EQUATION SHEET

Circle Geometry

Diameter: $d = 2r$

Circumference: $C = 2\pi r = \pi d$

Area: $A = \pi r^2$

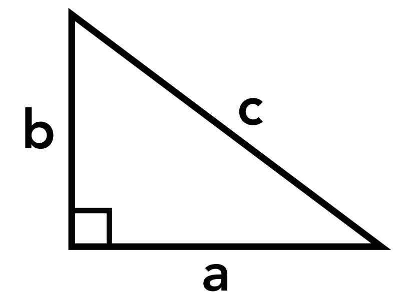
1 revolution = 360° = 2π radians

Triangle Geometry

Area: $A = \frac{1}{2}bh$

Angles: $\theta_1 + \theta_2 + \theta_3 = 180°$

Pythagorean theorem: $c^2 = a^2 + b^2$



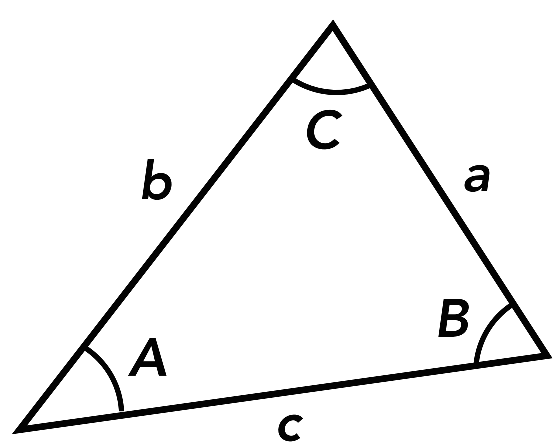
Trig identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

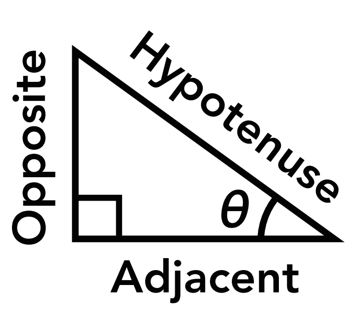


Law of sines

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) \qquad \theta = \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) \qquad \theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

Quadratic Formula

Quadratic formula for

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1D and 2D Kinematics

Average speed

$$v_{avg} = \frac{\text{total distance}}{\text{total time}}$$

delta

$$\Delta = \text{final} - \text{initial}$$

$$\Delta x = x_f - x_i$$

or

$$\Delta x = x - x_0$$

Variables		SI Unit
t	time	s
x	horizontal position	m
y	vertical position	m
v	velocity	$\frac{m}{s}$
a	acceleration	$\frac{m}{s^2}$

	Horizontal motion:	Vertical motion:
Displacement:	$\Delta x = x_f - x_i$	$\Delta y = y_f - y_i$
Velocity:	$v_x = \frac{\Delta x}{\Delta t}$	$v_y = \frac{\Delta y}{\Delta t}$
Velocity (rearranged):	$x_f = x_i + v_x \Delta t$	$y_f = y_i + v_y \Delta t$
Acceleration:	$a_x = \frac{\Delta v_x}{\Delta t}$	$a_y = \frac{\Delta v_y}{\Delta t}$
Acceleration (rearranged):	$v_{xf} = v_{xi} + a_x \Delta t$	$v_{yf} = v_{yi} + a_y \Delta t$
Kinematic equations for constant acceleration:	$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$	$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$
	$v_{xf}^2 = v_{xi}^2 + 2 a_x (x_f - x_i)$	$v_{yf}^2 = v_{yi}^2 + 2 a_y (y_f - y_i)$

Subscripts		
i	0	initial
f	_	final
x		horizontal
y		vertical

Projectile Motion

Range

$$\Delta x = v_i \cos(\theta) \frac{v_i \sin(\theta) + \sqrt{(v_i \sin(\theta))^2 + 2 g y_i}}{g}$$

Range
(if $y_i = y_f$)

$$\Delta x = \frac{v_i^2 \sin(2\theta)}{g}$$

Circular and Rotational Motion

Variables		SI Unit
s	tangential position	m
Δs	tangential displacement	m
v_t	tangential velocity	$\frac{m}{s}$
a_t	tangential acceleration	$\frac{m}{s^2}$

Variables		SI Unit
θ	angular position	rad
$\Delta\theta$	angular displacement	rad
ω	angular velocity	$\frac{rad}{s}$
α	angular acceleration	$\frac{rad}{s^2}$

Conversion
(Angular variable must use radians)

	Circular motion (tangential description)	↔	Rotational motion (angular description)
Position:	s m	$s = r\,\theta$	θ rad
Displacement:	$\Delta s = s_f - s_i$ m	$\Delta s = r\,\Delta\theta$	$\Delta\theta = \theta_f - \theta_i$ rad
Velocity:	$v_t = \frac{\Delta s}{\Delta t}$ $\frac{m}{s}$	$v_t = r\,\omega$	$\omega = \frac{\Delta\theta}{\Delta t}$ $\frac{rad}{s}$
Acceleration:	$a_t = \frac{\Delta v_t}{\Delta t}$ $\frac{m}{s^2}$	$a_t = r\,\alpha$	$\alpha = \frac{\Delta\omega}{\Delta t}$ $\frac{rad}{s^2}$
Kinematic equations with acceleration:	$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2$		$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
	$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i)$		$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$

Newton's 2nd Law of Motion

Newton's 2nd law of motion

$\vec{F}_{net} = m\vec{a}$ or $\sum\vec{F} = m\vec{a}$

$\sum$: the sum of __

Variables		SI Unit
F	force	$N = \frac{kg \cdot m}{s^2}$
m	mass	kg
a	acceleration	$\frac{m}{s^2}$
v	velocity	$\frac{m}{s}$

Gravitational Force & Weight

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg \cdot s^2}$	gravitational constant

Newton's Law of Universal Gravitation (gravitational force)

$$F_g = \frac{Gm_1m_2}{r^2} = F_{1\text{ on }2} = F_{2\text{ on }1}$$

Gravitational field strength or acceleration due to gravity

$$g = \frac{GM}{r^2}$$

Gravitational force on mass in gravitational field

$$F_g = mg \quad = \quad F_g = \frac{GMm}{r^2}$$

Variables		SI Unit
F_g	gravitational force	N
w	weight force	N
m	mass	kg
M	mass producing a field	kg
r	distance between centers	m
g	gravitational acceleration	$\frac{m}{s^2}$

Weight force

$$F_g = mg \quad \text{or} \quad w = mg$$

Friction

Maximum static friction force

$$f_{s\text{ max}} = \mu_s F_n$$

μ_s : coefficient of static friction

Kinetic friction force

$$f_k = \mu_k F_n$$

μ_k : coefficient of kinetic friction

Rolling friction force

$$f_r = \mu_r F_n$$

μ_r : coefficient of kinetic friction

Variables		SI Unit
f_s	static friction force	N
f_k	kinetic friction force	N
f_r	rolling friction force	N
μ_s	coefficient of static friction	
μ_k	coefficient of kinetic friction	
μ_r	coefficient of rolling friction	
F_n	normal force	N

Spring Force

Spring force (Hooke's Law)

$$F_{sp} = k\Delta x$$

Equivalent spring constant for springs in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

Equivalent spring constant for springs in parallel

$$k_{eq} = k_1 + k_2 + \ldots$$

Variables		SI Unit
F_{sp}	spring force	N
Δx	displacement	m
k	spring constant	$\frac{N}{m}$

Elasticity of Materials

"Spring constant" for a material

$$k = \frac{YA}{L}$$

Elastic Force

$$F = \frac{YA}{L}\Delta L$$

Stress

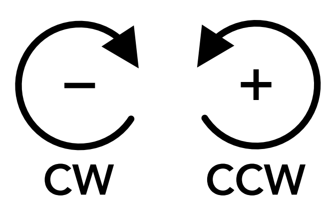
$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

Variables		SI Unit
F	force	N
k	spring constant	$\frac{N}{m}$
Y , E	Young's modulus	$\frac{N}{m^2}$
A	cross-sectional area	m^2
L	length	m

Torque

Torque

$$\tau = rF_{\perp}$$ or $$\tau = r_{\perp}F$$



Variables		SI Unit
τ	torque	N·m
F	force	N
r	distance from rotation axis	m

Rotational Dynamics

Newton's 2nd law of motion applied to rotation

$$\tau_{net} = I\alpha$$ or $$\sum\tau = I\alpha$$

$\sum$: the sum of __

Rotational inertia for a system of masses

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

Variables		SI Unit
τ	torque	N·m
I	rotational inertia	$kg \cdot m^2$
α	angular acceleration	$\frac{rad}{s^2}$
m	mass	kg
r	distance from rotation axis	m

Rotational inertia for common shapes:

Solid sphere (center)	Sphere shell (center)	Solid cylinder (center)	Cylinder shell (center)	Solid rod (center)	Solid rod (end)
$I = \frac{2}{5}mR^2$	$I = \frac{2}{3}mR^2$	$I = \frac{1}{2}mR^2$	$I = mR^2$	$I = \frac{1}{12}mL^2$	$I = \frac{1}{3}mL^2$

Center of Mass

x coordinate of center of mass of a system

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$

y coordinate of center of mass of a system

$$y_{COM} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots}$$

Variables		SI Unit
x	x position	m
y	y position	m
m	mass	kg

Uniform Circular Motion

Frequency

$$f = \frac{1}{T}$$

Tangential velocity

$$v = \frac{2\pi r}{T}$$

$$v = 2\pi r f$$

Variables		SI Unit
v	velocity	$\frac{m}{s}$
r	radius	m
T	period	s
f	frequency	$Hz = \frac{cycles}{s}$
ω	angular velocity	$\frac{rad}{s}$

Centripetal Acceleration and Force

Centripetal acceleration

$$\vec{a}_c = \frac{v^2}{r} \text{ (towards center of circle)}$$

v : tangential speed (m/s)
r : radius of circular path (m)

Variables		SI Unit
a_c	centripetal acceleration	$\frac{m}{s^2}$
a	acceleration	$\frac{m}{s^2}$
v	velocity	$\frac{m}{s}$
r	radius	m
t	time	s

Centripetal acceleration (other variables substituted for speed)

$$a_c = \frac{v^2}{r} = \omega^2 r = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

ω : angular speed (rad/s)
f : frequency (Hz = rev/s)
T : period (s)

Centripetal force

$$\vec{F}_c = m\frac{v^2}{r} \text{ (towards center of circle)}$$

Orbital Motion

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg \cdot s^2}$	gravitational constant

Orbital speed

$$v = \sqrt{\frac{GM}{r}}$$

Orbital period

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Orbital period for elliptical orbit

$$T = 2\pi\sqrt{\frac{a^3}{G(M+m)}}$$

Orbital period for elliptical orbit (assuming M is much larger than m)

$$T = 2\pi\sqrt{\frac{a^3}{GM}}$$

Variables		SI Unit
M	planet mass	kg
m	object mass	kg
R	planet radius	m
r	orbital radius	m
v	orbital speed	$\frac{m}{s}$
T	orbital period	s
F_g	gravitational force	N
F_c	centripetal force	N

Kinetic energy of object in a circular orbit

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Gravitational potential energy of two-mass system

$$U_g = -\frac{GMm}{r}$$

Total energy of object in a circular orbit

$$E = K + U_g = -\frac{GMm}{2r}$$

Variables		SI Unit
E	total energy	J
K	kinetic enerrgy	J
U_g	potential energy	J

Kinetic Energy

Kinetic energy

$$K = \frac{1}{2}mv^2$$

Variables		SI Unit
K	kinetic energy	J
m	mass	kg
v	speed	$\frac{m}{s}$

Rotational kinetic energy

$$K_{rot} = \frac{1}{2}I\omega^2$$

Variables		SI Unit
K_{rot}	rotational kinetic energy	J
I	rotational inertia	$kg \cdot m^2$
ω	angular speed	$\frac{rad}{s}$

Gravitational Potential Energy

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg \cdot s^2}$	gravitational constant

Gravitational potential energy of a two-mass system

$$U_g = -\frac{GMm}{r}$$

$U_g = 0$ at $r = \infty$

Change in gravitational potential energy of an object-earth system

$$\Delta U_g = mg\Delta y$$

Gravitational potential energy of an object-earth system
*relative to a reference point

$$U_g = mgy \qquad U_g = 0 \text{ at } y = 0$$

Variables		SI Unit
U_g	gravitational potential energy	J
M	planet mass	kg
m	object mass	kg
r	distance between centers	m
y	height	m
g	gravitational acceleration	$\frac{m}{s^2}$

Spring Potential Energy

Spring potential energy

$$U_{sp} = \frac{1}{2}k\Delta x^2$$

Δx or Δy

Variables		SI Unit
U_{sp}	spring potential energy	J
k	spring constant	$\frac{N}{m}$
Δx	displacement	m

Conservation of Energy

Conservation of energy (universe and isolated systems)

$$\Delta E_{total} = 0 \quad , \quad E_{total\ i} = E_{total\ f}$$

Variables		SI Unit
E	energy	J
K	kinetic energy	J
U_g	gravitational potential energy	J
U_{sp}	spring potential energy	J

Work

Work

$$\Delta E_{system} = W$$

Work

$$W = F_{\parallel} d$$

$F_{\parallel}$: component of force parallel to d
*F is an external force
d : displacement of the system

Variables		SI Unit
W	work	$J = N \cdot m$
E	energy	J
F	force	N
d	displacement	m

Power

Power

$$P = \frac{\Delta E}{\Delta t}$$

Power

$$P = \frac{W}{\Delta t} = F_{\parallel} v$$

$F_{\parallel}$: component of force parallel to v
*F is an external force
v : velocity of the system

Variables		SI Unit
P	power	$W = \frac{J}{s}$
E	energy	J
W	work	J
F	force	N
v	velocity	$\frac{m}{s}$

Momentum

Momentum

$$\vec{p} = m\vec{v}$$

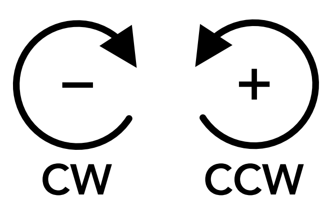
Momentum vector components

$$p_x = mv_x \qquad p_y = mv_y$$

Variables		SI Unit
p	momentum	$\frac{kg \cdot m}{s}$
m	mass	kg
v	velocity	$\frac{m}{s}$

Angular momentum

$$L = I\omega$$



Variables		SI Unit
L	angular momentum	$\frac{kg \cdot m^2}{s}$
I	rotational inertia	$kg \cdot m^2$
ω	angular velocity	$\frac{rad}{s}$

Impulse

Impulse

$$\vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$

F_{avg} : average force over time

Variables		SI Unit
J	impulse	$\frac{kg \cdot m}{s} = N \cdot s$
p	momentum	$\frac{kg \cdot m}{s}$
F	force	N
t	time	s

Rotational impulse

$$\Delta L = \tau_{avg} \Delta t$$

τ_{avg} : average torque over time

Variables		SI Unit
τ	torque	$N \cdot m$
L	angular momentum	$\frac{kg \cdot m^2}{s}$
I	rotational inertia	$kg \cdot m^2$
ω	angular velocity	$\frac{rad}{s}$

Conservation of Momentum

Law of conservation of momentum
(universe and isolated systems)

$$\Delta\vec{p}_{\text{total}} = 0 \quad , \quad \vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$

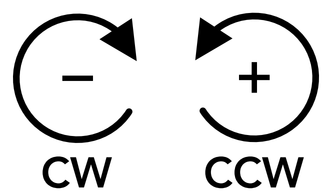
$$\Delta p_{\text{x total}} = 0 \quad , \quad p_{\text{xi total}} = p_{\text{xf total}}$$

$$\Delta p_{\text{y total}} = 0 \quad , \quad p_{\text{yi total}} = p_{\text{yf total}}$$

Variables		SI Unit
p	momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
m	mass	kg
v	velocity	$\frac{\text{m}}{\text{s}}$
J	impulse	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
F	force	N
t	time	s

Law of conservation of angular momentum
(universe and isolated systems)

$$\Delta\vec{L}_{\text{total}} = 0 \quad , \quad \vec{L}_{\text{total i}} = \vec{L}_{\text{total f}}$$



Variables		SI Unit
L	angular momentum	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$
I	rotational inertia	$\text{kg}\cdot\text{m}^2$
ω	angular velocity	$\frac{\text{rad}}{\text{s}}$

Simple Harmonic Motion

Period of a mass-spring oscillation

$$T_{\text{sp}} = 2\pi\sqrt{\frac{m}{k}}$$

Frequency of a mass-spring oscillation

$$f_{\text{sp}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Maximum velocity of a mass-spring oscillation

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$

Variables		SI Unit
T	period	s
f	frequency	$\text{Hz} = \frac{\text{cycles}}{\text{s}}$
A	amplitude	m
m	mass	kg
k	spring constant	$\frac{\text{N}}{\text{s}}$
U_{sp}	spring potential energy	J
K	kinetic energy	J

Period of a pendulum oscillation

$$T_{\text{p}} = 2\pi\sqrt{\frac{L}{g}}$$

Frequency of a pendulum oscillation

$$f_{\text{p}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Maximum velocity of a pendulum oscillation

$$v_{\text{max}} = \theta_{\text{max}}\sqrt{gL}$$

Variables		SI Unit
T	period	s
f	frequency	$\text{Hz} = \frac{\text{cycles}}{\text{s}}$
θ	angle	rad
L	length	m
g	grav. acceleration	$\frac{\text{m}}{\text{s}^2}$
U_{g}	grav. potential energy	J
K	kinetic energy	J

Waves

Wave speed

$$v = \lambda f = \frac{\lambda}{T}$$

Linear density

$$\mu = \frac{m}{L}$$

Speed of a wave in a string

$$v_{string} = \sqrt{\frac{T_s}{\mu}}$$

Variables		SI Unit
λ	wavelength	m
T	period	s
f	frequency	$Hz = \frac{cycles}{s}$
A	amplitude	m, ...
v	velocity	$\frac{m}{s}$

Sound

Constants		Unit	Name
I_0	1×10^{-12}	$\frac{W}{m^2}$	threshold of hearing

Constants		Unit	Name
R	8.3145	$\frac{J}{mol \cdot K}$	ideal gas constant

Speed of sound in a gas

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

Variables		SI Unit
v	velocity	$\frac{m}{s}$
γ	adiabatic index	
T	temperature	K
M	molar mass	$\frac{kg}{mol}$

Sound intensity

$$I = \frac{P_{source}}{4\pi r^2}$$

Sound intensity level

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right)$$

Variables		SI Unit
I	sound intensity	$\frac{W}{m^2}$
P	power	$\frac{J}{s}$
r	distance from source	m
β	sound intensity level	dB

Observed frequency, receding sound source

$$f_o = \frac{f_s}{1 + (v_s/v)}$$

Observed frequency, approaching sound source

$$f_o = \frac{f_s}{1 - (v_s/v)}$$

Observed frequency, receding observer

$$f_o = \left(1 - \frac{v_o}{v}\right)f_s$$

Observed frequency, approaching observer

$$f_o = \left(1 + \frac{v_o}{v}\right)f_s$$

Variables		SI Unit
f_s	source frequency	Hz
f_o	observed frequency	Hz
v_s	source speed	$\frac{m}{s}$
v_o	observer speed	$\frac{m}{s}$
v	speed of sound	$\frac{m}{s}$

Wave Interference

Variables		SI Unit
d	in-line path length	m
r	radial path length	m
λ	wavelength	m
m	number of wavelengths	

Beat frequency

$$f_{beat} = |f_1 - f_2|$$

In-line interference:

Constructive interference

$$\Delta d = m\lambda \quad m = 0, 1, 2, \ldots$$

Destructive interference

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, \ldots$$

Radial interference:

Constructive interference (point C)

$$\Delta r = m\lambda \quad m = 0, 1, 2, \ldots$$

Destructive interference (point D)

$$\Delta r = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, \ldots$$

Standing Waves

Variables		SI Unit
λ	wavelength	m
f	frequency	Hz
L	length	m
v	velocity	$\frac{m}{s}$
m	mode	

Both ends are either nodes or antinodes:

Wavelengths

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, \ldots$$

Frequencies

$$f_m = \frac{v}{\lambda_m} = m\left(\frac{v}{2L}\right) \quad m = 1, 2, 3, \ldots$$

One end is a node, one end is an antinode:

Wavelengths

$$\lambda_m = \frac{4L}{m} \quad m = 1, 3, 5, \ldots$$

Frequencies

$$f_m = \frac{v}{\lambda_m} = m\left(\frac{v}{4L}\right) \quad m = 1, 3, 5, \ldots$$

Fluids

Pressure unit conversions:

1 bar = 100,000 Pa

1 atm = 101,325 Pa

1 psi ≈ 6,894.757 Pa

1 torr = 1 mmHg = 1/760 atm ≈ 133.322 Pa

1 inHg = 25.4 mmHg ≈ 3,386.38 Pa

1 inH_2O = 2.54 cmH_2O ≈ 249.082 Pa

Values		Unit	Name
ρ_{water}	1,000	$\frac{kg}{m^3}$	density of water (4°C)
ρ_{ice}	916	$\frac{kg}{m^3}$	density of ice (0°C)
ρ_{merc}	13,600	$\frac{kg}{m^3}$	density of mercury (0°C)
P_{atm}	101,325	Pa	standard atmospheric pressure
g	9.8	$\frac{m}{s^2}$	gravitational acceleration

Density

$$\rho = \frac{m}{V}$$

Pressure

$$P = \frac{F}{A}$$

Variables		SI Unit
P_{abs}	absolute pressure	Pa
P_{gauge}	gauge pressure	Pa
P_0	reference pressure	Pa
P_{atm}	atmospheric pressure	Pa

Absolute pressure vs gauge pressure

$$P_{abs} = P_{gauge} + P_0 \quad \longleftrightarrow \quad P_{gauge} = P_{abs} - P_0$$

P_0 is usually P_{atm} (1 atm)

Variables		SI Unit
ρ	density	$\frac{kg}{m^3}$
m	mass	kg
V	volume	m³
P	pressure	Pa = $\frac{N}{m^2}$
F	force	N
A	area	m²
h	depth	m
v	velocity	$\frac{m}{s}$
t	time	s
y	height	m

Absolute pressure at depth below surface

$$P_{abs} = \rho g h + P_0$$

Gauge pressure at depth below surface

$$P_{gauge} = \rho g h$$

Pressure difference between two depths

$$\Delta P = \rho g \Delta h$$

Buoyant force on object from fluid

$$F_B = \rho_f V_f g$$

Flow rate

$$\frac{V}{\Delta t} = Av$$

Conservation of flow rate

$$A_1 v_1 = A_2 v_2$$

Bernoulli's equation

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Torricelli's theorem

$$v = \sqrt{2g\Delta y}$$