alpha+2,\ \beta+2$ (b) $\frac{1}{\alpha},\ \frac{1}{\beta}$

10. If α, β and γ are the three roots of the cubic equation $x^3-2x^2-4x+5=0$, find the values of:

(a) $\alpha\beta\gamma$ (b) $\alpha\beta+\alpha\gamma+\beta\gamma$ (c) $(\alpha+1)(\beta+1)(\gamma+1)$

11. If α and β are the roots of $x^2-px+3=0$ and $\alpha-\beta=3$ find the possible value(s) of p.

Part II – questions 12-15, calculator allowed

12. Consider the function $f(x)=\frac{1}{2}x^4+x^3-9x^2+8x+58$.

(a) Determine the domain and range of f.

(b) Find all real zeros of f.

13. Given that the graph of the function $h(x)=3x^3+bx^2+cx+20$ is tangent to the x-axis at $x=2$, find the value of b and the value of c.

14. Solve the inequality $2x^3-9x^2+8x+9<x+3$.

15. A line with a gradient of 2 intersects the parabola $y=3x^2-2x-6$ where $x=-1$. Find the exact coordinates of the point where the line intersects that parabola a second time.

Polynomial Functions & Equations

ANSWERS

1. $y=-\frac{1}{2}x^2+4x-8$

2. $y=\frac{1}{2}x^4-x^3-6x^2+4x+16$

3. $x=\frac{2}{3}$

4. $A=4,\ B=-6$

5. $2x^3-7x^2+6x+5$

7. $a=-2,\ c=18$

8. $b=2$

9. (a) $x^2-x-7=0$ (b) $5x^2-3x-1=0$

10. (a) -5 (b) -4 (c) -6

11. $p=\pm\sqrt{21}$

12. (a) domain: $x\in\mathbb{R}$, range: $y\geq -54$ (b) $x\approx -5.22,\ x\approx -2.15$

13. $b=-7,\ c=-8$

14. $x<-\frac{1}{2},\ 2<x<3$

15. $\left(\frac{7}{3},\frac{17}{3}\right)$