Topic: Definite integrals, odd functions

Question: Which of the following is true?

Definite integrals of odd functions evaluated on the interval $[-a, a]$...

Answer choices:

A will have different values depending on the function.

B will always equal 0.

C will never exist.

D will always equal ∞.

Solution: B

Odd functions are symmetric about the origin. If a function is symmetric about the origin, it means that any area above the x-axis in the first quadrant will be reflected below the x-axis in the third quadrant. Or that any area above the x-axis in the second quadrant will be reflected below the x-axis in the fourth quadrant.

Therefore, if we take the integral of an odd function on the interval $[-a, a]$, it means that the area above the x-axis will be equal to the area below the x-axis, and therefore that the value of the integral will always be 0.

If the interval is anything other than $[-a, a]$, we know that value of the integral will be non-zero.

Topic: Definite integrals, odd functions

Question: Evaluate the definite integral of the odd function.

$$\int_{-5}^{5} x^3 \, dx$$

Answer choices:

A 312.5

B ∞

C 0

D 156.25

Solution: C

Odd functions are symmetric about the origin. If a function is symmetric about the origin, it means that any area above the x-axis in the first quadrant will be reflected below the x-axis in the third quadrant. Or that any area above the x-axis in the second quadrant will be reflected below the x-axis in the fourth quadrant.

Therefore, if we take the integral of an odd function on the interval $[-a, a]$, it means that the area above the x-axis will be equal to the area below the x-axis, and therefore that the value of the integral will always be 0.

Since we're told that the given function is odd, and since the interval is $[-5,5]$ which matches the form $[-a, a]$, it means that the value of this definite integral is 0.