SPRING FORCE & ELASTICITY

Spring Force and Hooke's Law

1. The SI unit for the spring constant is…
   - A N
   - B N·m
   - C N/m
   - D m/N

2. Spring A has a higher stiffness (higher spring constant) than spring B. If the same force is applied to the ends of both springs, which spring will have a greater change in length?
   - A Spring A
   - B Spring B
   - C The have the same change in length
   - D Cannot be determined

3. A spring is hanging from the ceiling at its original length. If a 6 kg mass is attached to the bottom of the spring and a person pushes upwards on the mass with a force of 65 N, the spring will...
   - A get shorter
   - B get longer
   - C remain at its original length
   - D cannot be determined

4. A force of magnitude F pulls on a spring with a spring constant of k and original length of L. The new length of the spring is...
   - A L
   - B $L + \frac{F}{k}$
   - C $\frac{F}{k}$
   - D $L - \frac{F}{k}$

5. Spring A compresses by 20 cm when a 30 N force is applied to it. Spring B stretches by 30 cm when a 40 N force is applied to it. Which spring has a larger spring constant?
   - A The springs have the same spring constant
   - B Spring A
   - C Spring B
   - D Cannot be determined

6. A 30 cm long spring is hanging from the ceiling. When a 5 kg block is attached to the bottom of the spring, the length of the spring becomes 40 cm. How long is the spring if the 5 kg block is replaced with an 8 kg block?
   - A 16 cm
   - B 30 cm
   - C 38 cm
   - D 46 cm

7. A spring is attached to a wall at one end and a person pulls on the other end with a force of 50 N. The force exerted on the spring from the wall is…
   - A 0 N
   - B 25 N
   - C 50 N
   - D 100 N

8. A spring is attached to a wall at one end and a person pulls on the other end with a force of 50 N. If the spring constant is 100 N/m, how much does the spring stretch?
   - A 0 cm
   - B 25 cm
   - C 50 cm
   - D 100 cm

9. Two springs are attached to each other in series. Together they act as a single spring with an equivalent spring constant that is…
   - A greater than the spring constants of either individual spring
   - B less than the spring constants of either individual spring
   - C equal to the smaller of the two individual spring constants
   - D none of the above

10. One end of a spring is mounted to a wall and the other end is attached to a block which can slide along the floor. If the block is moved so that the spring gets shorter and then the block is released from rest, what is the direction of the block's acceleration at that moment?
    - A Away from the wall
    - B Towards the wall
    - C Upwards
    - D Downwards

11. Two springs are attached to the same objects at each end so that they're combined in parallel. Together they act as a single spring with an equivalent spring constant that is...
    - A greater than the spring constants of either individual spring
    - B less than the spring constants of either individual spring
    - C equal to the larger of the two individual spring constants
    - D none of the above

12. A block of mass m is hanging from a spring with spring constant k which is mounted to the ceiling. A separate force is applied to the block so that the spring becomes longer than its original unstretched length by an amount of Δx. Which of the following represents the net force acting on the block the moment the block is released (the separate force is removed), assuming up is the positive direction?

- A $k\Delta x$
- B $mg + k\Delta x$
- C $k\Delta x - mg$
- D $mg - k\Delta x$

13. A spring is mounted vertically to the floor. If a force is applied to the free end of the spring so that the spring gets longer, what is the direction of the force exerted on the spring from the floor?

- A Upwards
- B Downwards
- C The force is zero
- D Cannot be determined

14. Three springs are combined in series with each other and the end of the first spring is mounted to a wall. If a force is applied to the end of the last spring in the series, which of the following is true? (select all that apply)

- A each spring changes length by the same amount
- B the force exerted on each spring are the same
- C the change in length of each spring depends on its spring constant
- D the force exerted on each spring depends on its spring constant

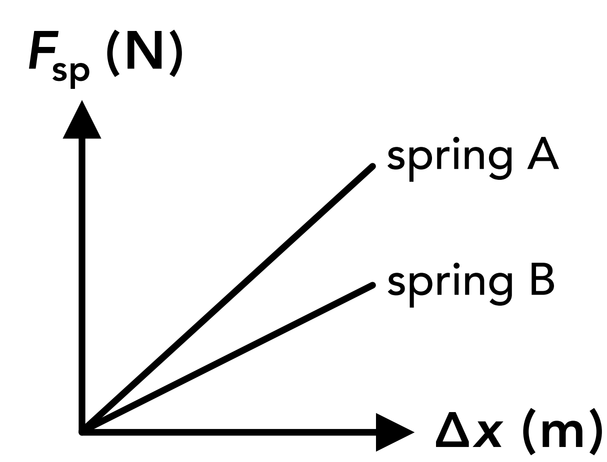
15. A spring is mounted vertically to the floor and 12 kg block is placed on the spring, compressing it by 6 cm. What is the magnitude of the force exerted on the spring from the floor?

- A 0 N
- B 2 N
- C 72 N
- D 118 N

16. Two springs are combined in parallel with each other and they are attached to the same objects at each end. If the springs are stretched...

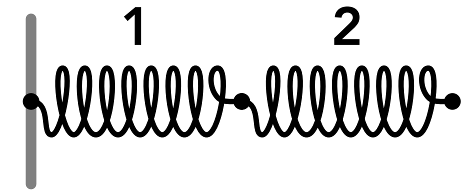
- A the force exerted on each spring is the same
- B the force exerted on each spring depends on the spring constant of each spring
- C the change in length of each spring depends on the spring constant of each spring
- D the change in length of each spring is the same

17. A graph of the spring force vs the displacement for two springs is shown on the right. Which of the following is true?



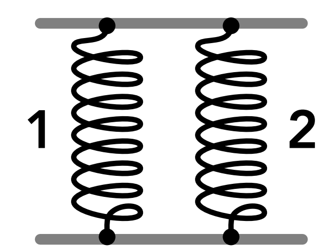
- A Spring A has a larger spring constant than spring B
- B Spring B has a larger spring constant than spring A
- C Springs A and B have the same spring constant
- D The relationship between the spring constants cannot be determined

18. Two springs are combined as shown on the right. Spring 1 has a spring constant of 150 N/m and spring 2 has a spring constant of 100 N/m. What is the equivalent spring constant for the combined springs?



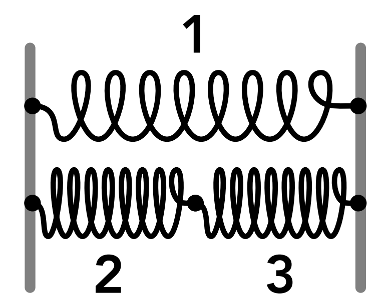
- A 150 N/m
- B 100 N/m
- C 60 N/m
- D 50 N/m

19. Two springs are combined as shown on the right. Spring 1 has a spring constant of 80 N/m and spring 2 has a spring constant of 60 N/m. What is the equivalent spring constant for the combined springs?



- A 60 N/m
- B 70 N/m
- C 80 N/m
- D 140 N/m

20. Three springs are combined as shown on the right. If the spring constant of each spring is 50 N/m, what is the equivalent spring constant for the combined springs?

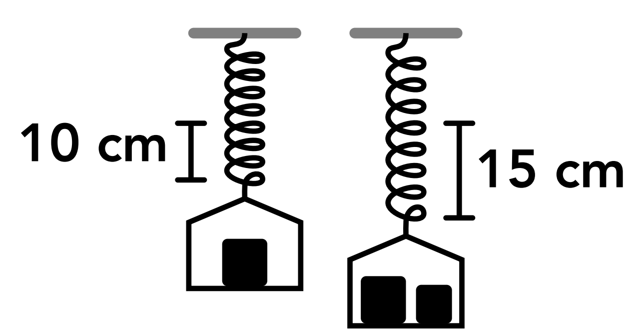


- A 50 N/m
- B 75 N/m
- C 100 N/m
- D 150 N/m

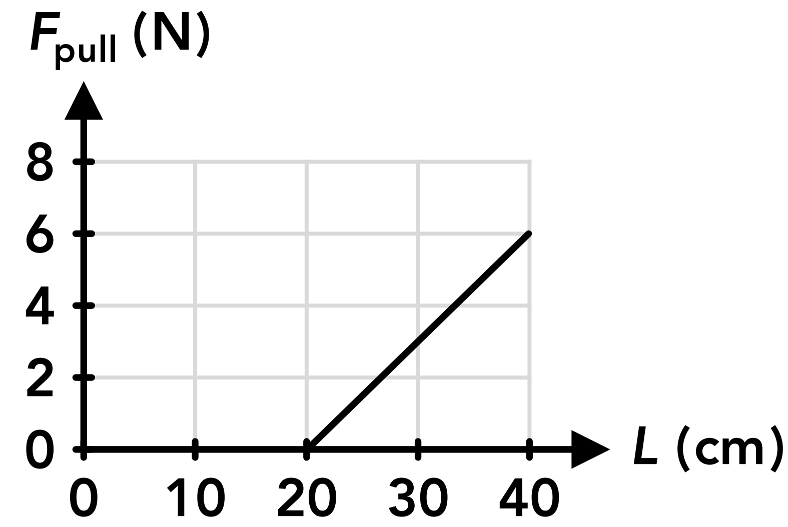
21. A spring of length 30 cm and spring constant 500 N/m is mounted vertically to the floor. A block is placed on top of the spring which compresses to a length of 20 cm. What is the mass of the block?

22. A 20 kg block is attached to one end of a spring with a spring constant of 200 N/m, and the other end of the spring is mounted to a wall. The block is moved and held so that the spring shortens by 15 cm. Assuming there is no friction on the block, what is the acceleration of the block the moment it's released?

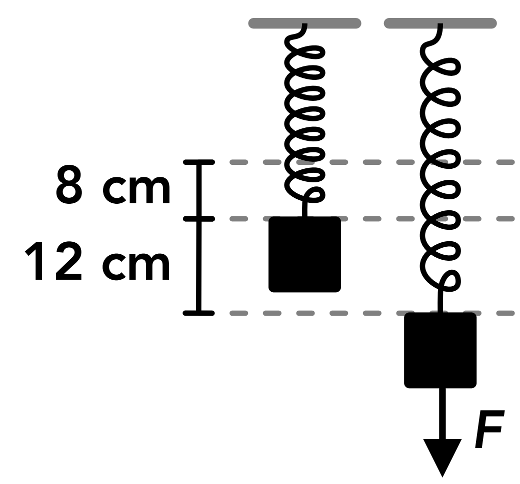
23. A 2 kg basket is hung from the ceiling by a spring and a 5 kg block is placed in the basket, causing the spring to stretch 10 cm from its original length when nothing was attached to it. A second block is added to the basket, causing the spring to stretch an additional 5 cm. What is the mass of the second block?



24. A pulling force is applied to a spring and the length of the spring vs the applied force is shown in the graph on the right. What is the spring constant of the spring in N/m?



25. A spring is hung from the ceiling and a 6 kg block is attached to the bottom of the spring, causing it to stretch 8 cm from its original length. A force is then applied to the block to pull it down and stretch the spring an additional 12 cm. When the force is removed, what is the acceleration of the block?



Elasticity of Materials

26. The SI unit for the Young's modulus (or elastic modulus) of a material is…

- A N
- B N/m
- C N/m²
- D N·m

27. The same force is applied to two metal rods which have the same length and Young's modulus, but one rod has twice the cross-sectional area as the other. The rod with the greater cross-sectional area…

- A will stretch twice as much as the other rod
- B will stretch half as much as the other rod
- C will stretch the same amount as the other rod
- D the relationship between the amount each rod stretches cannot be determined

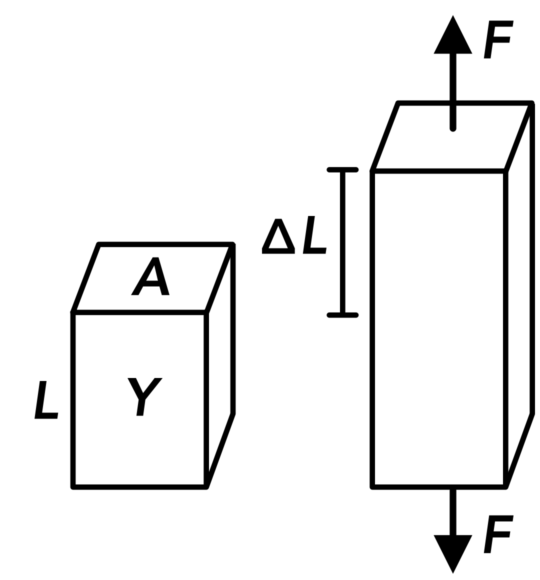
28. Two 100 kg masses are hung from two separate metal wires. The two wires are identical in every way except the original length of one wire is 1/3 as long as the other. The change in length (due to the 100 kg mass) of the longer wire is...

- A three times greater than the change in length of the shorter wire
- B 1/3 the change in length of the shorter wire
- C the same as the change in length of the shorter wire
- D the relationship between the change in length of each wire cannot be determined

29. Two solid plastic cylinders are connected end-to-end and compressed together, and the same force is applied to each cylinder (they are like springs connected in series). The two cylinders are identical in every way except one cylinder has a Young's modulus that is 10 times the Young's modulus of the other cylinder. The cylinder with the smaller Young's modulus will compress…

- A the same amount as the other cylinder
- B 1/10 the amount as the other cylinder
- C 10 times the amount as the other cylinder
- D the relationship between the change in length of each cylinder cannot be determined

30. A metal bar is pulled from each end as shown on the right. In terms of the other variables, which of the following represents the change in length?



- A $\frac{F}{YA}$
- B $\frac{F}{Y}$
- C $\frac{YA}{L}$
- D $\frac{FL}{YA}$

31. A 50 kg mass is hung from a fishing line with a Young's modulus of 3×10^9 N/m². The fishing line can be modeled as a thin cylinder with a diameter of 1 mm. If the original length of the fishing line was 50 cm, how long is the fishing line when the mass is attached?

32. A plastic rod with a Young's modulus of 4×10^9 N/m² and a length of 60 cm is pulled at both ends with a force of 200 N and stretches 3 cm. If the rod has a square cross-sectional area, what is the thickness of the rod?

33. An object is attached to the end of a 2 m long metal wire which is hanging from the ceiling. The diameter of the wire is 3 mm and the Young's modulus is 50×10^9 N/m². If the wire stretches 1 cm, what is the mass of the object?

34. A wire with a Young's modulus of 40×10^9 N/m² and a diameter of 2 mm is pulled at both ends with a force of 600 N, causing it to stretch to a new total length of 520 cm. What was the original, unstretched length of the wire?

35. A wire with a length of 1 m and a diameter of 1 mm is attached to the ceiling at one end and a 300 kg object is hung from the bottom end of the wire, causing to stretch to a length of 1.05 m. What is the Young's modulus of the wire?

Answers

1. C	11. A	21. 5.1 kg	31. 60 cm
2. B	12. C	22. 1.5 m/s²	32. 1 mm
3. A	13. B	23. 3.5 kg	33. 180 kg
4. B	14. B, C	24. 30 N/m	34. 517.5 cm
5. B	15. D	25. 14.7 m/s²	35. **7.49×10^{10} N/m²**
6. D	16. B, D	26. C	
7. C	17. A	27. B	
8. C	18. C	28. A	
9. B	19. D	29. C	
10. A	20. B	30. D	

Answers - Spring Force and Hooke's Law

1. **Answer: C**
   The SI unit for the spring constant is N/m, given by the units in Hooke's Law: $F_{sp} = k\Delta x$.

2. **Answer: B**
   From Hooke's Law, $F_{sp} = k\Delta x$, a greater spring constant will result in a smaller change in length if the force is the same for both springs.

3. **Answer: A**
   The net force applied to the bottom end of the spring is the 65 N acting upwards and the weight of the 6 kg mass acting downwards (58.8 N), so the net force acts upwards which compresses the spring.

4. **Answer: B**
   The pulling force causes the spring to get longer, so the new length is the original length plus the displacement.

   $F_{sp} = k\Delta x \qquad \Delta x = \frac{F}{k}$

5. **Answer: B**
   Spring A: $F_{sp} = k\Delta x \qquad (30\text{ N}) = k(0.2\text{ m}) \qquad k = 150\text{ N/m}$
   Spring B: $F_{sp} = k\Delta x \qquad (40\text{ N}) = k(0.3\text{ m}) \qquad k = 133\text{ N/m}$

6. **Answer: D**
   The force acting on the spring is the weight of the block attached to it. We need to find the spring constant first:
   5 kg block: $F_{sp} = k\Delta x \qquad (5\text{ kg})g = k(0.4\text{ m} - 0.3\text{ m}) \qquad k = 490\text{ N/m}$
   8 kg block: $F_{sp} = k\Delta x \qquad (8\text{ kg})g = (490\text{ N/m})\Delta x \qquad \Delta x = 16\text{ cm} \qquad L = (30\text{ cm}) + (16\text{ cm}) = 46\text{ cm}$

7. **Answer: C**
   If the spring is not accelerating (we assume that it's at rest) then the net force acting on the spring is zero. The person pulls on the spring in one direction with 50 N of force, so the wall must be pulling on the spring in the opposite direction with 50 N of force. We always assume the force applied at both ends of a spring is the same.

8. **Answer: C**
   The value for the spring force that we use in Hooke's Law is the force applied to one end of the spring (the force at each end is the same). We do not add the forces at each end together.
   $F_{sp} = k\Delta x \qquad (50\text{ N}) = (100\text{ N/m})\Delta x \qquad \Delta x = 50\text{ cm}$

9. **Answer: B**
When springs are combined in series, the equivalent spring constant is always less than any of the individual spring constants. The equation for the equivalent spring constant for a series of springs is:
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

10. **Answer: A**
The block is moved towards the wall and the spring gets shorter, so the force acting on the block from the spring points away from the spring and away from the wall, which causes the block to accelerate in that direction.

11. **Answer: A**
When springs are combined in parallel, the equivalent spring constant is equal to the sum of the individual spring constants so it's always greater than any individual spring constant:
$k_{eq} = k_1 + k_2 + \ldots$

12. **Answer: C**
The spring becomes longer that its unstretched length so the block was pulled downwards and the force exerted on the block from the spring acts upwards and is equal to $F_{sp} = k\Delta x$. There is also the weight force acting downwards on the block, $F_g = mg$.

13. **Answer: B**
The spring gets longer so the force exerted on the spring at each end must point away from the spring (the spring is pulled in both directions), so the force exerted on the spring from the floor is downwards.

14. **Answer: B, C**
When springs are combined in series the same force is applied to the ends of each individual spring because it's assumed the net force on each spring is zero (it does not accelerate) and the pair of forces that two consecutive springs exert on each other are equal and opposite (Newton's 3rd law). The change in length of each spring will depend on the spring constant of each spring.

15. **Answer: D**
If the spring is not accelerating after the block compresses it (we assume that it's at rest) then the net vertical force acting on the spring is zero. The force exerted upwards on the spring from the floor must be equal to the downwards weight force of the block exerted on the spring.
$F_g = mg = (12\text{ kg})g = 118\text{ N}$

16. **Answer: B, D**
When springs are combined in parallel they are attached to the same surfaces at each end so they will have the same displacement or change in length. The force exerted on each spring depends on the spring constant for each spring.

17. **Answer: A**
From Hooke's Law, $F_{sp} = k\Delta x$, a graph of the spring force on the vertical axis and displacement on the horizontal axis has a slope that is equal to the spring constant. The slope of the line for spring A is greater so spring A has a larger spring constant.

18. **Answer: C**
The two springs are combined in series:
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{150\text{ N/m}} + \frac{1}{100\text{ N/m}} \qquad k_{eq} = 60\text{ N/m}$$

19. **Answer: D**
The two springs are combined in parallel:
$k_{eq} = k_1 + k_2 = (80\text{ N/m}) + (60\text{ N/m}) = 140\text{ N/m}$

20. **Answer: B**
Springs 2 and 3 are first added in series:
$$\frac{1}{k_{2+3}} = \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{50\text{ N/m}} + \frac{1}{50\text{ N/m}} \qquad k_{2+3} = 25\text{ N/m}$$
The equivalent spring "2+3" and spring 1 are then added in parallel:
$k_{1+2+3} = k_1 + k_{2+3} = (50\text{ N/m}) + (25\text{ N/m}) = 75\text{ N/m}$

21. **Answer: 5.1 kg**
The weight force from the block is exerted on the spring and causes it to compress.
$F_{sp} = k\Delta x \qquad mg = (500\text{ N/m})(0.3\text{ m} - 0.2\text{ m}) \qquad m = 5.1\text{ kg}$

22. **Answer: 1.5 m/s²**
The acceleration of the block is caused by the force exerted by the spring (the only horizontal force).
$\sum F_x = ma_x \qquad F_{sp} = ma_x \qquad k\Delta x = ma_x \qquad (200\text{ N/m})(0.15\text{ m}) = (20\text{ kg})a_x \qquad a_x = 1.5\text{ m/s}^2$

23. **Answer: 3.5 kg**
The spring constant can be found from the first scenario and then used to find the mass of the second block.
1 block: $F_{sp} = k\Delta x \qquad (2\text{ kg} + 5\text{ kg})g = k(0.1\text{ m}) \qquad k = 686\text{ N/m}$
2 blocks: $F_{sp} = k\Delta x \qquad (2\text{ kg} + 5\text{ kg} + m)g = (686\text{ N/m})(0.15\text{ m}) \qquad m = 3.5\text{ kg}$

24. **Answer: 30 N/m**
The original length of the spring is 20 cm because that's the length of the spring when zero force is applied. The spring changes length as the applied force increases. The spring constant can be found from the slope of the diagonal line or by knowing that the original length is 20 cm and using the values at another point on the graph.
$$k = \frac{\Delta F}{\Delta L} = \frac{(6\text{ N}) - (0\text{ N})}{(0.4\text{ m}) - (0.2\text{ m})} = 30\text{ N/m}$$
$F_{sp} = k\Delta x \qquad (6\text{ N}) = k(0.4\text{ m} - 0.2\text{ m}) \qquad k = 30\text{ N/m}$

25. **Answer: 14.7 m/s²**
The spring constant can be found from the first scenario when the block is hanging from the spring:
$F_{sp} = k\Delta x \qquad (6\text{ kg})g = k(0.08\text{ m}) \qquad k = 735\text{ N/m}$
In the second scenario when the block is released and the applied force is removed, there is a spring force acting upwards on the block and its weight force acting downwards:
$\sum F_y = ma_y \qquad F_{sp} - F_g = ma_y \qquad (735\text{ N/m})(0.08\text{ m} + 0.12\text{ m}) - (6\text{ kg})g = (6\text{ kg})a_y \qquad a_y = 14.7\text{ m/s}^2$

Answers - Elasticity of Materials

26. **Answer: C**
The SI unit for the Young's modulus Y (also known as the elastic modulus E) is N/m².

27. **Answer: B**
In the equation for elastic force, if the cross-sectional area A is multiplied by 2 the change in length ΔL must be divided by 2 if every other variable is constant.
$$F = \frac{YA}{L}\Delta L$$

28. **Answer: A**
In the equation for elastic force, if the original length L is multiplied by 3 the change in length ΔL must also be multiplied by 3 if every other variable is constant. The force is constant because the same weight is used.

$$F = \frac{YA}{L}\Delta L$$

29. **Answer: C**
In the equation for elastic force, if the Young's modulus Y is multiplied by 1/10 the change in length ΔL must be multiplied by 10 if every other variable is constant.

$$F = \frac{YA}{L}\Delta L$$

30. **Answer: D**
The change in length ΔL can be found by rearranging the equation for elastic force:

$$F = \frac{YA}{L}\Delta L$$

31. **Answer: 60 cm**

$$A = \pi r^2 = \pi\left(\frac{0.001\text{ m}}{2}\right)^2 = 7.85 \times 10^{-7}\text{ m}^2$$

$$F = \frac{YA}{L}\Delta L \qquad (50\text{ kg})g = \frac{(3 \times 10^9\text{ N/m}^2)(7.85 \times 10^{-7}\text{ m}^2)}{(0.5\text{ m})}\Delta L \qquad \Delta L = 10\text{ cm}$$

$$L_{new} = (50\text{ cm}) + (10\text{ cm}) = 60\text{ cm}$$

32. **Answer: 1 mm**

$$F = \frac{YA}{L}\Delta L \qquad (200\text{ N}) = \frac{(4 \times 10^9\text{ N/m}^2)(t^2)}{(0.6\text{ m})}(0.03\text{ m}) \qquad t = 1\text{ mm}$$

33. **Answer: 180 kg**

$$A = \pi r^2 = \pi\left(\frac{0.003\text{ m}}{2}\right)^2 = 7.07 \times 10^{-6}\text{ m}^2$$

$$F = \frac{YA}{L}\Delta L \qquad mg = \frac{(50 \times 10^9\text{ N/m}^2)(7.07 \times 10^{-6}\text{ m}^2)}{(2\text{ m})}(0.01\text{ m}) \qquad m = 180\text{ kg}$$

34. **Answer: 517.5 cm**

$$A = \pi r^2 = \pi\left(\frac{0.002\text{ m}}{2}\right)^2 = 3.14 \times 10^{-6}\text{ m}^2$$

$$F = \frac{YA}{L}\Delta L \qquad (600\text{ N}) = \frac{(40 \times 10^9\text{ N/m}^2)(3.14 \times 10^{-6}\text{ m}^2)}{L}(5.2\text{ m} - L) \qquad L = 517.5\text{ cm}$$

35. **Answer: 7.49×10^{10} N/m²**

$$A = \pi r^2 = \pi\left(\frac{0.001\text{ m}}{2}\right)^2 = 7.85 \times 10^{-7}\text{ m}^2$$

$$F = \frac{YA}{L}\Delta L \qquad (300\text{ kg})g = \frac{Y(7.85 \times 10^{-7}\text{ m}^2)}{(1\text{ m})}(1.05\text{ m} - 1\text{ m}) \qquad Y = 7.49 \times 10^{10}\text{ N/m}^2$$