Mock Test 2

Part A: Knowledge and Understanding (15 marks)

1) Determine the derivative of $f(x) = \sqrt{x^2 + 1}$ from first principles. (5 marks)

2) If $y = 2u^3 + 3u^2$ and $u = x + \sqrt{x}$, find $\frac{dy}{dx}$ when $x = 1$. (4 marks)

3) If $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants, find a so that $f'''(x) = 9$. (4 marks)

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4) Determine $\frac{dy}{dx}$. Do not simplify. (3 marks)

$$y = \sqrt[5]{x^2} + \frac{2x+1}{x^2+1}$$

Part B: Application (14 marks)

1) A particle has a displacement of $d(t) = 20 - \frac{30}{\sqrt{9+2t}}$ metres after t minutes. When is the particle's speed 0.24 m/min? (4 marks)

2) Determine the equation of the tangent to the curve $y = \frac{x}{\sqrt{2x+3}}$ at $x = 3$. (5 marks)

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3) If $d(x) = f(g(h(x)))$, find $d'(3)$ if $h(3) = 2$, $h'(3) = 3$, $g(x) = x^2 + 2x$, and $f(x) = x^3 - 2x$. (5 marks)

Part C: Thinking (22 marks)

1) The function $f(x) = x^4 - 18x^2 + 3$ has a tangent at the point $(-3, -78)$. The same tangent line is tangent to another point(s) on $f(x)$. Find the point (s). (4 marks)

2) For which value(s) of x are the slopes of the tangents to $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{x+2}{x-2}$ equal? (5 marks)

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3) Find the points on $y = \frac{6x}{x-3}$ where the tangent is perpendicular to the line $2x - 9y - 3 = 0$. (4 marks)

4) If $f(4) = 3$ and $f'(4) = -5$, find the exact value of $g'(4)$ where $g(x) = \sqrt{x}\, f(x)$ (4 marks)

5) Let (a, b) be any point on the graph $f(x) = \frac{1}{x}$. Prove the triangle area formed by the tangent through (a, b) and the coordinate axes is 2. (5 marks)

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Part D: Communication (7 marks)

1) Jane has incorrectly differentiated the function $f(x) = 4(2x^2 + 5)^3$ as $f'(x) = 12(4x)^2$. Explain what her error was and what rule she should be using to correctly differentiate f(x). (2 marks)

2) What does it mean if the derivative of a function is always negative? Give an example of such a function. (2 marks)

3) Draw a potential function f(x) if the graph of f′(x) is below. (3 marks)