



A-LEVEL
MATHEMATICS

7357/1

Paper 1

Mark scheme

June 2018

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

| | |
|---|--|
| M | mark is for method |
| R | mark is for reasoning |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

Key to mark scheme abbreviations

| | |
|---------|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | Indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| sf | significant figure(s) |
| dp | decimal place(s) |

AS/A-level Maths/Further Maths assessment objectives

| AO | Description |
|------------|--|
| AO1 | AO1.1a Select routine procedures |
| | AO1.1b Correctly carry out routine procedures |
| | AO1.2 Accurately recall facts, terminology and definitions |
| AO2 | AO2.1 Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a Make deductions |
| | AO2.2b Make inferences |
| | AO2.3 Assess the validity of mathematical arguments |
| | AO2.4 Explain their reasoning |
| | AO2.5 Use mathematical language and notation correctly |
| AO3 | AO3.1a Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a Interpret solutions to problems in their original context |
| | AO3.2b Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 Translate situations in context into mathematical models |
| | AO3.4 Use mathematical models |
| | AO3.5a Evaluate the outcomes of modelling in context |
| | AO3.5b Recognise the limitations of models |
| | AO3.5c Where appropriate, explain how to refine models |

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

| Q1 | Marking Instructions | AO | Marks | Typical Solution |
|--------------|------------------------|--------|----------|----------------------------------|
| 1 | Circles correct answer | AO1.1b | B1 | $\frac{dy}{dx} = -\frac{2}{x^3}$ |
| Total | | | 1 | |

| Q2 | Marking Instructions | AO | Marks | Typical Solution |
|--------------|------------------------|--------|----------|--------------------|
| 2 | Circles correct answer | AO1.1b | B1 | $y = 5 \times 5^x$ |
| Total | | | 1 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|------------------------|--------|----------|------------------|
| 3 | Circles correct answer | AO1.1b | B1 | 4 |
| Total | | | 1 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|---|--------|----------|--------------------------------|
| 4 | Takes logs of an equation. Must be correct use of logs. | AO1.1a | M1 | $y = e^{x-4}$ |
| | Obtains correct inverse function in any correct form | AO1.1b | A1 | $\ln y = x - 4$ |
| | Deduces correct domain | AO2.2a | B1 | $4 + \ln y = x$ |
| Total | | | 3 | $f^{-1}(x) = 4 + \ln x, x > 0$ |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-------------|---|--------|----------|--|
| 5(a) | Differentiates 2^t or 2^{-t} to obtain $\pm A \ln 2 \times 2^{\pm t}$ | AO1.1a | M1 | $\frac{dy}{dt} = (3 \ln 2) 2^t$ $\frac{dx}{dt} = (-4 \ln 2) 2^{-t}$ $\frac{dy}{dx} = \frac{(3 \ln 2) 2^t}{(-4 \ln 2) 2^{-t}}$ $= -\frac{3}{4} \times 2^{2t}$ |
| | Obtains $\frac{dy}{dt} = (\pm A \ln 2) 2^t$ and $\frac{dx}{dt} = (\pm B \ln 2) 2^{-t}$ | AO1.1b | A1 | |
| | Uses chain rule with correct $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and completes rigorous argument to obtain fully correct printed answer | AO2.1 | R1 | |
| (b) | Rearranges to write 2^{-t} in terms of x or 2^t in terms of y Or Writes given expression in terms of t | AO3.1a | M1 | $2^t = \frac{y+5}{3}$ $2^{-t} = \frac{x-3}{4}$ $1 = \left(\frac{y+5}{3}\right)\left(\frac{x-3}{4}\right)$ $12 = xy + 5x - 3y - 15$ $xy + 5x - 3y = 27$ ALT $xy + ax + by = (4 \times 2^t + 3)(3 \times 2^t - 5) + a(4 \times 2^t + 3) + b(3 \times 2^t - 5)$ $= 12 - 15 + (4a - 20)2^t + (3b + 9)2^t + 3a - 5b$ $a = 5, b = -3$ $xy + 5x - 3y = -3 + 15 + 15$ $= 27$ |
| | Eliminates t Or Compares coefficients PI by $a=5$ or $b=-3$ | AO1.1a | M1 | |
| | Completes rigorous argument to obtain correct values of a , b and c and write the Cartesian equation in the required form ISW | AO2.1 | R1 | |
| | Total | | 6 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|----------------|--|--------|-----------|--|
| 6(a) | Writes in a form to which the binomial expansion can be applied Accept $A\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ | AO3.1a | M1 | $\frac{1}{\sqrt{4+x}} = \frac{1}{2}\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ $\approx \frac{1}{2} \left[1 + \left(-\frac{1}{2}\right)\frac{x}{4} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{4}\right)^2}{2!} \right]$ $\approx \frac{1}{2} \left[1 - \frac{x}{8} + \frac{3x^2}{128} \right]$ $\approx \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2$ |
| | Uses binomial expansion for their $(1 + kx)^{\pm\frac{1}{2}}$ with at least two terms correct (can be unsimplified) | AO1.1a | M1 | |
| | Obtains correct simplified answer No need to expand brackets CAO | AO1.1b | A1 | |
| (b) | Substitutes $-x^3$ in their three term expansion from part (a) | AO1.1a | M1 | $\frac{1}{\sqrt{4-x^3}} \approx \frac{1}{2} - \frac{1}{16}(-x^3) + \frac{3}{256}(-x^3)^2$ $\approx \frac{1}{2} + \frac{x^3}{16} + \frac{3x^6}{256}$ |
| | Obtains correct expansion. FT their (a) | AO1.1b | A1F | |
| (c) | Uses their three term expansion as the integrand ignore limits PI by next mark | AO1.1a | M1 | $\int_0^1 \frac{1}{\sqrt{4-x^3}} dx \approx \int_0^1 \left(\frac{1}{2} + \frac{x^3}{16} + \frac{3x^6}{256} \right) dx$ $\approx \left[\frac{x}{2} + \frac{x^4}{64} + \frac{3x^7}{1792} \right]_0^1$ $\approx \frac{1}{2} + \frac{1}{64} + \frac{3}{1792}$ ≈ 0.5172991 |
| | Integrates (at least two terms correct) | AO1.1a | M1 | |
| | Obtains correct value CAO | AO1.1b | A1 | |
| (d)(i) | Explains that each term in the expansion is positive | AO2.4 | E1 | Each term in the expansion is positive. So increasing the terms will increase the estimated value hence the value must be an underestimate. |
| | Deduces that increasing the number of terms will increase the estimated value and that the value must be an underestimate. (Condone inference if evidence given ie value calculated numerically and compared) | AO2.2a | R1 | |
| (d)(ii) | States the validity of their binomial expansion for part (b) Provided their $k \neq \pm 1$ | AO3.1a | B1F | The binomial expansion is valid for $ x < \sqrt[3]{4}$ $2 > \sqrt[3]{4}$ |
| | Compares integral lower limit with validity of correct expansion CAO | AO2.3 | E1 | |
| Total | | | 12 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|--|--------|----------|---|
| 7(a) | Uses a technique which could lead to showing two lines are perpendicular. Obtains at least one correct distance (or distance ²) or gradient. | AO3.1a | M1 | $AB^2 = (8-15)^2 + (17-10)^2$ $= 98$ $AC^2 = (8--2)^2 + (17--7)^2$ $= 676$ |
| | Obtains three correct distances (or distance ²) or two gradients. Lengths: $7\sqrt{2}, 17\sqrt{2}, 26$ $AB = -\frac{7}{7}, BC = \frac{17}{17}$ Gradients: | AO1.1b | A1 | $CB^2 = (15--2)^2 + (10--7)^2$ $= 578$ $AB^2 + BC^2 = 98 + 578$ $= 676$ $= AC^2$ |
| | Completes correct rigorous argument to show required result Uses Pythagoras OR Multiplies gradients to show product is -1 AND Writes a concluding statement. | AO2.1 | R1 | Angle ABC is a right angle. |
| (b)(i) | Explains why AC is a diameter Must reference angle subtended by diameter (condone "angle in a semi-circle") or give full explanation. | AO2.4 | E1 | The angle subtended by a diameter is $90^\circ \therefore AC$ must be a diameter of the circle |
| (b)(ii) | Deduces correct radius (or radius ²) | AO2.2a | B1 | Radius $\frac{\sqrt{676}}{2} = 13$ Centre $\left(\frac{8-2}{2}, \frac{17-7}{2}\right) = (3, 5)$ Distance from centre to D $(3--8)^2 + (5--2)^2 = 11^2 + 7^2$ $= 170 > 169$ So D lies outside the circle. |
| | Obtains mid-point of diameter | AO1.1b | B1 | |
| | Uses $D(-8, -2)$ to find the distance or (distance ²) from their centre OE | AO1.1a | M1 | |
| | Completes rigorous argument by comparing $\sqrt{170} > 13$ (or $170 > 169$) to show that D lies outside the circle | AO2.1 | R1 | |
| Total | | | 8 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|--|--------|----------|--|
| 8(a) | Uses $A = \frac{1}{2}ab \sin C$ for triangle OAC or OAB PI by equation | AO1.2 | B1 | $\frac{1}{2}r \times \frac{r}{2} \sin \theta = \frac{1}{4} \left(\frac{1}{2}r^2\theta \right)$ $\Rightarrow \frac{r^2}{4} \sin \theta = \frac{1}{8}r^2\theta$ $\Rightarrow 2r^2 \sin \theta = r^2\theta$ $\Rightarrow 2 \sin \theta = \theta$ AG |
| | Forms an equation relating the area of OAC and ABC in the form $Ar^2 \sin \theta = Br^2\theta$ | AO3.1a | M1 | |
| | Obtains fully correct equation ACF | AO1.1b | A1 | |
| | Simplifies to obtain required equation, only award if all working correct with rigorous argument. | AO2.1 | R1 | |
| (b) | Rearranges to the form $f(\theta) = 0$ PI by correct θ_2 or θ_3 | AO1.1a | M1 | $f(\theta) = \theta - 2 \sin \theta = 0$ $\theta_{n+1} = \theta_n - \frac{\theta_n - 2 \sin \theta_n}{1 - 2 \cos \theta_n}$ $\theta_2 = 2.094395\dots$ $\theta_3 = 1.913222\dots$ $\theta_3 = 1.91322 \text{ (5 d.p.)}$ |
| | Differentiates their $f(\theta)$ or uses calculator PI correct θ_2 or θ_3 | AO1.1b | A1 | |
| | Obtains correct θ_3 | AO1.1b | A1 | |
| (c) | Obtains percentage error for θ_3 AWRT 0.94% | AO3.2b | B1 | 0.935% |
| Total | | | 8 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-------------|---|--------|----------|---|
| 9(a) | Uses S_n for arithmetic sequence with $n = 6$ or $n = 36$ | AO1.1a | M1 | $S_6 = 3(2a + 5d)$ $= 6a + 15d$ |
| | Finds correct expressions for S_6 and S_{36} | AO1.1b | A1 | $S_{36} = 18(2a + 35d)$ $= 36a + 630d$ |
| | Forms equation in a and d using their $S_{36} = (their S_6)^2$ | AO3.1a | M1 | |
| | Expands quadratic and collects like terms to obtain printed answer Only award for completely correct solution with no errors | AO2.1 | R1 | $36a + 630d = (6a + 15d)^2$ $36a + 630d = 36a^2 + 90ad + 90ad + 225d^2$ $4a + 70d = 4a^2 + 20ad + 25d^2$ |
| (b) | Uses u_n for arithmetic sequence with $n = 6$ | AO1.1b | B1 | $a + 5d = 25 \Rightarrow d = \frac{25 - a}{5}$ |
| | Eliminates a or d using their ' $a + 5d = 25$ ' and the printed result in part (a) to obtain a quadratic in one variable | AO1.1a | M1 | $4a + 70\left(\frac{25 - a}{5}\right) = 4a^2 + 20a\left(\frac{25 - a}{5}\right) + 25\left(\frac{25 - a}{5}\right)^2$ $4a + 350 - 14a = 4a^2 + 100a - 4a^2 + 625 - 50a + a^2$ |
| | Obtains correct quadratic equation Need not be simplified | AO1.1b | A1 | $350 - 10a = 100a + 625 - 50a + a^2$ |
| | Solves their quadratic $a = -5, a = -55$ (or $d = 6, d = 16$) | AO1.1a | M1 | $a^2 + 60a + 275 = 0$ |
| | Deduces min value $a = -55$ NMS $a = -55$ $5/5$ | AO3.2a | A1 | $a = -5, a = -55$ (or $d = 6, d = 16$) $a = -55$ |
| | Total | | 9 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|---|--------|----------|---|
| 10(a) | Uses model to form an equation to find k with $t=5.7$, $m = \frac{1}{2} m_0$ | AO3.4 | M1 | $200 = 400e^{-kx 5.7}$ $k=0.1216047\dots$ $m = 400 e^{-0.1216\dots \times 4}$ $m = 250$ |
| | Obtains correct value of k | AO1.1b | A1 | |
| | Uses model to find m with $t=4$, $m_0 = 400$ and <i>their</i> k (Condone $m_0=200$) | AO3.4 | M1 | |
| | Obtains correct value of m CAO (245.9296...) AWRT 250 | AO1.1b | A1 | |
| (b) | Uses model to set up inequality or equation using <i>their</i> k and 280 | AO3.1b | M1 | $400e^{-0.1216t} \leq 280$ $e^{-0.1216t} \leq 0.7$ $-0.1216t \leq \ln(0.7)$ $t \geq 2.933$ |
| | Solves their inequality or equation to find t (Follow through their k only) (2.933067) | AO1.1b | A1F | |
| | Interprets <i>their</i> solution (Only follow through if time is earlier than 1:42 pm) | AO3.2a | A1F | |
| (c) | States any sensible reason such as: Different people eliminate caffeine at different rates The model is based on an average person The length of time taken to drink two cups of coffee may have been significant The amount of caffeine in a “strong cup of coffee” may vary | AO3.5b | B1 | Different people eliminate caffeine at different rates |
| | Total | | 8 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|------------|---|--------|-----------|--|
| 11(a)(i) | Uses model to form equation with $V=0$ | AO3.4 | M1 | $\therefore 10 + 100\left(\frac{T}{30}\right)^3 - 50\left(\frac{T}{30}\right)^4 = 0$ |
| | Rearranges to isolate T^4 term | AO1.1a | M1 | |
| | Completes rigorous and convincing argument to clearly show the required result. Need to see evidence of division by T to isolate T^3 term Must be an equation throughout AG | AO2.1 | R1 | $\Rightarrow 50\left(\frac{T}{30}\right)^4 = 10 + 100\left(\frac{T}{30}\right)^3$ $\Rightarrow \frac{T^4}{16200} = 10 + \frac{T^3}{270}$ $\Rightarrow \frac{T^3}{16200} = \frac{10}{T} + \frac{T^2}{270}$ $\Rightarrow T = \sqrt[3]{\frac{162000}{T} + 60T^2}$ |
| 11(a)(ii) | Calculates T_1 (44.96345.....) | AO1.1a | M1 | $T_1 = 44.963$ |
| | Calculates T_2 and T_3 (49.98742....) | | | $T_2 = 49.987$ |
| | Condones greater than 3dp (53.50407....) | AO1.1b | A1 | $T_3 = 53.504$ |
| 11(a)(iii) | Explains 38 in context | AO3.2a | B1 | 38 represents current year 2018 |
| 11(b) | Translates 2029 into $t=49$ | AO3.3 | B1 | $10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4 = 4.5 \times 1.063^t$ $\Rightarrow t = 49.009$ $1980 + 49 = 2029$ <p>Therefore use of oil and production of oil will be equal in the year 2029</p> |
| | Uses models to set up equation or evaluate both models at one value of t | AO3.4 | M1 | |
| | Obtains correct values for both models for two appropriate values of t . $t \in [49, 50]$ eg $t=49$ and $t=50$ $t=49$ gives: 89.89 and 89.81 $t=50$ gives: 87.16 and 95.47 Or Solves equation using any method to obtain AFWW 49.009 to 49.01 | AO1.1b | A1 | |
| | Explains that the use of oil and the production of oil are equal when $t = 49.009$ Or Uses a change of sign argument OE to explain that the value of each model for two appropriate values of t shows that the production of oil and the use of oil are the same for $t \in (49, 50)$ | AO2.4 | E1 | |
| | Total | | 10 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|--------------|--|--------|-----------|---|
| 12(a) | Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors | AO1.1a | M1 | $p\left(-\frac{1}{2}\right) = 30 \times \left(-\frac{1}{2}\right)^3 - 7 \left(-\frac{1}{2}\right)^2 - 7 \left(-\frac{1}{2}\right) + 2$ $= 0$ $\therefore 2x + 1 \text{ is a factor of } p(x)$ |
| | Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that $p(-1/2)=0$ and clearly state that this implies that $2x + 1$ is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement | AO2.1 | R1 | |
| (b) | Obtains quadratic factor PI | AO1.1a | M1 | $p(x) = (2x + 1)(15x^2 - 11x + 2)$ $= (2x + 1)(5x - 2)(3x - 1)$ |
| | Obtains second linear factor | AO1.1b | A1 | |
| | Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3 | AO1.1b | A1 | |
| (c) | Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$) | AO3.1a | M1 | $\frac{30 \sec^2 x + 2 \cos x}{7} = \sec x + 1$ $\Rightarrow 30 \sec^2 x + 2 \cos x = 7 \sec x + 7$ $\Rightarrow 30 \sec^3 x + 2 = 7 \sec^2 x + 7 \sec x$ $30 \sec^3 x - 7 \sec^2 x - 7 \sec x + 2 = 0$ $\Rightarrow (2 \sec x + 1)(5 \sec x - 2)(3 \sec x - 1) = 0$ $\Rightarrow \sec x = -\frac{1}{2}, \frac{2}{3}, \frac{1}{3}$ <p>These values do not fall within the range of $\sec x$ as they are between -1 and 1</p> $\therefore \frac{30 \sec^2 x + 2 \cos x}{7} = \sec x + 1 \text{ has no real solutions.}$ |
| | Equates to zero and uses result from (b) or factorises | AO1.1a | M1 | |
| | Deduces that if solutions exist they must be of the form $\sec x = -1/2$, $\sec x = 1/3$ or $\sec x = 2/5$ OE | AO2.2a | A1 | |
| | Explains that the range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$ OE OE for $\cos x$ | AO2.4 | E1 | |
| | Completes argument explaining that there cannot be any real solutions as values are outside of the function's range. | AO2.1 | R1 | |
| Total | | | 10 | |

| Q | Marking instructions | AO | Mark | Typical solution |
|---|--|--------|-----------|---|
| 13 | Identifies and clearly defines consistent variables for length and width. Can be shown on diagram. | AO3.1b | B1 | Width of rectangle = $2x$ Length of rectangle = $2y$ |
| | Models the area of rectangle with an expression of the correct dimensions | AO3.3 | M1 | $A = 4xy$ |
| | Eliminates either variable to form a model for the area in one variable. | AO1.1a | M1 | $x^2 + y^2 = 16$ |
| | Obtains a correct equation to model the area in one variable | AO1.1b | A1 | $A = 4x\sqrt{16 - x^2}$ |
| | Differentiates their expression for area. Condone one error | AO3.4 | M1 | $\frac{dA}{dx} = 4\sqrt{16 - x^2} - \frac{4x^2}{\sqrt{16 - x^2}}$ $\frac{dA}{dx} = \frac{64 - 8x^2}{\sqrt{16 - x^2}}$ For maximum point $\frac{dA}{dx} = 0$ |
| | Explains that their derivative equals zero for a maximum or stationary point. | AO2.4 | E1 | $\frac{64 - 8x^2}{\sqrt{16 - x^2}} = 0$ $x = 2\sqrt{2}$ |
| | Equates area derivative to zero and obtains correct value for either variable. CAO | AO1.1b | A1 | When $x = 2.8$, $\frac{dA}{dx} = 0.448$ When $x = 2.9$, $\frac{dA}{dx} = -1.191$ |
| | Completes a gradient test or uses second derivative of their area function to determine nature of stationary point | AO1.1a | M1 | Therefore maximum |
| | Deduces that the area is a maximum at $x = 2\sqrt{2}$ or $\theta = \frac{\pi}{4}$ Values need not be exact | AO2.2a | R1 | The maximum area is 32 sq in |
| Obtains maximum area with correct units AWRT 32 | AO3.2a | B1 | | |
| Total | | | 10 | |

| Q | Marking instructions | AO | Mark | Typical solution |
|--------------|--|--------|----------|---|
| 14(a) | Explains why $\angle EFQ = A$ Must be a fully correct explanation with reasons which may include: Vertically opposite angles and right angle implies similar triangles. | AO2.4 | E1 | $\angle OQR = \angle FQE$ vertically opposite angles $\angle ORQ = \angle FEQ = 90^\circ$ So $\angle EFQ = A$ |
| | Deduces $\frac{PF}{EF} = \cos(A)$ AND $\frac{EF}{OF} = \sin(B)$ Must have at least stated or implied that $\angle EFQ = A$ through similarity | AO2.2a | R1 | Since $\angle EFQ = A$ $\frac{PF}{EF} = \cos(A)$ And $\frac{EF}{OF} = \sin(B)$ in triangle OEF |
| 14(b) | Completes proof | AO2.2a | B1 | $\frac{DE}{OE} \times \frac{OE}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$ = sinAcosB + cos A sin B |
| 14(c) | Explains that the proof is based on right angled triangles which limits A and B to acute angles | AO2.3 | E1 | Since the proof is based on the diagram which uses right-angled triangles it is assumed that A and B are acute. Therefore, the proof only holds for acute angles. |
| 14(d) | Substitutes $-B$ into identity for $\sin(A+B)$ to give $\sin(A-B)$ | AO2.1 | R1 | $\sin(A-B) = \sin A \cos(-B) + \cos A \sin(-B)$ |
| | Recalls at least one of the identities $\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$ Must be explicitly stated | AO1.2 | B1 | $\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$ |
| | Deduces correct identity with no errors. This must be clearly deduced from a correct argument and not simply stated. | AO2.2a | R1 | Hence $\sin(A-B) = \sin A \cos B - \cos A \sin B$ |
| Total | | | 7 | |

| Q | Marking instructions | AO | Mark | Typical solution |
|--------------|---|--------|------------|---|
| 15(a) | Forms expression of the correct form for the gradient of the line AB condone sign error | AO1.1a | M1 | Gradient of AB $= \frac{(-4+h)^3 - 48(-4+h) - ((-4)^3 - 48(-4))}{h}$ |
| | Obtains correct expansion of $(-4+h)^3$ | AO1.1b | B1 | $= \frac{h^3 - 12h^2 + 48h - 64 - 48h + 192 - 128}{h}$ |
| | Obtains correct expansion of numerator | AO1.1b | A1 | $= \frac{h^3 - 12h^2}{h}$ |
| | Simplifies numerator and shows given result | AO2.1 | R1 | $= h^2 - 12h$ |
| 15(b) | Explains that as $h \rightarrow 0$ the gradient of the line AB \rightarrow the gradient of the curve or tangent to the curve Or gradient of curve is given by $\lim_{h \rightarrow 0} h^2 - 12h$ Must not use $h = 0$ | AO2.4 | E1 | The gradient of the curve is given by $\lim_{h \rightarrow 0} h^2 - 12h$ |
| | Explains that $\lim_{h \rightarrow 0} h^2 - 12h = 0$ therefore A must be a stationary point | AO2.4 | E1 | As $h \rightarrow 0$, $h^2 - 12h \rightarrow 0$ therefore A must be a stationary point |
| Total | | | 6 | |
| TOTAL | | | 100 | |