PRACTICE PROBLEMS

CONVERTING BETWEEN CIRCULAR & ROTATIONAL MOTION

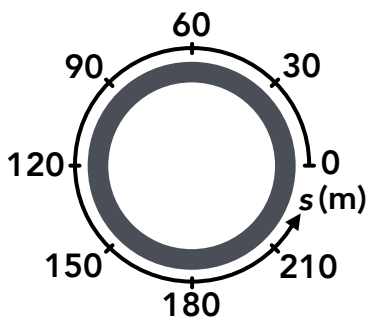
Position and displacement

1. 1 revolution is associated with: (Select all that apply)
   - A 1 circumference
   - B π radians
   - C 360°
   - D 2π radians

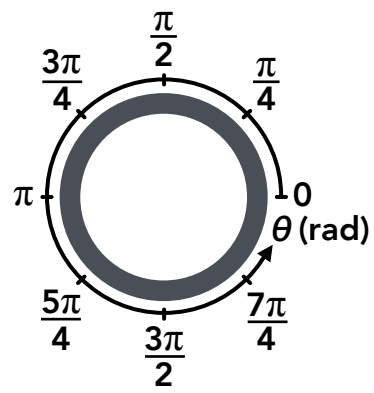
2. If the circumference of a circle is 20 m, what is the radius?
   - A 10 m
   - B 6.37 m
   - C 3.18 m
   - D 2.52 m

3. Which of the following axes represent the tangential description of motion? (Select all that apply)

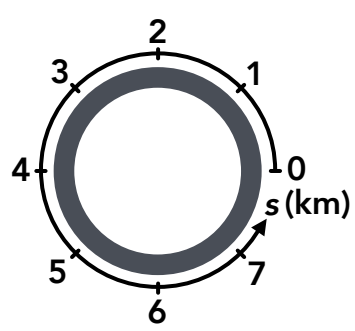
A



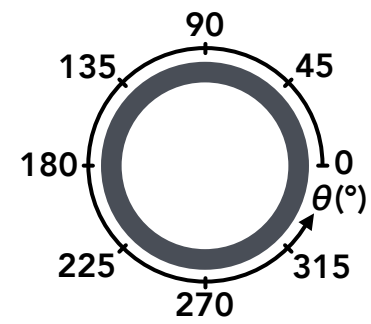
B



C

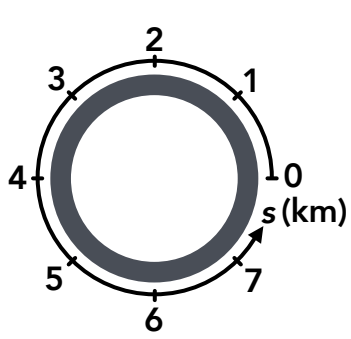


D

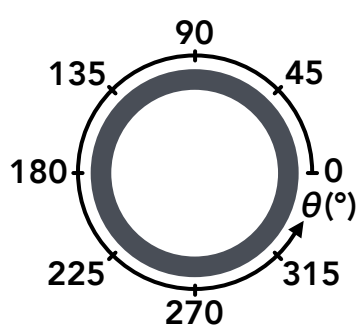


4. Which of the following axes represent the angular description of motion? (Select all that apply)

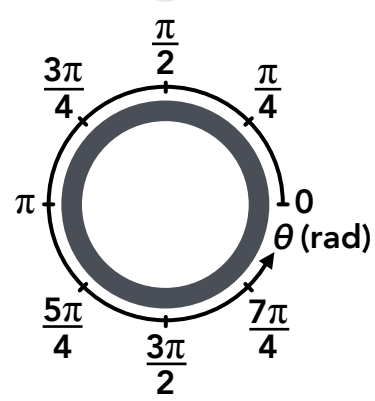
A



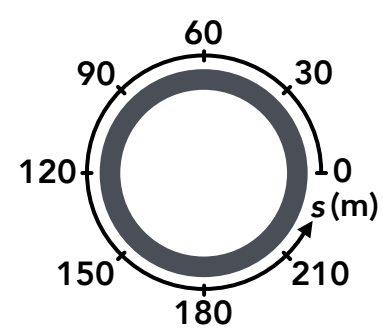
B



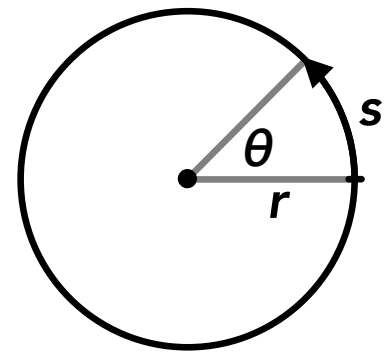
C



D



5. In the circle shown in Figure 1, what is ***s*** if θ is $\pi/4$ rad and ***r*** is 5 m?



(Not drawn to scale)

Figure 1

6. In the circle shown in Figure 1, what is θ if ***s*** is 2 m and ***r*** is 2.5 m?

7. In the circle shown in Figure 1, what is r if θ is $\pi/3$ rad and s is 12 m?

8. On a circular race track with a diameter of 20 m, what is the angular position (in rad) associated with a tangential position of 16 m?

9. On a circular race track, a tangential position of 30 m corresponds to an angular position of 120°. What is the radius of the track?

10. The handle on a door is 0.7 m from the door hinge. When the door is opened, if the handle moves an arc length of 0.9 m, what is the angular displacement of the door in deg?

11. A person is on an amusement park ride that rotates so that the riders follow a circular path with a diameter of 6 m. The ride begins and turns 12 revolutions clockwise, then stops and turns 7 revolutions counterclockwise, then stops and turns 4 revolutions clockwise. What is the tangential displacement of a person from the beginning to the end of the ride, in m?

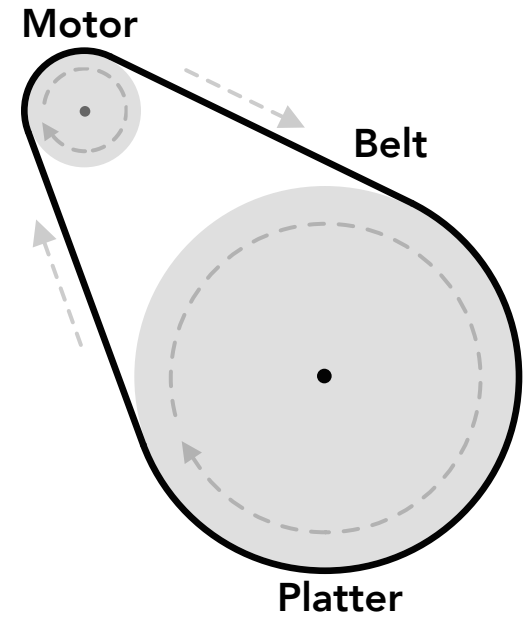
12. While a 30 cm diameter record is playing, a fly sitting on the outer edge of the record covers a tangential displacement of -35.7 m during a song. How many revolutions did the record turn during the song?

13. While tightening a bolt, a wrench rotates from an angular position of $2\pi/3$ rad to $\pi/3$ rad. What is the tangential displacement, in cm, of a point on the wrench that is 15 cm away from the center of the bolt?

14. Two people are riding on a large carousel. Person A is 5 m from the center and person B is 8 m from the center. If person A experiences a tangential displacement of 50 m during a period of time, what is the tangential displacement of person B during that time?

15. Car A is on a circular track with a radius of 0.6 km, and car B is on a circular track with a different radius. During a period of time, car A and car B both drive the same angular displacement. If car A travelled a tangential displacement of 1.2 km, and car B travelled a displacement of 1.8 km, what is the radius of car B's track in km?

16. A belt-driven record player like the one shown to the right (not drawn to scale) has a rubber belt wrapped around the outside of a motor wheel and the outside of the platter (a spinning plate that the record sits on while playing). As the motor wheel spins it moves the belt which turns the platter and the record. If the diameter of the motor wheel is 22 mm and the diameter of the platter is 330 mm, how many times does the motor wheel have to turn to get the platter to rotate once? (Consider how the belt's displacement relates to the motion of the motor wheel and platter, without slipping).



Speed and Velocity

17. Two flies are sitting on a spinning record at different spots but the same distance from the edge of the record. Which of the following is true about the motion of the flies?

    A They have the same tangential speed and the same angular speed
    B They have the same tangential speed but different angular speeds
    C They have different tangential speeds but the same angular speed
    D They have different tangential speeds and different angular speeds

18. Two cars are driving around the same, large circular track that only has one lane. If Car A has a higher tangential velocity than Car B, how are their angular velocities related?

    A $\omega_{\text{Car A}} > \omega_{\text{Car B}}$
    A $\omega_{\text{Car A}} = \omega_{\text{Car B}}$
    A $\omega_{\text{Car A}} < \omega_{\text{Car B}}$
    A It cannot be determined

19. A fly is sitting on a ceiling fan blade, 0.4 m from the center of rotation of the fan. If the tangential velocity of the fly is 8 m/s, what is the angular velocity of the fan in rad/s?

20. A small rock gets stuck in the outer edge of a car's tire, which is 0.6 m in diameter. If the angular velocity of the wheel is 6 rad/s, what is the tangential velocity of the rock (from the tire's rotation, ignoring the car's motion)?

21. A car is driving on a circular track with a tangential velocity of 18 m/s. If the angular velocity swept out by the car is 1.5 rad/s, what is the radius of the track?

22. A figure skater spins around with their arms stretched out and their hands 0.8 m from their center of rotation. If the tangential velocity of their hands is 17 m/s, what is the angular velocity of the skater in rpm?

23. If the minute hand on a (12-hour) clock is 120 mm long, what is the tangential speed of a point on the tip of the minute hand in mm/s?

24. If a 22 m diameter Ferris wheel is spinning at 5 rpm, what is the tangential displacement of a rider (at the edge of the Ferris wheel) over a period of 30 s?

25. The International Space Station (ISS) is in a circular orbit around the Earth. It takes 92.5 minutes for the ISS to circle the Earth and complete 1 orbit. If the radius of the Earth is 6371 km (assuming it's a sphere) and the tangential speed of the ISS is 7.66 km/s, how high is the ISS above the ground, in km?

26. Two flies are sitting on the same spinning object. Fly A is 4 cm from the center and fly B is 14 cm from the center. If fly A travels a tangential displacement of 44 cm in 2 s, how many cm does fly B travel in 3 s?

27. Two cars are on a circular track with 2 lanes. Car A follows the outside lane, a circular path with a radius of 100 m and car B follows the inside lane, a circular path with a radius of 95 m. The cars remain side by side as they drive around the track. If car A is travelling at a tangential speed of 65 km/h, what is car B's tangential speed in km/h?

Acceleration

28. A fly is sitting on a vinyl record when the "Start" button is pushed and the record accelerates at 2 rad/s². If the fly experiences a tangential acceleration of 0.2 m/s², how far is the fly from the center of the record?

29. An athlete performs a hammer throw by spinning around while holding a wire with a metal ball at the end, speeding up and eventually releasing the ball. If the ball is 1.5 m from the athlete's center of rotation and their angular acceleration is 4 rad/s², what is the tangential acceleration of the ball?

30. A person riding 3.5 m from the center of a carousel experiences a tangential acceleration of 2 m/s², what is the carousel's angular acceleration in rad/s² ?

31. A rider on a roller coaster travels through a circular loop with a radius of 9 m, slowing down as they move higher in the loop. If the tangential acceleration of the rider is -7 m/s², what is their angular acceleration in rad/s² ?

32. A pole vaulter runs and plants the end of the 5 m long pole in the ground, propelling them upwards along a circular path with an initial tangential velocity of 8 m/s. When the pole vaulter reaches the peak 2 s later, their tangential velocity is 5 m/s. What was the angular acceleration of the pole in rad/s² (assuming it was constant)?

33. You're using a medieval slingshot, which involves swinging around a length of string with a rock in a pouch at the end. At first you swing the rock around in a circle with a tangential speed of 25 m/s, but then decide it's a little too fast and slow it down to a tangential speed of 16 m/s over the course of 3 s. If the angular acceleration of the rock and string was -1.5 rad/s² during that time, what is the total length of the string and your arm (how far is the rock from the center of rotation)?

34. A car gets a small rock stuck in a tire while driving, which is 21 cm from the center of the wheel. The car is stopped at an intersection, and when the light turns green the car accelerates and the wheels speed up to 450 rpm in 5 s. What is the tangential acceleration of the rock during that period in cm/s² ?

35. A race car on a 0.6 km diameter circular track is going 54 km/h. When the car is 80 m from the finish line, the driver speeds up such that the car's angular acceleration around the track is a constant 0.01 rad/s². From the moment the car began accelerating, how long did it take to cross the finish line?

36. A vial spinning in a lab centrifuge is 0.1 m from the center of rotation. The technician sets the centrifuge to a higher speed, and the vial experiences a tangential acceleration of 2 m/s². During the 4 s it takes to go from one speed to the next, the centrifuge rotates 40 times. What was the original, slower angular speed of the centrifuge in rpm?

37. On a hot summer day, you walk over and turn up the speed of your ceiling fan from 200 rpm to 300 rpm. During the period of time that the fan is increasing in speed, a fly sitting 75 cm from the fan's center of rotation travels a tangential displacement of 15 m. What was the angular acceleration of the fan during that time in rad/s²?

Answers - Position and displacement

1. Answer: A, C, D
   When converting between tangential and angular quantities, we can use these relationships:
   $$\frac{\textbf{1 circumference}}{\textbf{1 revolution}}, \frac{\textbf{360°}}{\textbf{1 revolution}}, \frac{\textbf{2}\pi\textbf{ radians}}{\textbf{1 revolution}}$$

2. Answer: C
   $\boldsymbol{C = 2\pi r}$ $\quad$ $\boldsymbol{(20\text{ m}) = 2\pi r}$ $\quad$ $r = 3.18$ m

3. Answer: A, C
   The tangential description of motion refers to the motion along the circumference of a circle and uses a unit for length such as m or km.

4. Answer: B, C
   The angular description of motion refers to the angle swept out by a rotating object or an object in circular motion and uses a unit for angles such as degrees or radians.

5. Answer: 3.93 m
   $\boldsymbol{s = r\theta = (5\text{ m})(\pi/4\text{ rad})} = 3.93$ m

6. Answer: 0.8 rad
   $\boldsymbol{s = r\theta}$ $\quad$ $\boldsymbol{(2\text{ m}) = (2.5\text{ m})\theta}$ $\quad$ $\theta = 0.8$ rad

7. Answer: 11.46 m
   $\boldsymbol{s = r\theta}$ $\quad$ $\boldsymbol{(12\text{ m}) = r(\pi/3\text{ rad})}$ $\quad$ $r = 11.46$ m

8. Answer: 1.6 rad
   $\boldsymbol{s = r\theta}$ $\quad$ $\boldsymbol{(16\text{ m}) = (20\text{ m} / 2)\theta}$ $\quad$ $\theta = 1.6$ rad

9. Answer: 14.3 m
   $$\frac{\textbf{120°}}{} \times \frac{\textbf{2}\pi\textbf{ rad}}{\textbf{360°}} = 2\pi/3 \text{ rad}$$
   $\boldsymbol{s = r\theta}$ $\quad$ $\boldsymbol{(30\text{ m}) = r(2\pi/3\text{ rad})}$ $\quad$ $r = 14.3$ m

10. Answer: 74°
    $\boldsymbol{\Delta s = r\,\Delta\theta}$ $\quad$ $\boldsymbol{(0.9\text{ m}) = (0.7\text{ m})\Delta\theta}$ $\quad$ $\Delta\theta = 1.29$ rad
    $$\frac{\textbf{1.29 rad}}{} \times \frac{\textbf{360°}}{\textbf{2}\pi\textbf{ rad}} = 74°$$

11. Answer: -54π m or -169.6 m
    $\boldsymbol{\Delta\theta = (-12\text{ rev}) + (7\text{ rev}) + (-4\text{ rev})} = -9$ rev
    $$\frac{\textbf{-9 rev}}{} \times \frac{\textbf{2}\pi\textbf{ rad}}{\textbf{1 rev}} = -18\pi \text{ rad}$$
    $\boldsymbol{\Delta s = r\,\Delta\theta = (6\text{ m} / 2)(-18\pi\text{ rad})} = -54\pi$ m

12. Answer: -119/π rev or -37.9 rev

$\Delta s = r\,\Delta\theta \qquad (-35.7\text{ m}) = (0.3\text{ m} / 2)\Delta\theta \qquad \Delta\theta = -238\text{ rad}$

$$\frac{-238\text{ rad}}{} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = -119/\pi\text{ rev}$$

13. Answer: -5π cm or -15.7 cm

$\Delta s = r\,\Delta\theta = (15\text{ cm})(\pi/3\text{ rad} - 2\pi/3\text{ rad}) = -5\pi\text{ cm}$

14. Answer: 80 m

$\Delta s_A = r_A\Delta\theta_A \qquad (50\text{ m}) = (5\text{ m})\Delta\theta_A \qquad \Delta\theta_A = 10\text{ rad} = \Delta\theta_B$

$\Delta s_B = r_B\Delta\theta_B = (8\text{ m})(10\text{ rad}) = 80\text{ m}$

15. Answer: 0.9 km

$\Delta s_A = r_A\Delta\theta_A \qquad (1.2\text{ km}) = (0.6\text{ km})\Delta\theta_A \qquad \Delta\theta_A = 2\text{ rad} = \Delta\theta_B$

$\Delta s_B = r_B\Delta\theta_B \qquad (1.8\text{ km}) = r_B(2\text{ rad}) \qquad r_B = 0.9\text{ km}$

16. Answer: 15 rev

The linear displacement of the belt must match the tangential displacement of a point on the edge of the motor wheel and the platter. If the platter rotates once, the belt moves one circumference of the platter and the motor wheel turns so a point on its edge has a tangential displacement of one circumference of the platter.

$C_{Platter} = \pi d_{Platter} = 330\pi\text{ mm}$

$$\Delta s_{Motor} = r_{Motor}\Delta\theta_{Motor} \qquad (330\pi\text{ mm}) = (22\text{ mm} / 2)\Delta\theta_{Motor} \qquad \Delta\theta_{Motor} = 30\pi\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 15\text{ rev}$$

Answers - Speed and Velocity

17. Answer: A

Any two points on the same rotating object have the same angular speed. The two points have the same tangential speed if the radius of their circular path is the same (the points are the same distance from the center of rotation). The relationship is: $v_t = r\,\omega$

18. Answer: A

The relationship between the tangential velocity and the angular velocity is: $v_t = r\,\omega$. If the two cars are driving along the same circular track (with the same radius) then a greater tangential velocity means the angular velocity is also greater.

19. Answer: 20 rad/s

$v_t = r\,\omega \qquad (8\text{ m/s}) = (0.4\text{ m})\omega \qquad \omega = 20\text{ rad/s}$

20. Answer: 1.8 m/s

$v_t = r\,\omega = (0.6\text{ m} / 2)(6\text{ rad/s}) = 1.8\text{ m/s}$

21. Answer: 12 m

$v_t = r\,\omega \qquad (18\text{ m/s}) = r(1.5\text{ rad/s}) \qquad r = 12\text{ m}$

22. Answer: 202.9 rpm

$v_t = r\,\omega \qquad (17\text{ m/s}) = (0.8\text{ m})\omega \qquad \omega = 21.25\text{ rad/s}$

$$\frac{21.25\text{ rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 202.9\text{ rev/min (rpm)}$$

23. Answer: $\pi/15$ mm/s or 0.21 mm/s

$$\omega = \frac{1 \text{ rev}}{1 \text{ h}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/1800 \text{ rad/s}$$

$$v_t = r\omega = (120 \text{ mm})(\pi/1800 \text{ rad/s}) = \pi/15 \text{ mm/s}$$

24. Answer: 55π m or 172.8 m

$$\omega = \frac{5 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/6 \text{ rad/s}$$

$$v_t = r\omega = (22 \text{ m} / 2)(\pi/6 \text{ rad/s}) = 11\pi/6 \text{ m/s}$$

$$v_t = \frac{\Delta s}{\Delta t} \qquad (11\pi/6 \text{ m/s}) = \frac{\Delta s}{(30 \text{ s})} \qquad \Delta s = 55\pi \text{ m}$$

25. Answer: 395 km

$$\omega = \frac{1 \text{ rev}}{92.5 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/2775 \text{ rad/s}$$

$$v_t = r\omega \qquad (7.66 \text{ km/s}) = r(\pi/2775 \text{ rad/s}) \qquad r = 6766 \text{ km}$$

$(6766 \text{ km}) - (6371 \text{ km}) = 395 \text{ km}$

26. Answer: 231 cm

$$v_{t\,A} = \frac{\Delta s}{\Delta t} = \frac{44 \text{ cm}}{2 \text{ s}} = 22 \text{ cm/s}$$

$$v_{t\,A} = r_A\omega_A \qquad (22 \text{ cm/s}) = (4 \text{ cm})\omega_A \qquad \omega_A = 5.5 \text{ rad/s} = \omega_B$$

$$v_{t\,B} = r_B\omega_B = (14 \text{ cm})(5.5 \text{ rad/s}) = 77 \text{ cm/s}$$

$$v_{t\,B} = \frac{\Delta s}{\Delta t} \qquad (77 \text{ cm/s}) = \frac{\Delta s}{(3 \text{ s})} \qquad \Delta s = 231 \text{ cm}$$

27. Answer: 61.8 km/h

$v_{t\,A} = r_A\omega_A \qquad (65 \text{ km/h}) = (0.1 \text{ km})\omega_A \qquad \omega_A = 650 \text{ rad/h} = \omega_B$ (the cars remain side by side so $\omega_A = \omega_B$)

$$v_{t\,B} = r_B\omega_B = (0.095 \text{ km})(650 \text{ rad/h}) = 61.8 \text{ km/h}$$

Answers - Acceleration

28. Answer: 0.1 m

$a_t = r\alpha \qquad (0.2 \text{ m/s}^2) = r(2 \text{ rad/s}^2) \qquad r = 0.1 \text{ m}$

29. Answer: 6 m/s²

$$a_t = r\alpha = (1.5 \text{ m})(4 \text{ rad/s}^2) = 6 \text{ m/s}^2$$

30. Answer: 0.57 rad/s²

$a_t = r\alpha \qquad (2 \text{ m/s}^2) = (3.5 \text{ m})\alpha \qquad \alpha = 0.57 \text{ rad/s}^2$

31. Answer: -0.78 rad/s²

$a_t = r\alpha \qquad (-7 \text{ m/s}^2) = (9 \text{ m})\alpha \qquad \alpha = -0.78 \text{ rad/s}^2$

32. Answer: -0.3 rad/s²

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{(5\text{ m/s}) - (8\text{ m/s})}{(2\text{ s})} = -1.5\text{ m/s}^2$$

$$a_t = r\alpha \qquad (-1.5\text{ m/s}^2) = (5\text{ m})\alpha \qquad \alpha = -0.3\text{ rad/s}^2$$

33. Answer: 2 m

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{(16\text{ m/s}) - (25\text{ m/s})}{(3\text{ s})} = -3\text{ m/s}^2$$

$$a_t = r\alpha \qquad (-3\text{ m/s}^2) = r(-1.5\text{ rad/s}^2) \qquad r = 2\text{ m}$$

34. Answer: 63π cm/s² or 197.9 cm/s²

$$\omega_f = \frac{450\text{ rev}}{1\text{ min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 15\pi\text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(15\pi\text{ rad/s}) - (0\text{ rad/s})}{(5\text{ s})} = 3\pi\text{ rad/s}^2$$

$$a_t = r\alpha = (21\text{ cm})(3\pi\text{ rad/s}^2) = 63\pi\text{ cm/s}^2$$

35. Answer: 3.9 s

$$a_t = r\alpha = (300\text{ m})(0.01\text{ rad/s}^2) = 3\text{ m/s}^2$$

$$v_{ti} = \frac{54\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{60\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 15\text{ m/s}$$

$$\Delta s = v_{ti}t + \frac{1}{2}a_t t^2 \qquad (80\text{ m}) = (15\text{ m/s})t + \frac{1}{2}(3\text{ m/s}^2)t^2 \qquad t = 3.9\text{ s}$$

36. Answer: 218.0 rpm

$$\Delta\theta = \frac{40\text{ rev}}{} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 80\pi\text{ rad}$$

$$a_t = r\alpha \qquad (2\text{ m/s}^2) = (0.1\text{ m})\alpha \qquad \alpha = 20\text{ rad/s}^2$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (80\pi\text{ rad}) = \omega_i(4\text{ s}) + \frac{1}{2}(20\text{ rad/s}^2)(4\text{ s})^2 \qquad \omega_i = 22.83\text{ rad/s}$$

$$\frac{22.83\text{ rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 218.0\text{ rev/min (rpm)}$$

37. Answer: 13.7 rad/s²

$$\Delta s = r\,\Delta\theta \qquad (15\text{ m}) = (0.75\text{ m})\Delta\theta \qquad \Delta\theta = 20\text{ rad}$$

$$\omega_i = \frac{200\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 20\pi/3\text{ rad/s}$$

$$\omega_f = \frac{300\text{ rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 10\pi\text{ rad/s}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \qquad (10\pi\text{ rad/s})^2 = (20\pi/3\text{ rad/s})^2 + 2\alpha(20\text{ rad}) \qquad \alpha = 13.7\text{ rad/s}^2$$