Quiz1 - Probability of Events (v2) SOLUTION KEY

[total marks on quiz: 40 marks]

1. Events A and B are given such that $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{6}$. Find $P(B)$.

[4 marks]

Solution:

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\frac{2}{3}=\frac{1}{2}+P(B)-\frac{1}{6} \quad \Rightarrow \quad P(B)=\frac{2}{3}-\frac{1}{2}+\frac{1}{6}=\frac{4}{6}-\frac{3}{6}+\frac{1}{6}=\frac{2}{6} \quad \text{thus, } P(B)=\frac{1}{3}$$

2. In a group of 100 students taking the IB examinations, 65 take History, 44 take Economics and 25 take both History and Economics.

(a) If a student is selected at random, what is the probability that the student

(i) takes either History or Economics?

(ii) takes either History or Economics, but not both?

(iii) does not take either History or Economics?

(b) Given that a student takes History, what is the probability that the student takes Economics?

[9 marks]

Solution:

(a) $P(H)=0.65,\ P(E)=0.44,\ P(H\cap E)=0.25$

(i) $P(H\cup E)=P(H)+P(E)-P(H\cap E)$

$$P(H\cup E)=0.65+0.44-0.25=0.84$$

(ii) probability History or Economics but not both $=P(H\cup E)-P(H\cap E)=0.84-0.25=0.59$

(iii) probability does not take History or Economics $=1-P(H\cup E)=1-0.84=0.16$

(b) $P(H\cap E)=P(H)\cdot P(E|H) \quad \Rightarrow \quad P(E|H)=\frac{P(H\cap E)}{P(H)}$

$$P(E|H)=\frac{P(H\cap E)}{P(H)}=\frac{0.25}{0.65}\approx 0.385$$

3. There are 10 marbles in a bag. Four marbles are green, and six marbles are yellow. Two marbles are randomly chosen without replacement.

(a) Write down the probability that the first marble chosen is green.

(b) Find the probability that the two marbles chosen are both green.

(c) Find the probability that the two marbles chosen are different colours. **[6 marks]**

Solution:

(a) $P(G)=\frac{4}{10}=\frac{2}{5}$

(b) $P(GG)=\frac{2}{5}\cdot\frac{3}{9}=\frac{2}{5}\cdot\frac{1}{3}=\frac{2}{15}$

(c) $P(GY)+P(YG)=\frac{2}{5}\cdot\frac{6}{9}+\frac{6}{10}\cdot\frac{4}{9}=\frac{2}{5}\cdot\frac{2}{3}+\frac{3}{5}\cdot\frac{4}{9}=\frac{4}{15}+\frac{1}{5}\cdot\frac{4}{3}=\frac{8}{15}$

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4. Two fair six-sided dice are tossed. The sum of the two numbers showing is the *score*. Find the probability that

(a) the *score* is 4.

(b) the *score* is at least 4. **[5 marks]**

Solution:

(a) $P(score = 4) = \frac{3}{36} = \frac{1}{12}$

(b) $P(score \geq 4) = 1 - P(score < 4)$

$= 1 - \left[P(score = 2) + P(score = 3) \right]$

$= 1 - \left[\frac{1}{36} + \frac{2}{36} \right] = 1 - \frac{1}{12} = \frac{11}{12}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

5. Events A and B are such that $P(A) = 0.25$ and $P(B) = 0.56$. Find the value of $P(A \cup B)$ when

(a) A and B are mutually exclusive;

(b) A and B are independent. **[5 marks]**

Solution:

(a) mutually exclusive events: $P(A \cup B) = P(A) + P(B) = 0.25 + 0.56 = 0.81$

(b) independent events: $P(A \cap B) = P(A) \cdot P(B) = (0.25)(0.56) = 0.14$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.56 - 0.14 = 0.67$

6. A box contains 3 balls. Each ball has a different number written on it – 1, 2 or 3. A ball is randomly chosen from the box and the number on it is recorded and it is placed back in the box. In this way, three balls are randomly chosen with replacement. What is the probability that the same number is chosen each time? **[3 marks]**

Solution:

the # of items in the sample space $= 3 \cdot 3 \cdot 3 = 27$; 3 different #s can occur on each of the 3 selections there are 3 ways to get the same # each time, i.e. 111, 222, or 333

thus, probability of choosing the same # three consecutive times $\frac{3}{27} = \frac{1}{9}$ [note: other methods possible]

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7. For events A and B, the probabilities are $P(A)=\frac{4}{13}$ and $P(B)=\frac{5}{13}$. Find the value of $P(A\cap B)$ if:

(a) $P(A\cup B)=\frac{7}{13}$; (b) events A and B are independent. **[4 marks]**

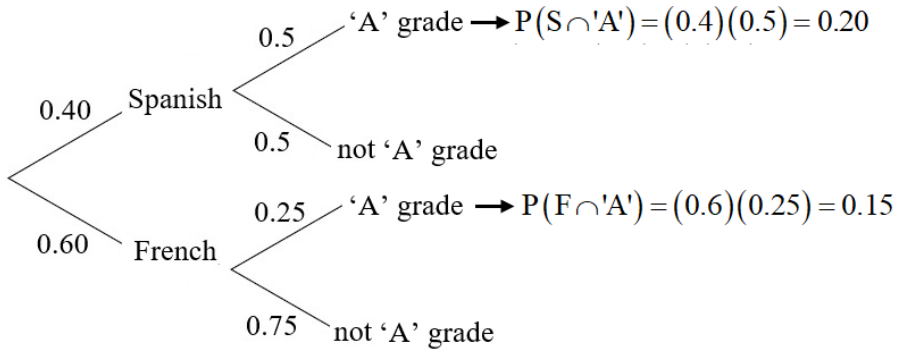
Solution:

(a) $P(A\cup B)=P(A)+P(B)-P(A\cap B) \Rightarrow \frac{7}{13}=\frac{4}{13}+\frac{5}{13}-P(A\cap B) \Rightarrow P(A\cap B)=\frac{2}{13}\approx 0.154$

(b) independent events: $P(A\cap B)=P(A)\cdot P(B)=\frac{4}{13}\cdot\frac{5}{13}=\frac{20}{169}\approx 0.118$

8. All of the students at a school take either Spanish or French, but not both. 40% of the students take Spanish, and half of the students who take Spanish are earning an 'A' grade. Only one-quarter of the students taking French are earning an 'A' grade. Given that a randomly chosen student is earning an 'A' grade, what is the probability that this student is taking French? **[4 marks]**

Solution:



$P('A')=P(S\cap 'A')+P(F\cap 'A')$

$P(F|'A')=\frac{P(F\cap 'A')}{P('A')}=\frac{0.15}{0.20+0.15}=\frac{0.15}{0.35}=\frac{3}{7}\approx 0.429$