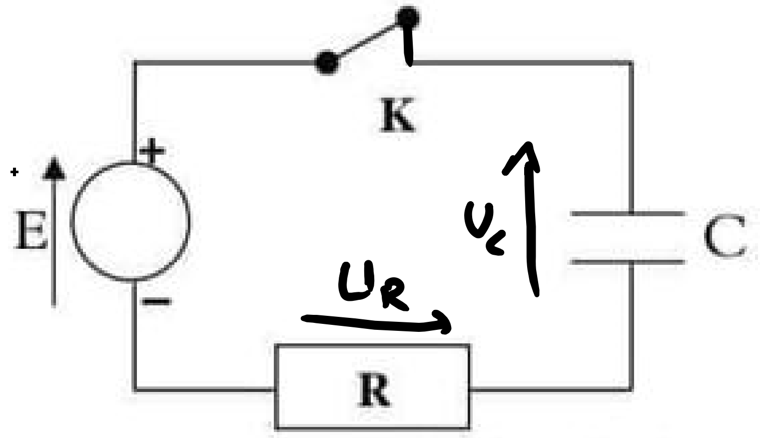
Analyse des courbes en dipôle RC

La charge

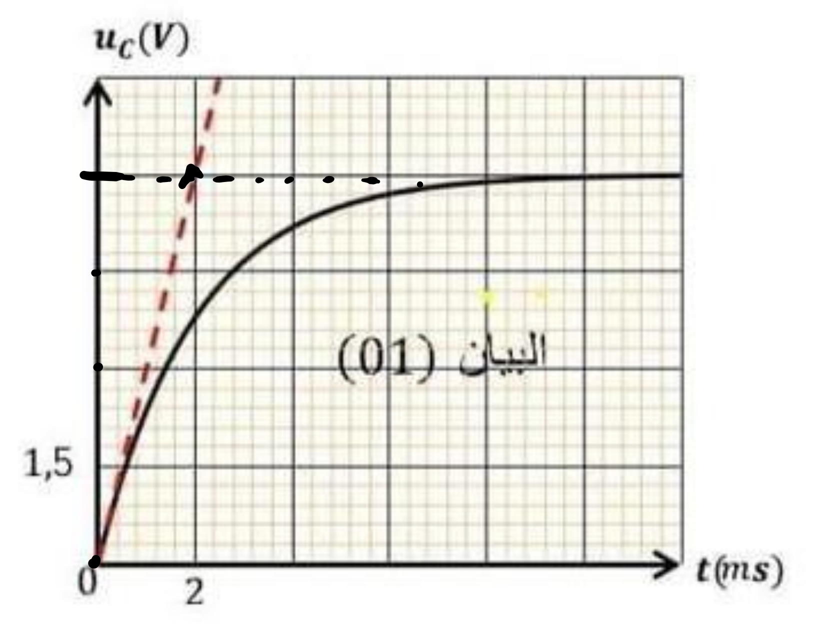
Partie 1 :



Données : $R = 1\,k\Omega$

1) Déterminer E et τ dans chaque courbe.

2) Calculer la capacité C et l'intensité du courant maximale : I_0.



1)

* $E = 4 \times 1,5 \Rightarrow E = 6V.$

* $\tau = 2\,ms$

2) * $\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{2 \times 10^{-3}}{1 \times 10^{3}}$

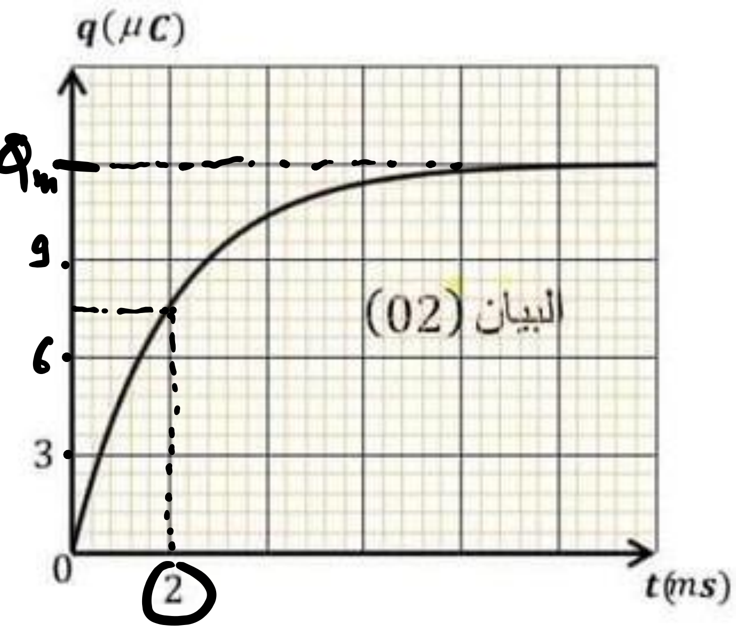
$C = 2.10^{-6}\,F \Rightarrow \boxed{C = 2\,\mu F}$

* $I_0 = i(0)$: La loi d'add des tensions : $u_C + u_R = E$

à $t=0$: $\cancel{u_C(0)} + u_R(0) = E \Rightarrow u_R(0) = E \Rightarrow R.i(0) = E$

$\cancel{u_C(0)}$: 0

$i(0) = \frac{E}{R} \Rightarrow I_0 = \frac{E}{R} = \frac{6}{1.10^3} = 6.10^{-3}A \Rightarrow \boxed{I_0 = 6\,mA}$



① on sait que $q = C.U_c$

au régime permanent $t \to \infty$

$q(\infty) = Q_m$ et $U_c(\infty) = E$

$\Rightarrow Q_m = C.E$

$\Rightarrow E = \frac{Q_m}{C} \times \frac{R}{R}$ C ??

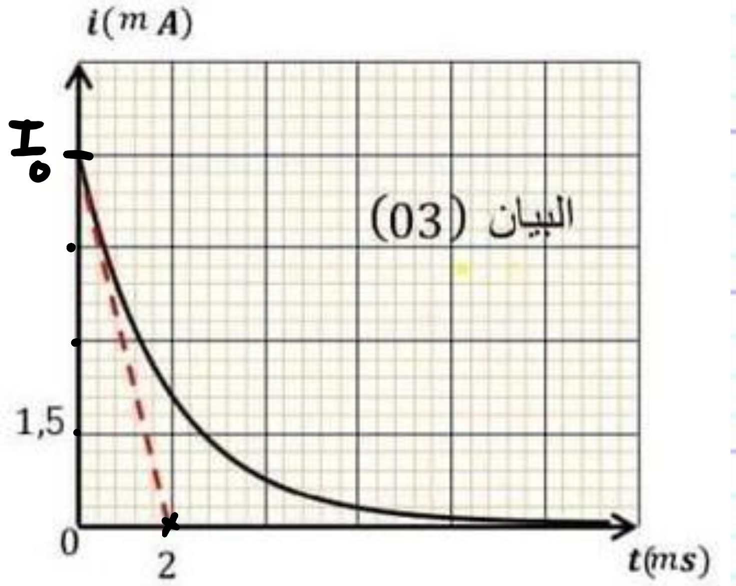
$\Rightarrow E = \frac{Q_m \times R}{\tau}$ → Nouveau

Graphiqt $q(\tau) = 0{,}63\,Q_m = 0{,}63 \times 12 = 7{,}56\ \mu C$

Projection $\boxed{\tau = 2ms}$

donc $E = \frac{12 \times 10^{-6} \times 1 \times \cancel{10^{3}}}{2 \times \cancel{10^{-3}}} \Rightarrow \boxed{E = 6V}$

② $C = \frac{\tau}{R} = \frac{2 \times 10^{-3}}{1 \times 10^{3}} = 2\mu F$; $I_0 = \frac{E}{R} = \frac{6}{1 \times 10^{3}} \Rightarrow \boxed{I_0 = 6\,mA}$



Graphiquement

on a $I_0 = 6\,mA$

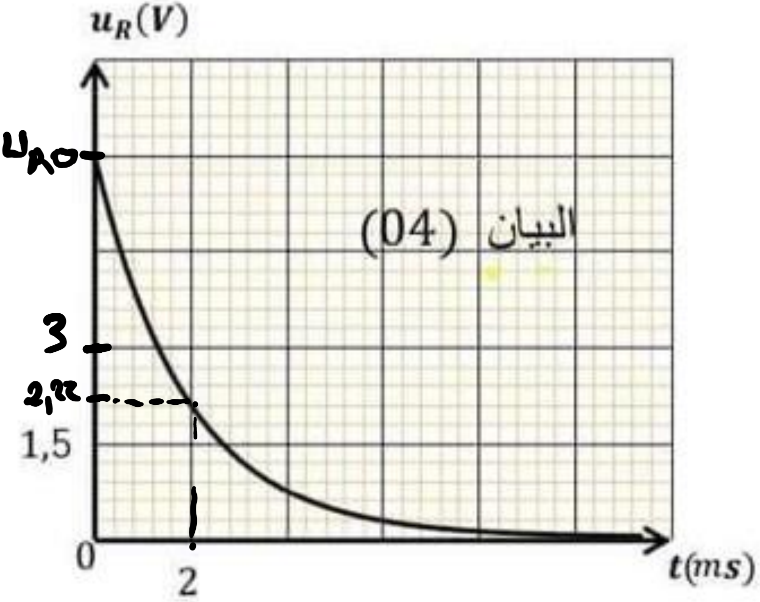
et $\tau = 2\,ms$

donc $I_0 = \frac{E}{R} \Rightarrow \boxed{E = R.I_0}$

AN: $E = 1 \times \cancel{10^{3}} \times 6 \times \cancel{10^{-3}}$

$\boxed{E = 6}$

$\boxed{C = \frac{\tau}{R} = 2\,\mu F}$



Graphiquement $U_{R0} = 6V$

* Déterminons I_0 :

Loi d'ohm : $U_{R0} = R.I_0 \Rightarrow \boxed{I_0 = \frac{U_{R0}}{R}}$

$I_0 = \frac{6}{1.10^3} \Rightarrow \boxed{I_0 = 6.10^{-3} A = 6mA}$

* Déterminons E :

→ Méthode 1 : $I_0 = \frac{E}{R} \Rightarrow E = R.I_0 = 1.10^3 \times 6 \times 10^{-3} \Rightarrow E = 6V$

→ Méthode 2 : loi d'additivité des tensions : $\cancel{U_C(0)} + U_R(0) = E$

à $t=0$ le condensateur est déchargé $U_C(0) = 0$

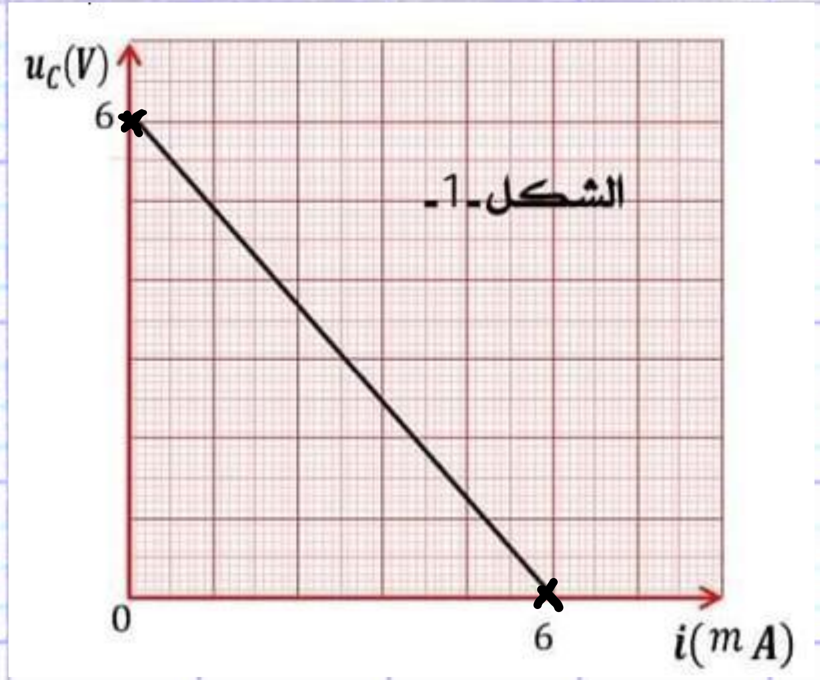
$\Rightarrow U_R(0) = E$

$\Rightarrow \boxed{E = 6V}$.

* Détermination de τ : $U_R(\tau) = 0,37.U_{R0} = 0,37 \times 6 = 2,22V \Rightarrow \boxed{\tau = 2ms}$

Partie 2 :

Déterminer τ, E, R et C dans chaque cas :



L'équation de la courbe $u_c = f(i)$

la fct est affine :

$$\boxed{u_c(i) = a.i + b} \quad (*)$$

a : la pente

b : l'ordonnée à l'origine $\underline{i=0}$

* La loi d'add des tension $u_c + u_R = E \Rightarrow u_c + Ri = E$

$$\Rightarrow \boxed{u_c = -Ri + E} \quad (**)$$

$$i = C\frac{du_c}{dt}$$

Par identification de (*) et (**) on trouve :

$-R = a$ et $b = E$

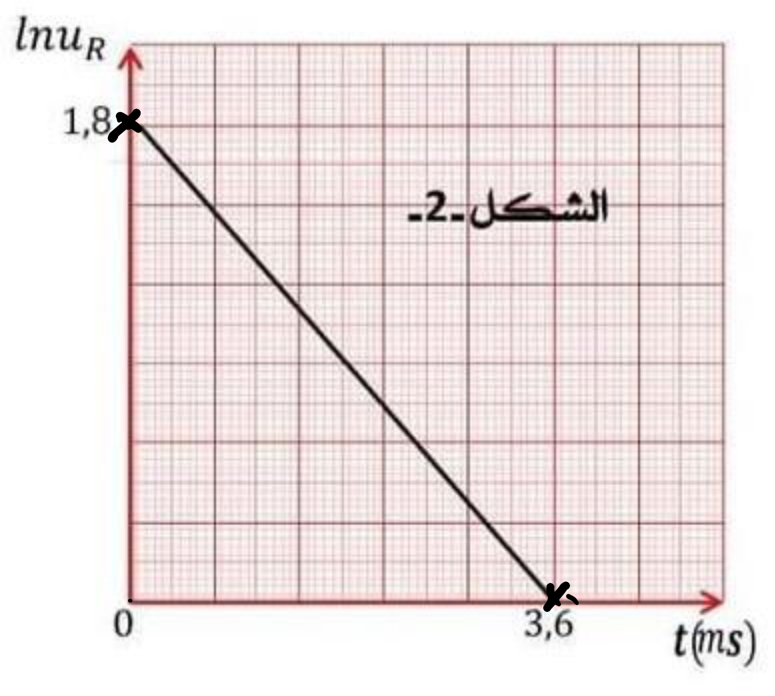
$R = -a$ et $E = b$

$\boxed{E = 6V}$

$$R = -\frac{6 - 0}{(0 - 6)\times 10^{-3}}$$

$\boxed{R = 1000\ \Omega}$

$\boxed{R = 1\,K\Omega}$



$la\ fct\ \ \ln u_R = f(t)$

$$\bullet\ \boxed{\ln u_R = at + b}\ (*)$$

$la\ charge\ :\ \ u_C(t) = E(1 - e^{-t/\tau})$

$u_R + u_C = E$

$$u_R = E - u_C$$

$$u_R = E - E(1 - e^{-t/\tau})$$

$$= \cancel{E} - \cancel{E} + Ee^{-t/\tau}$$

$$u_R = Ee^{-t/\tau}$$

$\ln(a.b) = \ln a + \ln b$

$$\ln u_R = \ln(E.e^{-t/\tau}) = \ln E + \cancel{\ln} e^{-t/\tau} = (\ln E) - \frac{t}{\tau}$$

$$\Rightarrow \ln u_R = -\frac{1}{\tau}.t + \ln E\ \ (**)$$

Identification $\quad a = -\dfrac{1}{\tau}\quad$ et $\quad b = \ln E$

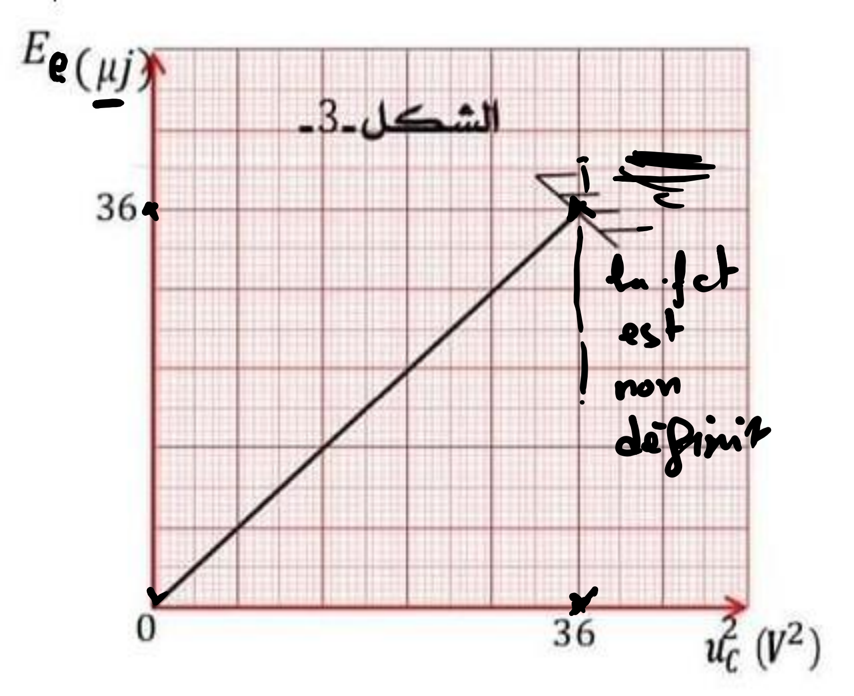
$\Rightarrow \tau = -\frac{1}{a}$ et $E = e^b$

graphiquement $\boxed{b = 1{,}8}$; $a = \frac{1{,}8 - 0}{(0 - 3{,}6) \times 10^{-3}\,s}$

$\boxed{a = -500\,s^{-1}}$

Donc $\tau = \cancel{-}\frac{1}{\cancel{-}500\,s^{-1}} = 2.10^{-3}\,s \Rightarrow \boxed{\tau = 2ms}$

et $E = e^b = e^{1{,}8} \Rightarrow \boxed{E = 6V}$



* $Ee = f(U_c^2)$ est fct linéaire

$$\boxed{E_e = a \cdot u_c^2}$$

Cours : $\boxed{Ee = \frac{1}{2} c \cdot U_c^2}$

Identification : $\frac{1}{2} c = a \Rightarrow \boxed{c = 2a}$

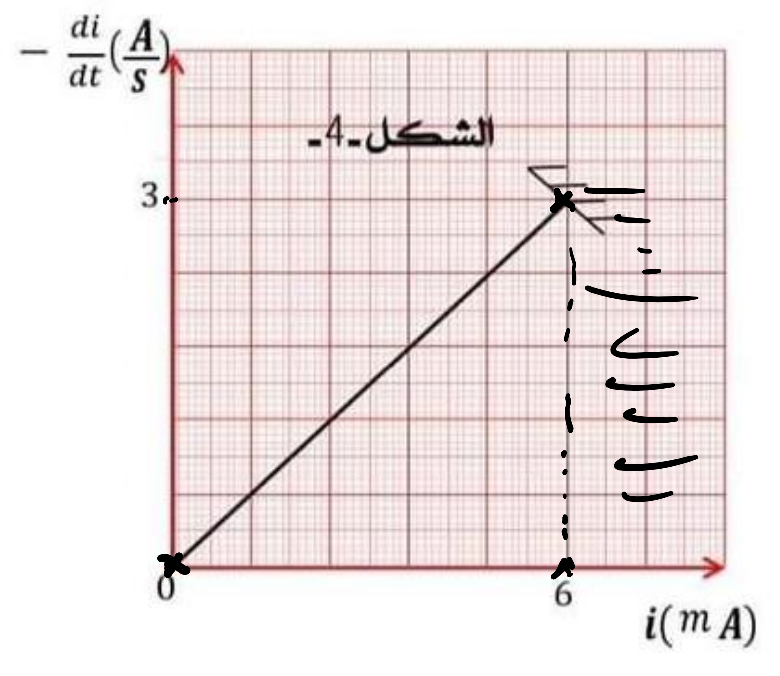
Donc $c = 2 \times \frac{(36 - 0) \times 10^{-6}}{36 - 0} = 2 \cdot 10^{-6}\,F$

$$\boxed{C = 2\,\mu F}$$

* La valeur de E. Graphiquement $U_{cmax}^2 = 36\,V^2$

$\Rightarrow U_{cmax} = \sqrt{36} = 6\,V$

$$\boxed{U_{cmax} = E = 6\,V}$$



* Calculer la valeur de τ graphiquement.

La fct est linéaire :

$$\boxed{-\frac{di}{dt} = \alpha . i} \quad (*)$$

* L'éq. diff vérifiée par $i(t)$:

La loi d'add des tension $\quad u_R + u_C = E$

dérivée : $\frac{du_R}{dt} + \frac{du_C}{dt} = \cancel{\frac{dE}{dt}} \to 0 \Rightarrow \frac{d(Ri)}{dt} + \frac{du_C}{dt} = 0$

$\Rightarrow \left(R\frac{di}{dt} + \frac{du_C}{dt} = 0\right) \times C \qquad i = C\frac{du_C}{dt}$

$\Rightarrow RC\frac{di}{dt} + C\frac{du_C}{dt} = 0$

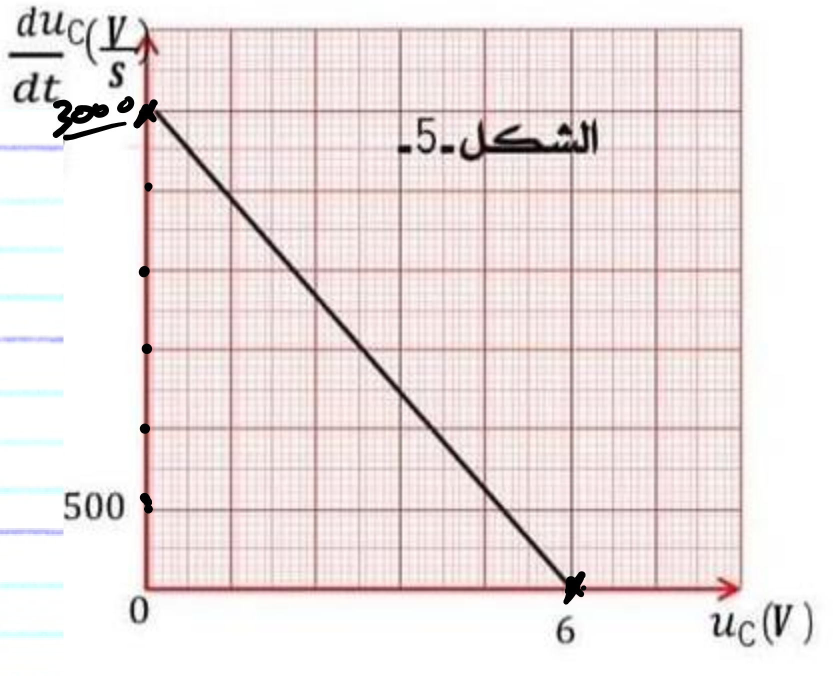
$\Rightarrow \tau\frac{di}{dt} + i = 0 \Rightarrow i = -\tau\frac{di}{dt} \Rightarrow \boxed{-\frac{di}{dt} = \frac{1}{\tau} i} \quad (**)$

Par identification $\frac{1}{\tau} = \alpha \Rightarrow \tau = \frac{1}{\alpha}$

AN: $$\tau = \frac{1}{\frac{3-0}{(6-0)\times 10^{-3}}}$$

$$\tau = 2.10^{-3}\,s$$

$$\boxed{\tau = 2\,ms}$$



• La valeur de τ et E :

La fct : $\frac{du_c}{dt} = f(u_c)$ est affine :

• s'écrit comme $\frac{du_c}{dt} = a u_c + b$ (*)

• L'équation diff. vérifiée par u_c :

$$u_c + u_R = E$$

$$\Rightarrow u_c + Ri = E$$

$$\Rightarrow \left(u_c + RC\frac{du_c}{dt} = E\right) \times \frac{1}{RC}$$

$$\Rightarrow \frac{u_c}{RC} + \frac{du_c}{dt} = \frac{E}{RC} \Rightarrow \frac{du_c}{dt} = -\frac{u_c}{RC} + \frac{E}{RC}$$

$$\frac{du_c}{dt} = -\frac{1}{RC}u_c + \frac{E}{RC} \quad (**)$$

Identification

$$-\frac{1}{RC} = a \Rightarrow -\frac{1}{\tau} = a$$

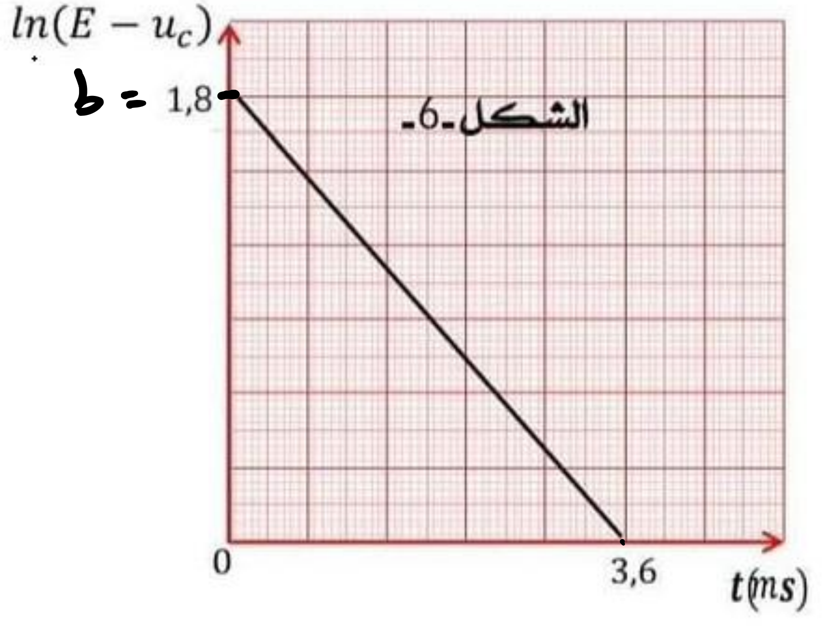
$$\Rightarrow \boxed{\tau = -1/a}$$

et $\quad \underline{b} = \frac{E}{RC} \Rightarrow b = \frac{E}{\tau} \Rightarrow \boxed{E = \tau . b}$

donc , $\quad \tau = -\dfrac{1}{\dfrac{3000-0}{0-6}} \Rightarrow \tau = 2 \times 10^{-3}\, s \Rightarrow \boxed{\tau = 2\,ms}$

et $\quad E = \tau . b = 2 \times \cancel{10^{-3}} \times 3\cancel{000}$

$\boxed{E = 6V}$



Déterminer E et τ.

la fct : $\boxed{\ln(E-U_c) = \alpha . t + b}$ (affine).

Dans le cas de la charge :

$$U_c(t) = E(1 - e^{-t/\tau})$$

$$U_c = E - Ee^{-t/\tau}$$

$$Ee^{-t/\tau} = E - U_c$$

$$\ln(Ee^{-t/\tau}) = \ln(E - U_c)$$

$$\ln E + \cancel{\ln} e^{-t/\tau} = \ln(E - U_c)$$

$$\ln E - \frac{t}{\tau} = \ln(E - U_c)$$

$$\boxed{\ln(E - U_c) = -\frac{1}{\tau} . t + \ln E}$$

Identification $b = \ln E \Rightarrow E = e^b = e^{1,8} = 6\,V$

et $\alpha = -\frac{1}{\tau} \Rightarrow \tau = -\frac{1}{\alpha}$

$$\tau = \frac{-1}{\frac{1,8 - 0}{(0 - 3,6) \times 10^{-3}}}$$

$$\tau = 2.10^{-3}\,s$$

$$\boxed{\tau = 2\,ms}$$