Proof Essential Practice

Skill: Disproof by counterexample

Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. Disprove the following statement by counter-example:
   "all prime numbers are odd"

2. Prove that the following statement is false:
   "for all integers n greater than or equal to 1,
   $n^2 + 3n + 1$ is a prime number"

3. The numbers $a, b \in \mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \dots\}$.
   Show, by counter-example, that the statement
   "If $a^2 = b^2$, then $a = b$"
   is false.

4. The numbers $x, y \in \mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \dots\}$.
   Show, by counter-example, that the statement
   "If $x^2 > y^2$, then $x > y$"
   is false.

5. The numbers $x, y, z \in \mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \dots\}$.
   Show, by counter-example, that the statement
   "If $x > y$, then $xz > yz$"
   is false.

6. The numbers $m, n \in \mathbb{N} = \{1,2,3,4,5, \dots\}$
   Show, by counter-example, that the statement
   "If $m + n$ is even, then both m and n are even"
   is false.

7. Show, by counter-example, that the statement
   $$tan2\theta \equiv 2tan\theta$$
   is false.