Integration by Substitution - 2

Evaluate each indefinite integral and check your result by differentiation.

1. $\int 2x\sqrt{1-x^2}\,dx$

2. $\int x^2\left(x^3+2\right)^4 dx$

3. $\int x\,\mathrm{e}^{x^2+1}dx$

4. $\int \frac{x^2}{5-x^3}\,dx$

5. $\int x\sin\left(x^2\right)dx$

6. $\int \frac{3x}{\left(x^2+6\right)^3}\,dx$

7. $\int x\sqrt[3]{8-x^2}\,dx$

8. $\int \mathrm{e}^{\sin x}\cos x\,dx$

9. $\int \frac{\sin x}{\cos x}\,dx$

10. $\int \frac{\ln x}{x}\,dx$

Integration by Substitution - 2

Worked Solutions

1. $\int 2x\sqrt{1-x^2}\,dx$

Let $u=1-x^2$, then $du=-2x\,dx \Rightarrow -du=2x\,dx$

Substituting: $\int 2x\sqrt{1-x^2}\,dx=\int \sqrt{u}\,(-du)=-\int u^{1/2}du=-\frac{2}{3}u^{3/2}+C$

Substituting back in terms of x: $\int 2x\sqrt{1-x^2}\,dx=-\frac{2}{3}\sqrt{\left(1-x^2\right)^3}+C$

2. $\int x^2\left(x^3+2\right)^4dx$

Let $u=x^3+2$, then $du=3x^2\,dx \Rightarrow \frac{1}{3}du=x^2\,dx$

Substituting: $\int x^2\left(x^3+2\right)^4dx=\int u^4\left(\frac{1}{3}du\right)=\frac{1}{3}\int u^4du=\frac{1}{3}\left(\frac{1}{5}u^5\right)+C=\frac{1}{15}u^5+C$

Substituting back in terms of x: $\int x^2\left(x^3+2\right)^4dx=\frac{1}{15}\left(x^3+2\right)^5+C$

3. $\int x\,\mathrm{e}^{x^2+1}dx$

Let $u=x^2+1$, then $du=2x\,dx \Rightarrow \frac{1}{2}du=x\,dx$

Substituting: $\int x\,\mathrm{e}^{x^2+1}dx=\int \mathrm{e}^u\left(\frac{1}{2}du\right)=\frac{1}{2}\int \mathrm{e}^u du=\frac{1}{2}\mathrm{e}^u+C$

Substituting back in terms of x: $\int x\,\mathrm{e}^{x^2+1}dx=\frac{1}{2}\mathrm{e}^{x^2+1}+C$

4. $\int \frac{x^2}{5-x^3}dx$

Let $u=5-x^3$, then $du=-3x^2\,dx \Rightarrow -\frac{1}{3}du=x^2dx$

Substituting: $\int \frac{x^2}{5-x^3}dx=\int \frac{1}{u}\left(-\frac{1}{3}du\right)=-\frac{1}{3}\int \frac{1}{u}du=-\frac{1}{3}\ln|u|+C$

Substituting back in terms of x: $\int \frac{x^2}{5-x^3}dx=-\frac{1}{3}\ln\left|5-x^3\right|+C$

5. $\int x\sin\left(x^2\right)dx$

Let $u=x^2$, then $du=2x\,dx \Rightarrow \frac{1}{2}du=x\,dx$

Substituting: $\int x\sin\left(x^2\right)dx=\int \sin u\left(\frac{1}{2}du\right)=\frac{1}{2}\int \sin u\,du=\frac{1}{2}(-\cos u)+C=-\frac{1}{2}\cos u+C$

Substituting back in terms of x: $\int x\sin\left(x^2\right)dx=-\frac{1}{2}\cos\left(x^2\right)+C$

Integration by Substitution - 2

Worked Solutions (continued)

6. $\int \frac{3x}{(x^2+6)^3}\,dx$

Let $u = x^2+6$, then $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

Substituting: $\int \frac{3x}{(x^2+6)^3}\,dx = 3\int \frac{1}{u^3}\left(\frac{1}{2}du\right) = \frac{3}{2}\int u^{-3}\,du = \frac{3}{2}\left(-\frac{1}{2}u^{-2}\right)+C = -\frac{3}{4u^2}+C$

Substituting back in terms of x: $\int \frac{3x}{(x^2+6)^3}\,dx = -\frac{3}{4(x^2+6)^2}+C$

7. $\int x\sqrt[3]{8-x^2}\,dx$

Let $u = 8-x^2$, then $du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$

Substituting: $\int x\sqrt[3]{8-x^2}\,dx = \int \sqrt[3]{u}\left(-\frac{1}{2}du\right) = -\frac{1}{2}\int u^{1/3}du = -\frac{1}{2}\left(-\frac{3}{4}u^{4/3}\right)+C = -\frac{3}{8}\sqrt[3]{u^4}+C$

Substituting back in terms of x: $\int x\sqrt[3]{8-x^2}\,dx = -\frac{3}{8}\sqrt[3]{(8-x^2)^4}+C$

8. $\int e^{\sin x}\cos x\,dx$

Let $u = \sin x$, then $du = \cos x\,dx$

Substituting: $\int e^{\sin x}\cos x\,dx = \int e^u\,du = e^u + C$

Substituting back in terms of x: $\int e^{\sin x}\cos x\,dx = e^{\sin x}+C$

9. $\int \frac{\sin x}{\cos x}\,dx$

Let $u = \cos x$, then $du = -\sin x\,dx \Rightarrow -du = \sin x\,dx$

Substituting: $\int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{u}(-du) = -\int \frac{1}{u}du = -\ln|u|+C$

Substituting back in terms of x: $\int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x|+C$

10. $\int \frac{\ln x}{x}\,dx$

Let $u = \ln x$, then $du = \frac{1}{x}dx$

Substituting: $\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{1}{2}u^2+C$

Substituting back in terms of x: $\int \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2+C$