H

National
Qualifications
2018

2018 Mathematics

Higher - Paper 2

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•¹ state an integral to represent the shaded area	•¹ $\int_{-1}^{3}\left(3+2x-x^2\right)dx$	4
	•² integrate	•² $3x+\frac{2x^2}{2}-\frac{x^3}{3}$	
	•³ substitute limits	•³ $\left(3\times3+\frac{2\times3^2}{2}-\frac{3^3}{3}\right)-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$	
	•⁴ evaluate integral	•⁴ $\frac{32}{3}$ (units²)	

Notes:

1. •¹ is not available to candidates who omit 'dx'.
2. Limits must appear at the •¹ stage for •¹ to be awarded.
3. Where a candidate differentiates one or more terms at •², then •³ and •⁴ are unavailable.
4. Candidates who substitute limits without integrating, do not gain •³ or •⁴.
5. Do not penalise the inclusion of '$+c$'.
6. Do not penalise the continued appearance of the integral sign after •¹.
7. If •⁴ is only given as a decimal then it must be given correct to 1 decimal place.

Commonly Observed Responses:

Candidate A		Candidate B	
$\int_{-1}^{3} 3+2x-x^2$	•¹ ✗	$\int\left(3+2x-x^2\right)dx$	•¹ ✗
$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓	$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓
	•³ ^	$=9-\left(-\frac{5}{3}\right)$	•³ ✓
$=\frac{32}{3}$	•⁴ $\boxed{✓ 1}$	$=\frac{32}{3}$	•⁴ ✓

Commonly Observed Responses:	
Candidate C $\int \left(3+2x-x^2\right)dx$ •[1] ✘ $= 3x+\frac{2x^2}{2}-\frac{x^3}{3}$ •[2] ✓ $=\left(3\times 3+\frac{2\times 3^2}{2}-\frac{3^3}{3}\right)$ $-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$ •[3] ✓ $=\frac{32}{3}$ •[4] ✓	**Candidate D** $\int_3^{-1}\left(3+2x-x^2\right)dx$ •[1] ✓ … •[2] ✓•[3] ✓ $=-\frac{32}{3}$, hence area is $\frac{32}{3}$ •[4] ✓ However $-\frac{32}{3}=\frac{32}{3}$ does not gain •[4].

Question		Generic scheme	Illustrative scheme	Max mark
2.	(a)	•¹ find $\mathbf{u.v}$	•¹ 24	**1**

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•² find $\lvert\mathbf{u}\rvert$ •³ find $\lvert\mathbf{v}\rvert$ •⁴ apply scalar product •⁵ calculate angle	•² $\sqrt{26}$ •³ $\sqrt{138}$ •⁴ $\cos\theta° = \frac{24}{\sqrt{26}\sqrt{138}}$ •⁵ $66\cdot38...°$ or $1\cdot16...$radians	**4**

Notes:

1. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{-1^2+4^2-3^2}=\sqrt{26}$ or $\sqrt{-1^2+4^2+-3^2}=\sqrt{26}$, •² is awarded.
2. •⁴ is not available to candidates who simply state the formula $\cos\theta° = \frac{\mathbf{u.v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$.
3. Accept answers which round to $66°$ or $1\cdot2$ radians (or 73·8 gradians).
4. Do not penalise the omission or incorrect use of units.
5. •⁵ is only available for a single angle.
6. For a correct answer with no working award 0/4.

Commonly Observed Responses:

Candidate A

$\lvert\mathbf{u}\rvert=\sqrt{26}$	•² ✓
$\lvert\mathbf{v}\rvert=\sqrt{138}$	•³ ✓
$\frac{24}{\sqrt{26}\sqrt{138}}$	•⁴ ^
$\theta=66\cdot38...°$	•⁵ $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ differentiate $\bullet^2$ evaluate derivative at $x=2$ $\bullet^3$ interpret result	$\bullet^1$ $3x^2-7$ $\bullet^2$ 5 $\bullet^3$ $(f$ is$)$ increasing	**3**

Notes:

1. $\bullet^2$ and $\bullet^3$ are only available as a consequence of working with a derivative.
2. Accept $f'(2)>0$ for $\bullet^2$.
3. $f'(x)>0$ with no evidence of evaluating the derivative at $x=2$ does not gain $\bullet^2$ or $\bullet^3$. See candidate B.
4. Do not penalise candidates who use y in place of $f(x)$.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$3x^2-7$	$\bullet^1$ ✓	$3x^2-7$	$\bullet^1$ ✓
$\begin{array}{c\|c} x & 2 \\ \hline f'(x) & + \end{array}$	$\bullet^2$ ✓	$f'(x)>0$	$\bullet^2$ ^
increasing	$\bullet^3$ ✓	f is increasing	$\bullet^3$ ^

Question	Generic scheme	Illustrative scheme	Max mark
4.	**Method 1**	**Method 1**	3
	•1 identify common factor	•1 $-3\left(x^2+2x\ldots\right.$ stated or implied by •2	
	•2 complete the square	•2 $-3(x+1)^2\ldots$	
	•3 process for c	•3 $-3(x+1)^2+10$	
	Method 2	**Method 2**	
	•1 expand completed square form	•1 $ax^2+2abx+ab^2+c$	
	•2 equate coefficients	•2 $a=-3,\ 2ab=-6\ \ ab^2+c=7$	
	•3 process for b and c and write in required form	•3 $-3(x+1)^2+10$	

Notes:

1. $-3(x+1)^2+10$ with no working gains •1 and •2 only; however, see Candidate E.
2. •3 is only available for a calculation involving both multiplication and addition of integers.

Commonly Observed Responses:

Candidate A		Candidate B	
$-3\left(x^2+2\right)+7$	exception in General marking principle (h)	$-3\left(\left(x^2-6x\right)+7\right)$	•1 ✗
$-3\left(\left(x+1\right)^2-1\right)+7$	•1 ✓ •2 ✓	$-3\left(\left(x-3\right)^2-9\right)+7$	•2 ✓ 1
$-3(x+1)^2+10$	•3 ✓	$-3(x-3)^2+34$	•3 ✓ 1

Candidate C		Candidate D	
$a(x+b)^2+c=ax^2+2abx+ab^2+c$	•1 ✓	$ax^2+2abx+ab^2+c$	•1 ✓
$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$	•2 ✓	$a=-3,\ \ 2ab=-6,\ \ ab^2+c=7$	•2 ✓
$b=1,\ \ c=10$	•3 ✓	$b=1,\ \ c=10$	•3 ✗
•3 is awarded as all working relates to completed square form		•3 is lost as no reference is made to completed square form	

Commonly Observed Responses:	
Candidate E $-3(x+1)^2+10$ Check: $=-3(x^2+2x+1)+10$ $=-3x^2-6x-3+10$ $=-3x^2-6x+7$ Award 3/3	**Candidate F** $-3x^2-6x+7$ $=-3(x+1)^2-1+7$ $\bullet^1$ ✓ $\bullet^2$ ✓ $=-3(x+1)^2+6$ $\bullet^3$ ✗
Candidate G $-3x^2-6x+7$ $=x^2+2x-\frac{7}{3}$ $\bullet^1$ ✗ $=(x+1)^2-\frac{10}{3}$ $\bullet^2$ ✗ $=-3(x+1)^2+10$ $\bullet^3$ ✗	

Question		Generic scheme	Illustrative scheme	Max mark
5.	(a)	•¹ find the midpoint of PQ •² calculate m_{PQ} and state perp. gradient •³ find equation of L_1 in a simplified form	•¹ $(6,1)$ •² $-1 \Rightarrow m_{perp} = 1$ •³ $y = x - 5$	**3**

Notes:

1. •³ is only available as a consequence of using a perpendicular gradient and a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at •² or •³ stage for •³ to be awarded.
3. At •³, accept $x - y - 5 = 0$, $y - x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•⁴ determine y coordinate •⁵ state x coordinate	•⁴ 5 •⁵ 10	**2**

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(c)	•⁶ calculate radius of the circle •⁷ state equation of the circle	•⁶ $\sqrt{50}$ stated or implied by •⁷ •⁷ $(x-10)^2 + (y-5)^2 = 50$	**2**

Notes:

4. Where candidates have calculated the coordinates of C incorrectly, •⁶ and •⁷ are available for using either PC or QC for the radius.
5. Where incorrect coordinates for C appear without working, only •⁷ is available.
6. Do not accept $\left(\sqrt{50}\right)^2$ for •⁷.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(i)	$\bullet^1$ start composite process $\bullet^2$ substitute into expression	$\bullet^1$ $f(2x)$ $\bullet^2$ $3+\cos 2x$	2
		(ii)	$\bullet^3$ state second composite	$\bullet^3$ $2(3+\cos x)$	1

Notes:

1. For $3+\cos 2x$ without working, award both $\bullet^1$ and $\bullet^2$.
2. Candidates who interpret the composite function as either $g(x)\times f(x)$ or $g(x)+f(x)$ do not gain any marks.

Commonly Observed Responses:

Candidate A - interpret $f(g(x))$ as $g(f(x))$		**Candidate B - interpret $f(g(x))$ as $g(f(x))$**	
(i) $2(3+\cos x)$	$\bullet^1$ ✗ $\bullet^2$ ✓ 1	(i) $f(2x)=2(3+\cos x)$	$\bullet^1$ ✓ $\bullet^2$ ✗
(ii) $3+\cos 2x$	$\bullet^3$ ✓ 1	(ii) $3+\cos(2x)$	$\bullet^3$ ✓ 1

Question	Generic scheme	Illustrative scheme	Max mark
6. (b)	$\bullet^4$ equate expressions from (a) $\bullet^5$ substitute for $\cos 2x$ in equation $\bullet^6$ arrange in standard quadratic form $\bullet^7$ factorise $\bullet^8$ solve for $\cos x$ $\bullet^9$ solve for x	$\bullet^4$ $3+\cos 2x = 2(3+\cos x)$ $\bullet^5$ $3+2\cos^2 x - 1 = 2(3+\cos x)$ $\bullet^6$ $2\cos^2 x - 2\cos x - 4 = 0$ $\bullet^7$ $2(\cos x - 2)(\cos x + 1)$ $\bullet^8$ $\bullet^9$ $\bullet^8$ $\cos x = 2$ $\cos x = -1$ $\bullet^9$ $\cancel{\cos x = 2}$ or eg 'no solution' $x = \pi$	**6**

Notes:

3. Do not penalise absence of common factor at $\bullet^7$.
4. $\bullet^5$ cannot be awarded until the equation reduces to a quadratic in $\cos x$.
5. Substituting $2\cos^2 \mathrm{A} - 1$ or $2\cos^2 \alpha - 1$ at $\bullet^5$ stage should be treated as bad form provided the equation is written in terms of x at $\bullet^6$ stage. Otherwise, $\bullet^5$ is not available.
6. '$=0$' must appear by $\bullet^7$ stage for $\bullet^6$ to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at $\bullet^6$ stage for $\bullet^6$ to be awarded.
7. For candidate who do not arrange in standard quadratic form, eg $-2\cos x + 2\cos^2 x - 4 = 0$ $\bullet^6$ is only available if $\bullet^7$ has been awarded.
8. $\bullet^7$ $\bullet^8$ and $\bullet^9$ are only available as a consequence of solving a quadratic with distinct real roots.
9. $\bullet^7$ $\bullet^8$ and $\bullet^9$ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. $\bullet^9$ is not available to candidates who work in degrees and do not convert their solution(s) into radian measure.
11. Answers written as decimals should be rounded to no fewer than 2 significant figures.
12. $\bullet^9$ is not available for any solution containing angles outwith the interval $0 \le x < 2\pi$.

Commonly Observed Responses:	
Candidate C **Quadratic expressed in terms of c or x.** $3+\cos 2x = 2(3+\cos x)$ $\bullet^4$ ✓ $3+2\cos^2 x - 1 = 2(3+\cos x)$ $\bullet^5$ ✓ $2\cos^2 x - 2\cos x - 4 = 0$ $\bullet^6$ ✓ $2c^2 - 2c - 4 = 0$ $2(c-2)(c+1) = 0$ $\bullet^7$ ✓ $c = 2$, $c = -1$ $\bullet^8$ ✗ no solution, $x = \pi$ $\bullet^9$ ✓ However, $\bullet^4$ ✓ $\bullet^5$ ✓ $\bullet^6$ ✓ $2(c-2)(c+1) = 0$ $\bullet^7$ ✓ $\cos x = 2$ $\cos x = -1$ $\bullet^8$ ✓ Solution stated in terms of $\cos x$ explicitly	**Candidate D** $3+\cos 2x = 2(3+\cos x)$ $\bullet^4$ ✓ $3+2\cos^2 x - 1 = 2(3+\cos x)$ $\bullet^5$ ✓ $2\cos^2 x - 2\cos x = 4$ $\bullet^6$ ^ $\cos^2 x - \cos x = 2$ $\cos x(\cos x - 1) = 2$ $\bullet^7$ $\boxed{✓\ 2}$ $\cos x = 2,\ \cos x - 1 = 2$ $\cos x = 2,\ \cos x = 3$ $\bullet^8$ ✗ no solutions $\bullet^9$ ✗ see note 9
Candidate E - reading $\cos 2x$ as $\cos^2 x$ $3+\cos^2 x = 2(3+\cos x)$ $\bullet^4$ ✗ $\bullet^5$ ^ - no substitution required $\cos^2 x - 2\cos x - 3 = 0$ $\bullet^6$ $\boxed{✓\ 1}$ $(\cos x - 3)(\cos x + 1)$ $\bullet^7$ $\boxed{✓\ 1}$ $\cos x = 3$, $\cos x = -1$ $\bullet^8$ $\boxed{✓\ 1}$ no solution, $x = \pi$ $\bullet^9$ $\boxed{✓\ 1}$	**Candidate F - using quadratic formula** ^{4} ✓ $\bullet^5$ ✓ $2\cos^2 x - 2\cos x - 4 = 0$ $\bullet^6$ ✓ $\cos x = \dfrac{2 \pm \sqrt{36}}{4}$ or $\cos x = \dfrac{1 \pm \sqrt{9}}{2}$ $\bullet^7$ ✓

Question	Generic scheme	Illustrative scheme	Max mark
7. (a) (i)	•[1] use ‘2’ in synthetic division or in evaluation of cubic •[2] complete division/evaluation and interpret result	•[1] $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & & & \\ \hline & 2 & & & \end{array}$ or $2\times(2)^3-3(2)^2-3\times(2)+2$ •[2] $\begin{array}{r\|rrr\|r} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & 0 \end{array}$ Remainder $=0$ $\therefore$ $(x-2)$ is a factor or $f(2)=0$ $\therefore$ $(x-2)$ is a factor	**2**
(ii)	•[3] state quadratic factor •[4] complete factorisation	•[3] $2x^2+x-1$ •[4] $(x-2)(2x-1)(x+1)$ stated explicitly	**2**

Notes:

1. Communication at •[2] must be consistent with working at that stage i.e. a candidate’s working must arrive legitimately at 0 before •[2] can be awarded.
2. Accept any of the following for •[2] :
   - ‘ $f(2)=0$ so $(x-2)$ is a factor’
   - ‘since remainder = 0, it is a factor’
   - the 0 from any method linked to the word ‘factor’ by e.g. ‘so’, ‘hence’, ‘$\therefore$’, ‘$\rightarrow$’, ‘$\Rightarrow$’
3. Do not accept any of the following for •[2]:
   - double underlining the zero or boxing the zero without comment
   - ‘ $x=-2$ is a factor’, ‘$(x+2)$ is a factor’, ‘$(x+2)$ is a root’, ‘ $x=2$ is a root’, ‘$(x-2)$ is a root’, ‘ $x=-2$ is a root’
   - the word ‘factor’ only, with no link.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
7. (b)	•[5] demonstrate result	•[5] $u_6=a(2a-3)-1=2a^2-3a-1$ leading to $u_7=a(2a^2-3a-1)-1$ $=2a^3-3a^2-a-1$	**1**

Notes:

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
7. (c) (i)	•6 equate u_5 and u_7 and arrange in standard form •7 solve cubic •8 discard invalid solutions for a	•6 $2a^3 - 3a^2 - 3a + 2 = 0$ •7 $a = 2,\ a = \frac{1}{2},\ a = -1$ •8 $a = \frac{1}{2}$	3
(ii)	•9 calculate limit	•9 -2	1

Notes:

4. Where •6 has been awarded, •7 is available for solutions in terms of x appearing in a(ii). However, see Candidates B and C. BEWARE: Candidates who make a second attempt at factorising the cubic obtained in c(i) and do so incorrectly cannot be awarded mark 7 for solutions appearing in a(ii).
5. •8 is only available as a result of a valid strategy at •7.
6. $x = \frac{1}{2}$ does not gain •8.
7. For candidates who do not state the cubic equation at •6, and adopt a guess and check approach, using solutions for x found in a(ii), may gain 3/3. See Candidate D.

Commonly Observed Responses:

Candidate A

$2a^3 - 3a^2 - 3a + 2 = 0$ •6 ✓

$x = 2,\ x = \frac{1}{2},\ x = -1$ in a(ii) •7 ✓ •8 ^

Candidate B - missing '$=0$' from equation

$2a^3 - 3a^2 - 3a + 2$ •6 ^

$\cancel{x=2},\ x = \frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 $\boxed{✓ 1}$

$a = \frac{1}{2}$ •8 $\boxed{✓ 1}$

Candidate C - missing '$=0$' from equation

$2a^3 - 3a^2 - 3a + 2$ •6 ^

$\cancel{x=2},\ x = \frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 ^

$\frac{1}{2}$ •8 ^

No clear link between a and x.

Candidate D - $x = -1,\ x = \frac{1}{2}$ and $x = 2$ identified in a(ii)

$u_5 = 2\left(\frac{1}{2}\right) - 3 = -2$ •6 ✓

$u_7 = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -2$ •7 ✓

$a = \frac{1}{2}$ because $-1 < a < 1$ •8 ✓

Question		Generic scheme	Illustrative scheme	Max mark
8.	(a)	•1 use compound angle formula •2 compare coefficients •3 process for k •4 process for a and express in required form	•1 $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ stated explicitly •2 $k\cos a^\circ = 2$ and $k\sin a^\circ = -1$ stated explicitly •3 $k=\sqrt{5}$ •4 $\sqrt{5}\cos(x-333\cdot4\ldots)^\circ$	4

Notes:

1. Accept $k(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ)$ for •1. Treat $k\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ as bad form only if the equations at the •2 stage both contain k.
2. Do not penalise the omission of degree signs.
3. $\sqrt{5}\cos x^\circ\cos a^\circ + \sqrt{5}\sin x^\circ\sin a^\circ$ or $\sqrt{5}(\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ)$ is acceptable for •1 and •3.
4. •2 is not available for $k\cos x^\circ = 2$, $k\sin x^\circ = -1$, however •4 may still be gained.
5. •3 is only available for a single value of k, $k>0$.
6. •4 is not available for a value of a given in radians.
7. Accept any value of a which rounds to 333°
8. Candidates may use any form of the wave function for •1, •2 and •3, however, •4 is only available if the wave is interpreted in the form $k\cos(x-a)^\circ$.
9. Evidence for •4 may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A

•1 ^

$\sqrt{5}\cos a^\circ = 2$

$\sqrt{5}\sin a^\circ = -1$ •2 ✓ •3 ✓

$\tan a^\circ = -\dfrac{1}{2}$

$a = 333\cdot4$

$\sqrt{5}\cos(x-333\cdot4)^\circ$ •4 ✓

Candidate B

$k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ •1 ✓

$\cos a^\circ = 2$

$\sin a^\circ = -1$ •2 ✗

$\tan a^\circ = -\dfrac{1}{2}$

$a = 333\cdot4$ Not consistent with equations at •2.

$\sqrt{5}\cos(x-333\cdot4)^\circ$ •3 ✓ •4 ✗

Candidate C

$\cos x^\circ\cos a^\circ + \sin x^\circ\sin a^\circ$ •1 ✗

$\cos a^\circ = 2$

$\sin a^\circ = -1$ •2 ✗

$k=\sqrt{5}$ •3 ✓

$\tan a^\circ = -\dfrac{1}{2}$

$a = 333\cdot4$

$\sqrt{5}\cos(x-333\cdot4)^\circ$ •4 ✗

Responses with the correct expansion of $k\cos(x-a)^\circ$ but errors for either $\bullet^2$ or $\bullet^3$:		
Candidate D $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ $\bullet^1$ ✓ $k\cos a^\circ = 2$ $k\sin a^\circ = -1$ $\bullet^2$ ✓ $\tan a^\circ = -2$ $\bullet^3$ ^ $\bullet^4$ ✗ $a = 296\cdot6$	**Candidate E** $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ $\bullet^1$ ✓ $k\cos a^\circ = -1$ $k\sin a^\circ = 2$ $\bullet^2$ ✗ $\tan a^\circ = -2$ $a = 116\cdot6$ $\sqrt{5}\cos(x-116\cdot6)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$	**Candidate F** $k\cos x^\circ\cos a^\circ + k\sin x^\circ\sin a^\circ$ $\bullet^1$ ✓ $k\cos a^\circ = 2$ $k\sin a^\circ = 1$ $\bullet^2$ ✗ $\tan a^\circ = \frac{1}{2}$ $a = 26\cdot6$ $\sqrt{5}\cos(x-26\cdot6)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$
Commonly Observed Responses:		
Responses with the incorrect labelling, $k(\cos A\cos B + \sin A\sin B)$ from the formula list:		
Candidate G $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos a^\circ = 2$ $k\sin a^\circ = -1$ $\bullet^2$ ✓ $\tan a^\circ = -\frac{1}{2}$ $a = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)^\circ$ $\bullet^3$ ✓ $\bullet^4$ ✓	**Candidate H** $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos x^\circ = 2$ $k\sin x^\circ = -1$ $\bullet^2$ ✗ $\tan x^\circ = -\frac{1}{2}$ $x = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$	**Candidate I** $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos B^\circ = 2$ $k\sin B^\circ = -1$ $\bullet^2$ ✗ $\tan B^\circ = -\frac{1}{2}$ $B = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)	(i)	•5 state minimum value	•5 $-3\sqrt{5}$ or $-\sqrt{45}$	1
		(ii)	**Method 1** •6 start to solve •7 state value of x **Method 2** •6 start to solve •7 state value of x	**Method 1** •6 $x-333\cdot4=180$ leading to $x=513\cdot4$ •7 $x=153\cdot4...$ **Method 2** •6 $x-333\cdot4=-180$ •7 $x=153\cdot4...$	2

Notes:

10. •7 is only available for a single value of x.
11. •7 is only available in cases where $a<-180$ or $a>180$. See Candidate J

Commonly Observed Responses:

Candidate J - from $\sqrt{5}\cos(x-26\cdot6)°$ $x-26\cdot6=180$ $x=206\cdot6$ •6 ✓ 1 •7 ✓ 2 Similarly for $\sqrt{5}\cos(x-116\cdot6)°$	**Candidate K - from 'minimum' of eg $-\sqrt{5}$** $3\sqrt{5}\cos(x-333\cdot4)°=-\sqrt{5}$ $\cos(x-333\cdot4)°=-\frac{1}{3}$ $x-333\cdot4=109\cdot5,\ 250\cdot5$ $x=442\cdot9,\ 583\cdot9$ •6 ✓ 1 $x=82\cdot9,\ 223\cdot9$ •7 ✗

Question	Generic scheme	Illustrative scheme	Max mark
9.	$\bullet^1$ express P in differentiable form	$\bullet^1$ $2x+128x^{-1}$	6
	$\bullet^2$ differentiate	$\bullet^2$ $2-\dfrac{128}{x^2}$	
	$\bullet^3$ equate expression for derivative to 0	$\bullet^3$ $2-\dfrac{128}{x^2}=0$	
	$\bullet^4$ process for x	$\bullet^4$ 8	
	$\bullet^5$ verify nature	$\bullet^5$ table of signs for a derivative (see next page) $\therefore$ minimum or $P''(8)=\dfrac{1}{2}>0$ $\therefore$ minimum	
	$\bullet^6$ evaluate P	$\bullet^6$ $P=32$ or min value $=32$	

Notes:

1. For a numerical approach award 0/6.
2. For candidates who integrate any term at the $\bullet^2$ stage, only $\bullet^3$ is available on follow through for setting their 'derivative' to 0.
3. $\bullet^4$, $\bullet^5$ and $\bullet^6$ are only available for working with a derivative which contains an index ≤ -2.
4. At $\bullet^2$ accept $2-128x^{-2}$.
5. Ignore the appearance of -8 at $\bullet^4$.
6. $\sqrt{\dfrac{128}{2}}$ must be simplified at $\bullet^4$ or $\bullet^5$ for $\bullet^4$ to be awarded.
7. $\bullet^5$ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 8.
8. $\bullet^6$ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at $\bullet^5$.
9. $\bullet^5$ and $\bullet^6$ are not available to candidates who state that the minimum exists at a negative value of x.

Commonly Observed Responses:

Candidate A - differentiating over more than one line		Candidate B - differentiating over more than one line	
	$\bullet^1$ ^	$P(x)=2x+128x^{-1}$	$\bullet^1$ ✓
$P'(x)=2+128x^{-1}$		$P'(x)=2+128x^{-1}$	
$P'(x)=2-128x^{-2}$	$\bullet^2$ ✗	$P'(x)=2-128x^{-2}$	$\bullet^2$ ✗
$2-128x^{-2}=0$	$\bullet^3$ $\boxed{✓ 1}$	$2-128x^{-2}=0$	$\bullet^3$ $\boxed{✓ 1}$

Table of signs for a derivative

Accept:

x	-8^-	-8	-8^+
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	8^-	8	8^+
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

x	$\rightarrow$	-8	$\rightarrow$
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	8	$\rightarrow$
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Here, for exemplification, tables of signs considering both roots separately have been displayed. However, in this question, it was only expected that candidates would consider the positive root for $\bullet^5$. Do not penalise the consideration of the negative root.

Arrows are taken to mean 'in the neighbourhood of'

x	a	-8	b	c	8	d
$P'(x)$	$+$	0	$-$	$-$	0	$+$
Shape or Slope	/	—	\	\	—	/

Where: $a < -8$, $-8 < b < 0$, $0 < c < 8$, $d > 8$

Do not Accept:

x	a	-8	b	8	c
$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8 < b < 8$' is not acceptable.

$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8 \rightarrow 8$' is not acceptable.

General Comments:

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $P'(x)$ in the table is an acceptable alternative to writing '$+$' or '$-$' signs. Values must be checked for accuracy.
  - The only acceptable variations of $P'(x)$ are: P', $\frac{dP}{dx}$ and $2-\frac{128}{x^2}$.

Question	Generic scheme	Illustrative scheme	Max mark
10.	•[1] use the discriminant •[2] identify roots of quadratic expression •[3] apply condition •[4] state range with justification	•[1] $(m-3)^2-4\times1\times m$ •[2] 1, 9 •[3] $(m-3)^2-4\times1\times m>0$ •[4] $m<1,\ m>9$ with eg sketch or table of signs	4

Notes:

1. If candidates have the condition '$\text{discriminant}<0$', '$\text{discriminant}\leq0$' or '$\text{discriminant}\geq0$', then •[3] is lost but •[4] is available.
2. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
3. For candidates who have identified expressions for a, b, and c and then state $b^2-4ac>0$ award •[3]. See Candidate A.
4. For the appearance of x in any expression for •[1], award 0/4.

Commonly Observed Responses:

Candidate A

$(m-3)^2-4\times1\times m$	•[1] ✓
$m^2-10m+9=0$	
$m=1,\ m=9$	•[2] ✓
$b^2-4ac>0$	•[3] ✓
$m<1,\ m>9$	•[4] ^

Expressions for a, b, and c implied at •[1]

Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•1 substitute for P and t •2 arrange equation in the form $A = e^{kt}$ •3 simplify •4 solve for k	•1 $50 = 100\left(1 - e^{3k}\right)$ •2 $0 \cdot 5 = e^{3k}$ or $-0 \cdot 5 = -e^{3k}$ •3 $\ln 0 \cdot 5 = 3k$ •4 $k = -0 \cdot 231$	**4**

Notes:

1. •2 may be assumed by •3.
2. Any base may be used at •3 stage. See Candidate D.
3. Accept any answer which rounds to $-0 \cdot 2$.
4. •3 must be consistent with the equation of the form $A = e^{kt}$ at its first appearance.
5. For candidates whose working would (or should) arrive at $\log(\text{negative})$ •4 is not available.
6. Where candidates use a ‘rule’ masquerading as a law of logarithms, •3 and •4 is not available.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$50 = 100\left(1 - e^{3k}\right)$	•1 ✓	$0 \cdot 5 = 100\left(1 - e^{3k}\right)$	•1 ✗
$0 \cdot 5 = -e^{3k}$	•2 ✗	$0 \cdot 995 = e^{3k}$	•2 $\boxed{✓\ 1}$
$\ln(0 \cdot 5) = 3k$	•3 ✗	$\ln(0 \cdot 995) = 3k$	•3 $\boxed{✓\ 1}$
$k = -0 \cdot 231$	•4 ✗	$k = -0 \cdot 0017\ldots$	•4 $\boxed{✓\ 1}$
68·5	•5 $\boxed{✓\ 1}$	$P = 0 \cdot 8319\ldots$	•5 $\boxed{✓\ 1}$
31·5% still queueing	•6 $\boxed{✓\ 1}$	99·2% still queuing	•6 $\boxed{✓\ 1}$
Candidate C		**Candidate D**	
$50 = 100\left(1 - e^{3k}\right)$	•1 ✓	$50 = 100\left(1 - e^{3k}\right)$	•1 ✓
$-0 \cdot 5 = -e^{3k}$	•2 ✓	$0 \cdot 5 = e^{3k}$	•2 ✓
$\ln(-0 \cdot 5) = \ln\left(-e^{3k}\right)$	•3 ✗	$\log_{10}(0 \cdot 5) = 3k \log_{10} e$	•3 ✓
$k = -0 \cdot 231$	•4 ✗	$k = -0 \cdot 231$	•4 ✓
68·5	•5 $\boxed{✓\ 1}$		
31·5% still queueing	•6 $\boxed{✓\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•5 evaluate P for $t = 5$ •6 interpret result	•5 68·5 •6 31·5% still queueing	**2**

Notes:

7. •5 and •6 are not available where $k \geq 0$.
8. •6 is only available where the value of P in •5 was obtained by substituting into an exponential expression.

Commonly Observed Responses:

Candidate D - $k = -0 \cdot 2$	
63·2	•5 ✓
36·8% still queueing	•6 ✓

Question	Generic scheme	Illustrative scheme	Max mark
12. (a) (i)	•[1] write down coordinates of centre	•[1] $(13,-4)$	**1**
(ii)	•[2] substitute coordinates and process for c	•[2] $13^2+(-4)^2+14\times13-22\times(-4)\ldots$ leading to $c=-455$	**1**

Notes:

1. Accept $x=13$, $y=-4$ for •[1].
2. Do not accept $g=13$, $f=-4$ or $13,\,-4$ for •[1].
3. For those who substitute into $r=\sqrt{g^2+f^2-c}$, working to find r must be shown for •[2] to be awarded.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b) (i)	•[3] calculate two key distances •[4] state ratio	•[3] two from $r_2=25$, $r_1=10$ and $r_2-r_1=15$ •[4] $3:2$ or $2:3$	**2**
(ii)	•[5] identify centre of C_2 •[6] state coordinates of P	•[5] $(-7,11)$ or $\begin{pmatrix}-7\\11\end{pmatrix}$ •[6] $(5,2)$	**2**

Notes:

4. The ratio must be consistent with the working for r_2-r_1
5. Evidence for •[3] may appear on a sketch.
6. For $3:2$ or $2:3$ with no working, award 0/2.
7. At •[6], the ratio used to identify the coordinates of P must be consistent with the sizes of the circles in the original diagram for •[6] to be available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(c)	•[7] state equation	•[7] $(x-5)^2+(y-2)^2=1600$ or $x^2+y^2-10x-4y-1571=0$	**1**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]