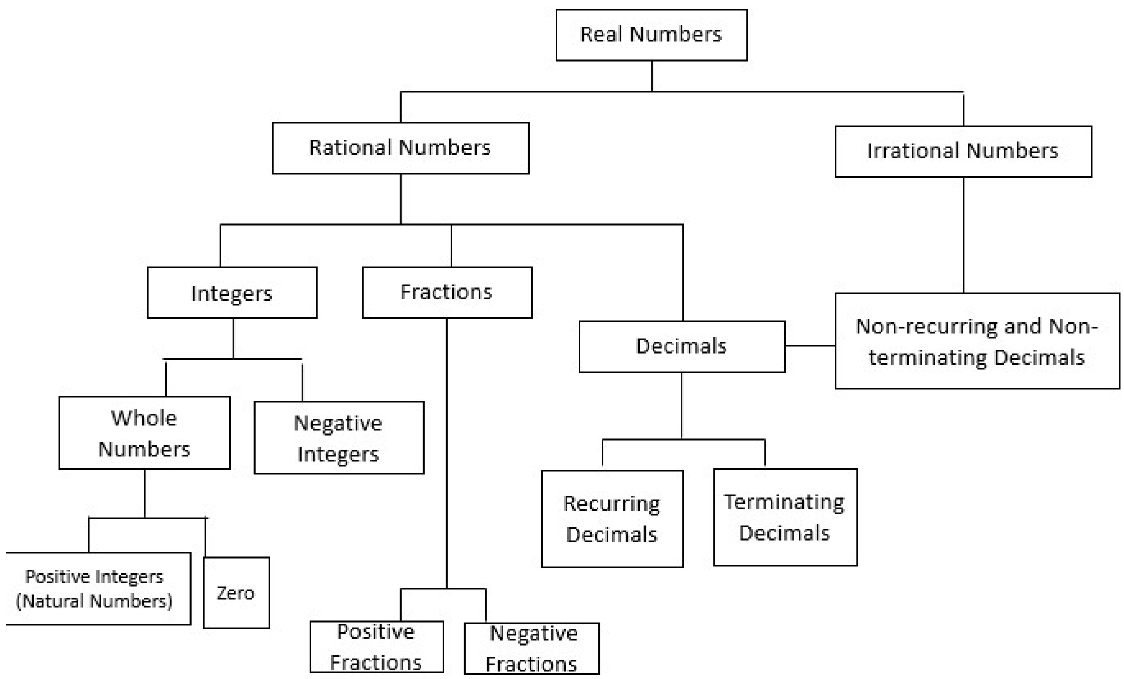
Topic 2: Integers, Rational Numbers and Real Numbers

Notes:

Number System:

1.



2. **Real numbers** consist of **both rational and irrational numbers**.

3. **Irrational numbers** are numbers that cannot be expressed as exact fractions.

   For an example: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, etc…

4. **Rational numbers** are numbers that can be expressed in an exact fraction, where the numerator and denominator are integers, and the denominator is not 0. Rational numbers include all integers and fractions.

   For an example: -1, -0.5, 0, 1, 2, 2.5, 4.973, etc…

5. The set of **integers** consists of …, -3, -2, -1, 0, 1, 2, …

6. The set of **whole numbers** consists of 0, 1, 2, 3, 4, 5, …

7. The set of **natural numbers** consists of 1, 2, 3, 4, 5, 6, …

8. The set of **negative integers** consists of …, -3, -2, and -1.

Ordering of Positive Rational Numbers:

9. To compare two positive rational numbers, express them in fractions and check if the following conditions are met or not.

Both fractions are of	Conclusion	Example
same denominator	Fraction with the larger numerator is the larger fraction	$\frac{7}{8} > \frac{3}{8}$
same numerator	Fraction with the smaller numerator is the larger fraction	$\frac{5}{7} > \frac{5}{12}$
different denominator and numerator	Convert to like fractions with the same denominator	$\frac{2}{5} = \frac{14}{35}$ and $\frac{3}{7} = \frac{15}{35}$ Thus, $\frac{3}{7} > \frac{2}{5}$

Mathematical Symbols for Comparison:

10.

Symbol	Meaning	Example
=	is equal to	$0.1 = \frac{1}{10}$
≠	is not equal to	$0.11 \neq \frac{1}{10}$
>	is greater than	$0.1 > 0.01$
<	is less than	$0.005 < 0.05$
≥	is greater or equal than	$a \geq 6$
≤	is lower or equal than	$b \leq 5$

Decimal Numbers:

11. There are three types of decimal numbers, and they are listed accordingly in the table below.

Type of Decimal	Meaning	Example(s)
Terminating Decimals	A decimal number that has **finite** decimal digits. (A definite / fixed number of decimals)	1.23, 1.425, 5.7354
Recurring (or easier; Repeating) Decimals	A decimal number that has several decimal digits that **repeat infinitely** (forever)	$0.\overline{6}$, $7.\overline{12}$, $0.\overline{375}$ (You are recommended to use dots instead of bar notation)

Non-Terminating and Non-Recurring Decimals	A decimal number that has decimals which **do not repeat in any particular order / pattern**.	$\sqrt{2}$ = 1.414 213 562… π = 3.141 592 653 589 79…

12. To obtain a recurring decimal from a fraction, use long division or evaluate using a calculator. The table below lists examples of fractions computed and represented as recurring decimals (bar notation.)

Fractions		**Recurring Decimals**
$\frac{2}{3}$	=	$0.\overline{6}$
$\frac{5}{12}$	=	$0.\overline{416}$
$\frac{13}{6}$	=	$2.\overline{16}$
$\frac{5}{99}$	=	$0.\overline{05}$
$\frac{4}{33}$	=	$0.\overline{12}$
$\frac{3}{7}$	=	$0.\overline{428571}$

13. To express a recurring decimal into a fraction, use a calculator or do the following steps:

Example: $0.\overline{126}$

Step 1: Take the strip of pattern and put it on the numerator of a fraction.

Step 2: The denominator of the fraction should be “9” but should have the corresponding number of digits as the strip of pattern.

Step 3: Simplify the fraction.

$0.\overline{126} = \frac{126}{999}$ (since 126 is 3 digits, 999 should be the denominator) = $\frac{14}{111}$

$0.\overline{126} = \frac{14}{111}$

Representation on a Number Line:

14. A number line is a straight line with at least one **arrowhead** pointing to the right indicating that the numbers continue in the same way / pattern **indefinitely**.

15. Regular number intervals are represented by markings of equal spacing and the value of the numbers is written below the line.

16. Coloured dots are used to represent the numbers on the number line.

17. The number on the right of the number line always must be greater than the number on the left beside and more.

Addition and Subtraction of Integers:

18.

No.	Case	Example	Explanation	Multiplication of Signs
1	Adding a negative integer.	+ (-3) = -3	Overall value is negative.	(+) x (-) = (-)
2	Subtracting a positive integer.	– (5) = -5	Overall value is negative.	(-) x (+) = (-)
3	Subtracting a negative integer.	– (-4) = 4	Overall value is positive.	(-) x (-) = (+)
4	Adding 2 positive integers.	3 + 5 = 8	Just take the sum of the digits; overall value is positive.	
5	Adding 2 negative integers.	-3 + (-4) = - (3+4) = -7	Just take the sum of the digits; overall value is negative.	
6	Adding a positive and a negative integer.	-6 + 7 = 7 – 6 = 1 OR 6 + (-7) = 6 – 7	Take the difference between the values of the digits and the result will follow the sign of the digit	… … … OR (+) x (-) = (-)

		= -1	which is numerically greater.	
7	Subtracting a positive integer from a positive integer.	8 – 3 = 5 OR 3 – 8 = -5	Take the difference between the two positive numbers, - if the first positive number is larger (in value), the result is positive. - If not, the result is negative.	
8	Subtracting a negative integer from a positive integer.	8 – (-3) = 8 + 3 = 11	Follow Case 3; the result is the same as adding a positive integer to the first positive integer.	(-) x (-) = (+)
9	Subtracting a negative integer from a	-2 – (-3) = -2 + 3 = 1	Follow Case 3; the result is the same as above.	(-) x (-) = (+)

	negative integer.	OR -4 – (-3) = -4 + 3 = -1		

Multiplication and Division of Numbers:

19. (Multiplication)

- (+) no. x (+) no. = (+) no.
- (+) no. x (-) no. = (-) no.
- (-) no. x (+) no. = (-) no.
- (-) no. x (-) no. = (+) no.

(Division)

(+) no. / (+) no. = (+) no.
(+) no. / (-) no. = (-) no.
(-) no. / (+) no. = (-) no.
(-) no. / (-) no. = (+) no.

Order of Operations:

20. The arithmetic operation of numbers follows the **BODMAS Rule** shown below. There are also **BIDMAS** and **PEMDAS rules**, but we will only show you BODMAS since it is recommended you use this.

(B) rackets – Simplify equations in the brackets first before going to something else. If there is more than one set of brackets intertwined with each other, start with the innermost set of brackets, and then make your way doing the outer ones.

Example: {3 x [4+2 / (3+2) x 12] + 3}

(O) f – Evaluate the Of, (power, roots) if they exist next.

(D) ivision, **(M)** ultiplication – These two come together as a set, but then perform Multiplication and Division from left to right of the equation.

(A)ddition, **(S)** ubtraction – These two also come as a set together, but perform Addition and Subtraction after Brackets, Of, Division and Multiplication, from the left to the right of the equation.