Sequences Essential Practice

Skill: Summing infinite geometric series

Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. Find the sum to infinity of each of the following geometric series:
   a. $1 + \frac{1}{4} + \frac{1}{16} + \cdots$
   b. $5 - 0.5 + 0.05 - \cdots$
   c. $100 - 80 + 64 - \cdots$
   d. $-1 - 0.2 - 0.04$

2. Find a simplified expression for the sum to infinity of each of the following geometric progressions. You may assume that the common ratio is such that the sum to infinity exists:
   a. $1 + x + x^2 + \cdots$
   b. $a - \frac{a^2}{b} + \frac{a^3}{b^2} + \cdots$
   c. $1 + 2x + 4x^2 + \cdots$
   d. $e^x + e^{2x} + e^{3x} + \cdots$
   e. $r + ar + a^2r + \cdots$

3. Find first term of a geometric series for which $r = \frac{2}{5}$ and $S_\infty = 20$.
4. Find the third term of a geometric series that has a common ratio of $-\frac{1}{3}$ and a sum to infinity of 18.
5. A geometric series has first term 6 and an infinite sum of 60. Find its common ratio.
6. The sum to infinity of a geometric series is three times the first term. Find the common ratio.
7. Find the value for r for which the sum to infinity of:

$$1 + r + r^2 + r^3 + \cdots$$

   is double the sum to infinity of

$$1 - r + r^2 - r^3 + \cdots$$

8. A geometric progression has first term $3A$ (where $A \neq 0$) and common ratio R. A second geometric progression has first term A and common ratio R^2. Given that the two progressions converge and have equal sums to infinity, find the value of R.