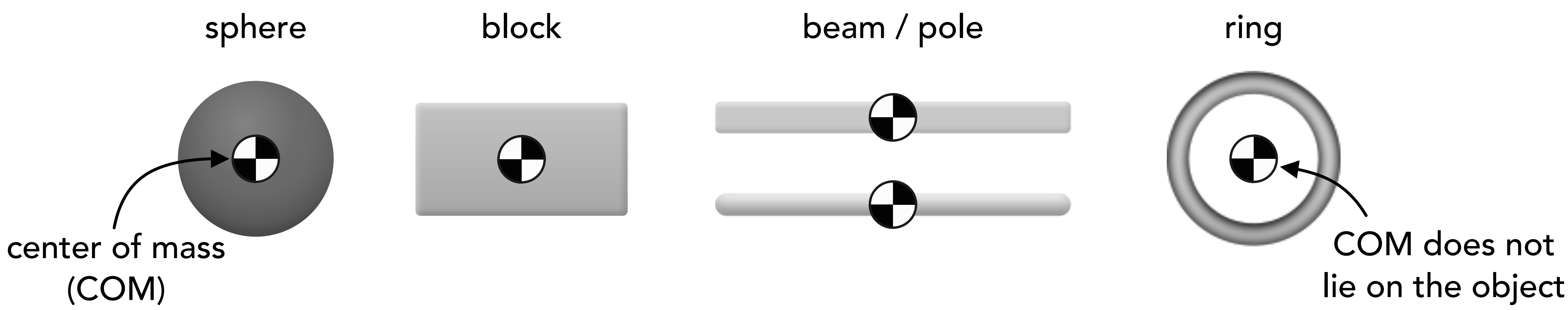
CENTER OF MASS

Center of Mass

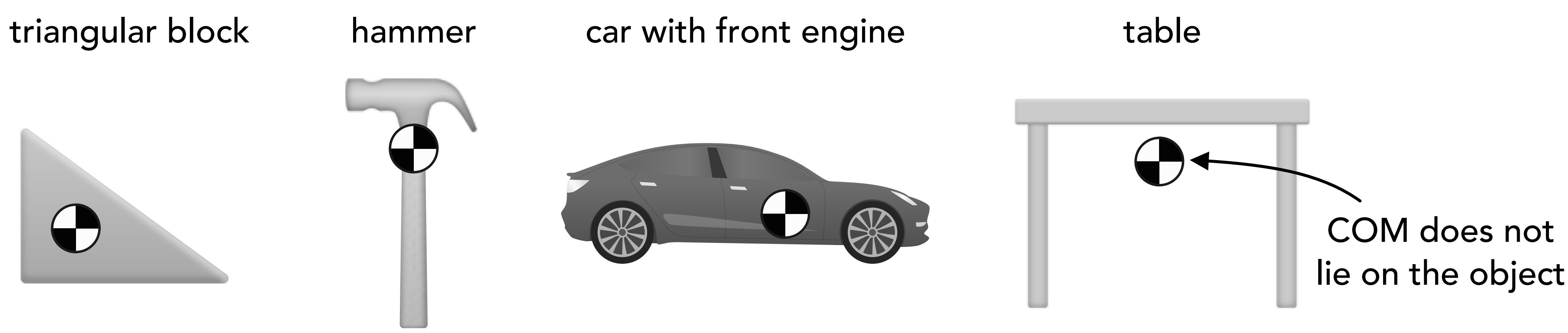
Variables		SI Unit
x	x position	m
y	y position	m
m	mass	kg

- The **center of mass (COM)** of an object or a system of masses is the mass-weighted average position of an object or a system of masses.
- If an object is symmetrical and has a uniform density, the center of mass is located at the center of the object (the center of the width, length and height of the object).
- If an object is not symmetrical or does not have a uniform density, the center of mass moves closer to areas of the object with more mass.
- The center of mass of an object or a sytem does not have to lie directly on the object and can be located in the empty space near the physical object(s).

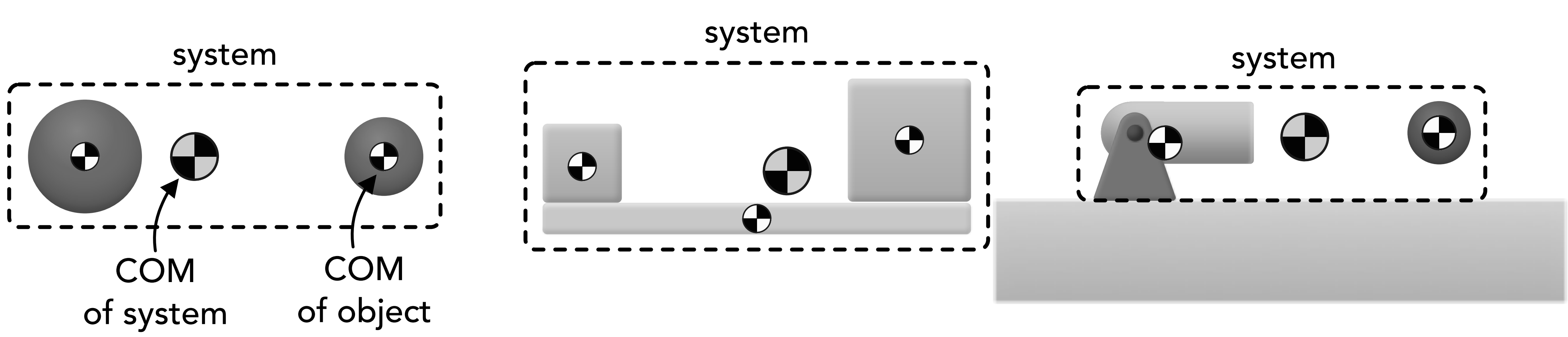
The center of mass of a symmetrical object with uniform density is located at the center of the object



The center of mass of an asymmetrical object is located closer to areas with more mass



The center of mass of a system depends on the location and mass of each object



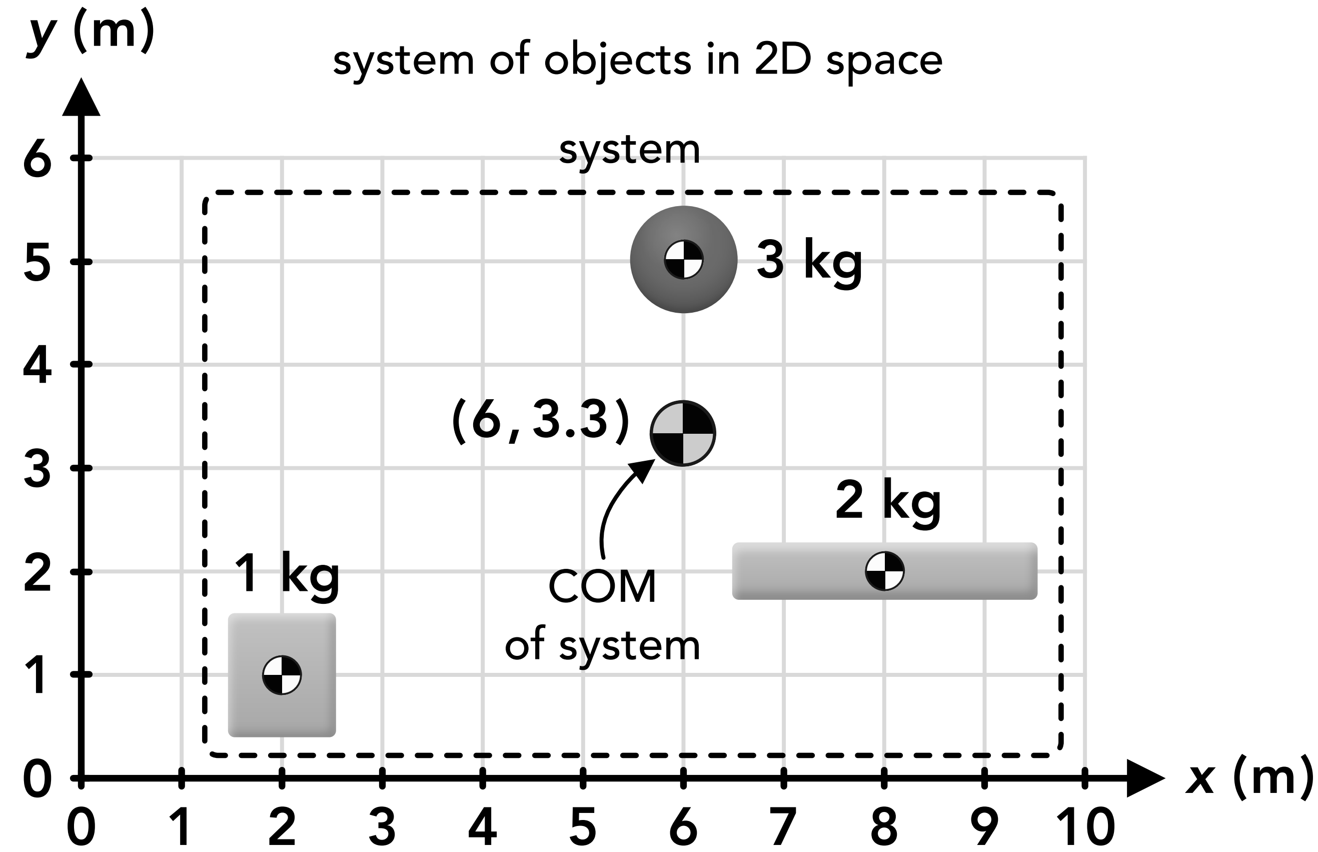
- The center of mass of a system or group of masses can be calculated using the equations below.
- If all of the masses lie on a single line only one coordinate (x) is needed. If the masses are distributed in 2D space then two coordinates (x, y) are needed. Although not covered here, (x, y, z) would be used in 3D space.
- **The coordinates used for each object are the coordinates of that object's own center of mass**, which will be in the middle of the object if the object is symmetrical.
- The origin of the coordinate system is arbitrary. If it's not given you can choose the origin (where x and y are zero).

x coordinate of center of mass of a system

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$

y coordinate of center of mass of a system

$$y_{COM} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots}$$



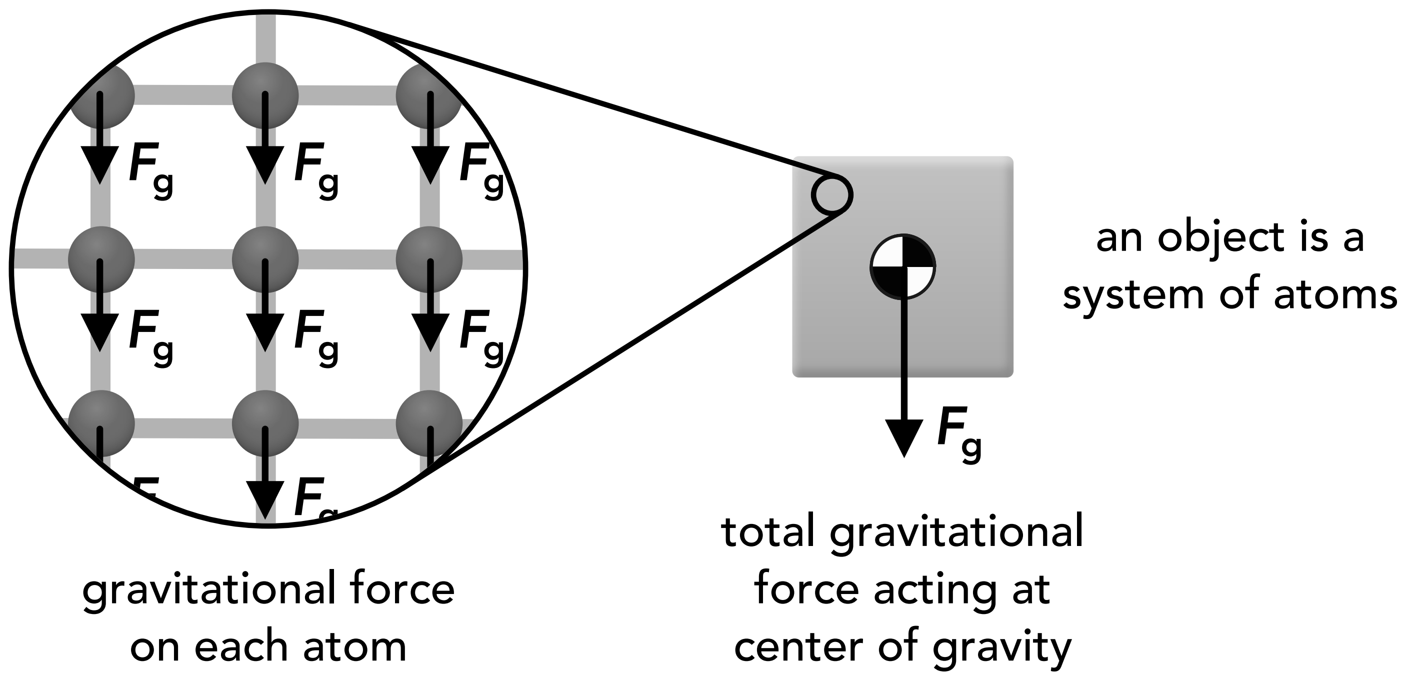
$$x_{COM} = \frac{(1\ \text{kg})(2\ \text{m}) + (2\ \text{kg})(8\ \text{m}) + (3\ \text{kg})(6\ \text{m})}{(1\ \text{kg}) + (2\ \text{kg}) + (3\ \text{kg})} = 6\ \text{m}$$

$$y_{COM} = \frac{(1\ \text{kg})(1\ \text{m}) + (2\ \text{kg})(2\ \text{m}) + (3\ \text{kg})(5\ \text{m})}{(1\ \text{kg}) + (2\ \text{kg}) + (3\ \text{kg})} = 3.3\ \text{m}$$

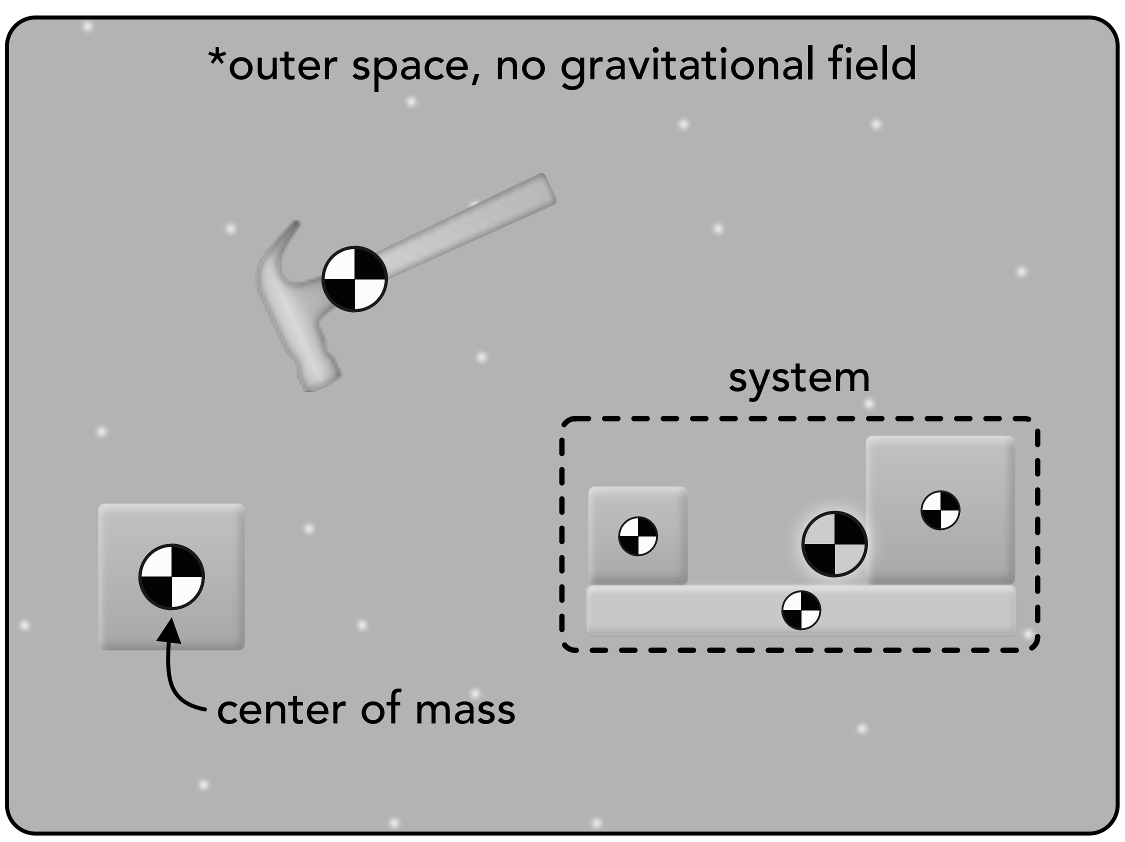
COM of system: **(6, 3.3)**

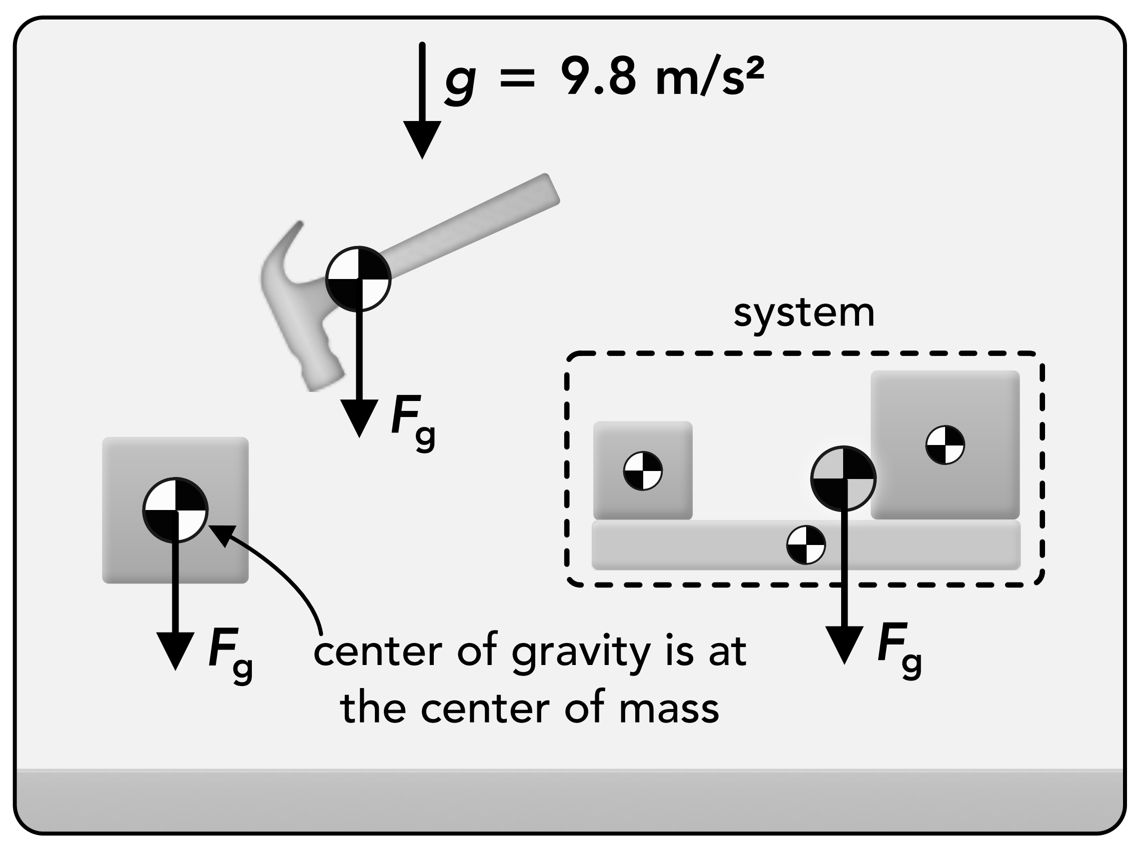
- Objects and systems also have a **center of gravity (COG)**. When an object or system is in a gravitational field, there is a gravitational force acting on every particle in the object or every mass in the system. However, we can treat all of those forces as **a single, total gravitational force acting on a single point - the center of gravity** (this is what we usually do).
- If we assume all of the object or system is within a uniform gravitational field (meaning the acceleration due to gravity ***g*** is the same everywhere across the object or system, which is a good approximation) then **the center of mass and the center of gravity are at the same point**.

We can treat the gravitational forces acting on each particle as a single, total gravitational force acting at the object's center of mass



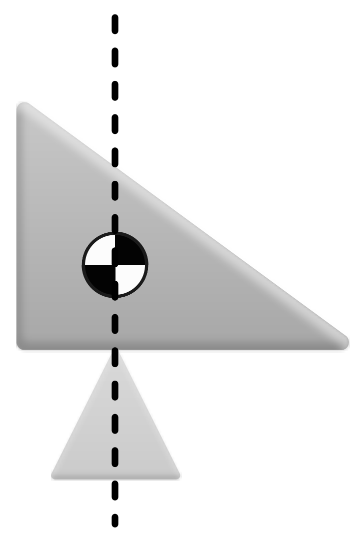
When in a uniform gravitational field, the center of gravity is located at the center of mass



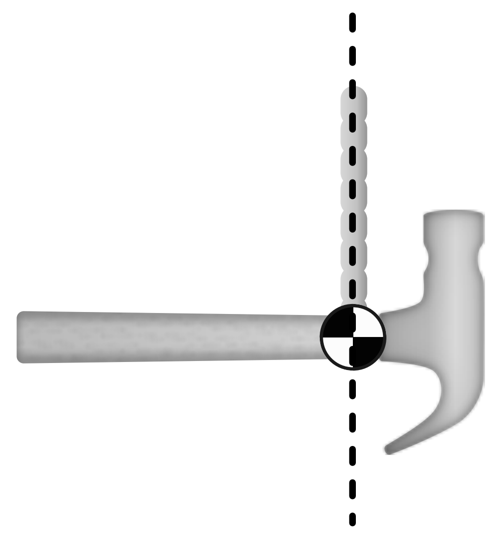


- An object or a system will **balance on its center of mass (center of gravity)** on a pivot point or when suspended from above. Since we can treat the individual gravitational forces acting on each object as a single gravitational force acting on the system's center of mass, that single gravitational force **will not generate a torque if it's directly above or below the point of rotation**.

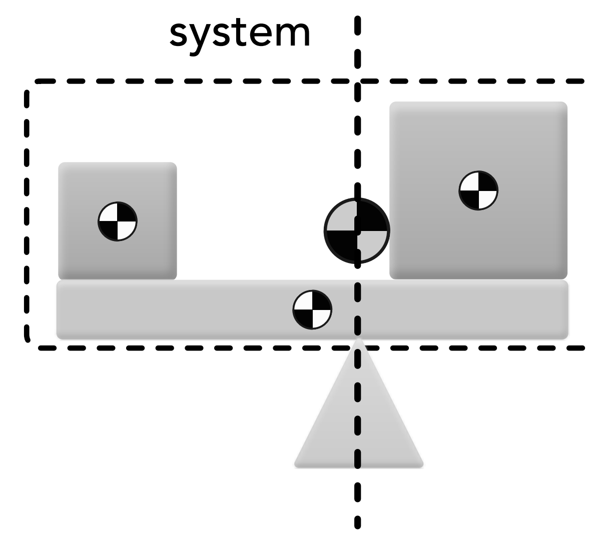
Objects and systems will be balanced (in rotational equilibrium) when the pivot point or suspension point is in vertical alignment with the center of mass



pivot is directly below the object's COM

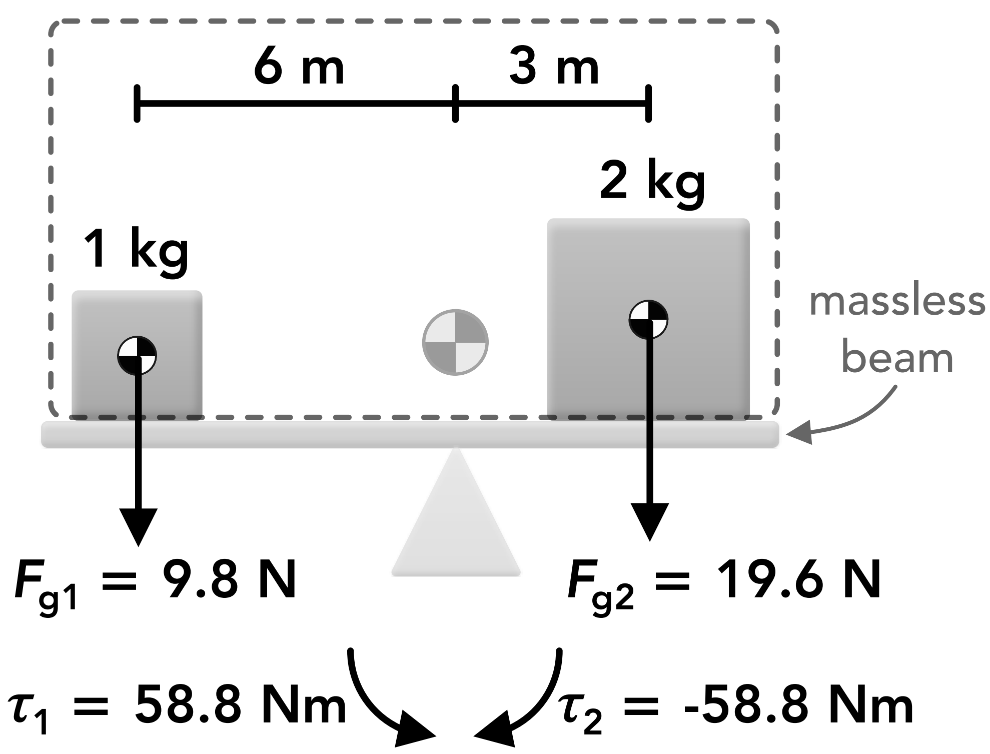


object is suspended above its COM



pivot is directly below the system's COM

If we look at each object separately, the individual gravitational forces generate individual torques about the pivot point. If the pivot point is directly below the system's center of mass, the net torque will be zero so the system is balanced.



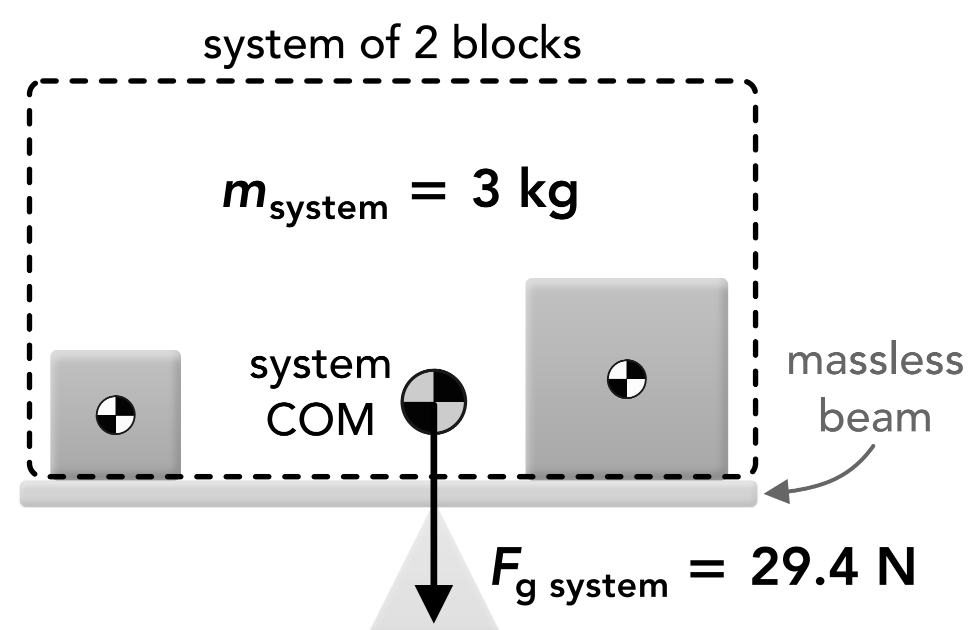
$$\tau_1 = r_1 F_1 = (6\text{ m})(9.8\text{ N})$$
$$\tau_2 = r_2 F_2 = -(3\text{ m})(19.6\text{ N})$$

$$\sum\tau = I\alpha$$
$$(58.8\text{ Nm}) - (58.8\text{ Nm}) = I\alpha$$
$$(0\text{ Nm}) = I(0\text{ rad/s}^2)$$

net torque is zero

angular acceleration is zero

If we treat the objects as a system, a single gravitational force acts at the system's center of mass. If the pivot point is directly below the system's center of mass, that single gravitational force points directly at the point of rotation and does not generate a torque, so the net torque on the system is zero and the system is balanced.



$$\tau_{system} = rF = (0\text{ m})(29.4\text{ N}) = 0\text{ Nm}$$

force is in line with point of rotation, $r = 0$

$$\sum\tau = I\alpha$$
$$(0\text{ Nm}) = I(0\text{ rad/s}^2)$$

net torque is zero

angular acceleration is zero