Conditional Probability Teacher Notes

Skill:	Conditional Probability and Venn Diagrams

Key Teaching Point: What is Conditional Probability

Conditional probability is probability where the outcome of one event affects the probability of the outcome of another event.

Notation:

The vertical bar "|" means "given that".

$P(A|B)$ should be interpreted as "the probability that event A occurs given that we know event B has already occurred". This means we should only consider possibilities involving event B having occurred as being the only possible outcomes.

Formula:

The formula for conditional probability is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A common-sense interpretation of this formula is that the denominator of the formula is the total probability of any outcome that involves the event B occurring. The numerator is the total probability of the overlap of A and B. The next example will illustrate.

Example 1

Work through these examples with the class to prepare them for the worksheet questions:

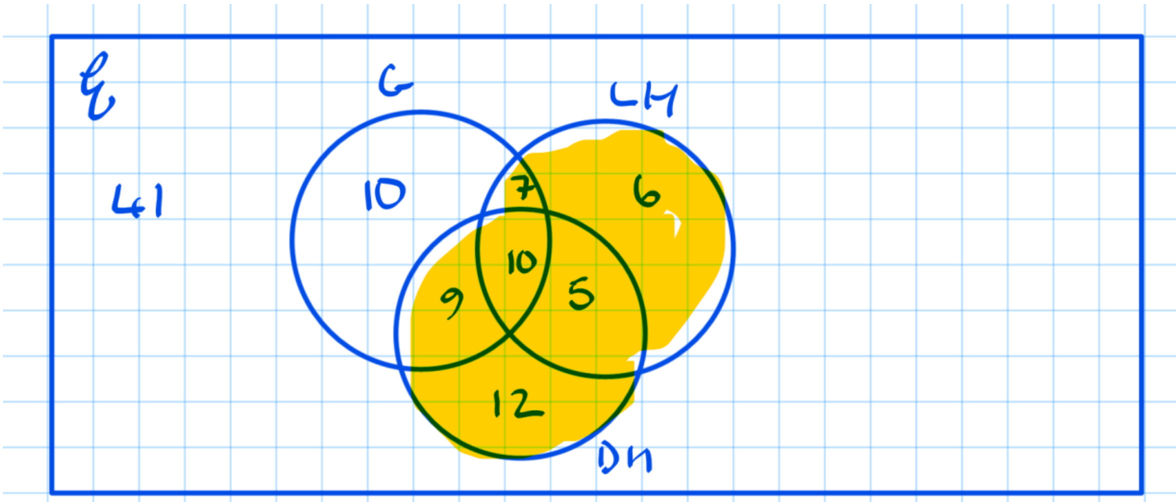
Teacher Note:

When a "given that" clause is used the total number of possible outcomes changes. We should ONLY consider numbers for the event(s) that we are "given" has happened. For example in the diagram below, the highlighted region corresponds to (ii)(d). In this example, we should only consider the events that fulfil the "given that" clause.

A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left handed and having dark hair. Glasses were worn by 36 people, 28 were left handed and 36 had dark hair. There were 17 who wore glasses and were left handed, 19 who wore glasses and had dark hair and 15 who were left handed and had dark hair. Only 10 people wore glasses, were left handed and had dark hair.

Use 'G' to denote the event that a chosen person wears glasses, 'LH' to denote the event that a person is left handed and 'DH' to denote the event that a person has dark hair.

i. Draw a Venn diagram to represent this information.



ii. State in words what the following expressions represent and determine the corresponding probabilities:

a. $P(G|DH)$
b. $P(DH|G)$
c. $P(LH|G')$
d. $P(G|LH \text{ or } DH)$

Example 1 (continued)

Teacher Note

The following can be attempted using the conditional formula or the approach given in the tutorial video. Naturally, I prefer the approach in the tutorial video as it helps students understand the process rather than just use the formula.

a. $P(G|DH)$ The prob that someone wore glasses given they had dark hair $\frac{19}{36}$

b. $P(DH|G)$ The prob they had dark hair given they wore glasses $\frac{19}{36}$

c. $P(LH|G')$ The prob they were left handed given they did not wear glasses $\frac{11}{64}$

d. $P(G|LH \text{ or } DH)$ The prob they wore glasses given they either were left handed or dark haired $\frac{26}{49}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 2

Teacher Note

Get the students to make a note of the following definitions from Year 1 as a reminder:

- Two events are independent if $P(A \cap B) = P(A) \times P(B)$
- Two events are mutually exclusive if they do not overlap (i.e $P(A \cap B) = 0$.

Example 2 Continued on Next Page…

Example 2 (continued)

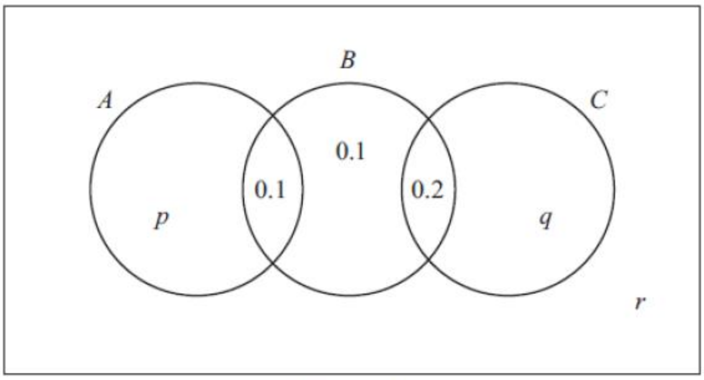


Figure 1

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p.

(3)

Given that $P(B|C) = \frac{5}{11}$

(b) find the value of q and the value of r.

(4)

(c) Find $P(A \cup C|B)$.

(2)

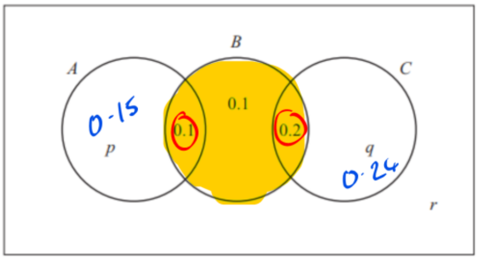


Figure 1

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent. $P(A \cap B) = P(A)P(B)$

(a) Find the value of p.

(3)

Given that $P(B|C) = \frac{5}{11}$

(b) find the value of q and the value of r.

(4)

(c) Find $P(A \cup C|B)$.

(2)

a/ $0.1 = (0.1 + p) \times 0.4$

$\Rightarrow 0.25 = 0.1 + p$

$\Rightarrow p = 0.15$

b/ $\frac{0.2}{0.2 + q} = \frac{5}{11}$

$\Rightarrow 0.2 = \frac{5}{11}(0.2 + q)$

$\Rightarrow \frac{11}{25} = 0.2 + q \Rightarrow q = 0.24$

$r = 1 - 0.15 - 0.1 - 0.1 - 0.2 - 0.24 = 0.21$

c/ $\frac{0.1 + 0.2}{0.1 + 0.1 + 0.2} = \frac{3}{4}$