A#4 DOMAIN & RANGE

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

State the range of $f(x) = 3 - e^{1-2x}$

a. if the domain of $f(x)$ is $x \in \mathbb{R}$
b. if the domain of $f(x)$ is $x \geq 0$

EXPERT

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \ x \geq 0$$

a. Show that $g(x) = \dfrac{x+1}{x-2}, \ x \geq 0$
b. Find the range of $g(x)$

MASTER

A function $f(x)$ is defined as $f(x) = \begin{cases} -x & -4 \leq x \leq 0 \\ x^2 & 0 \leq x < 2 \\ 10 - 3x & 2 \leq x < 4 \end{cases}$

a. The equation $f(x) = k$ has one solution. State the range of possible values of k.
b. State the range of $f(x)$

A#5 COMPOSITE FUNCTIONS

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APPRENTICE

The function g is defined by $g(x) = \dfrac{1}{2x-3}$, for $x \neq 1.5$

Find an expression for $gg(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers.

EXPERT

The function f is defined by $f(x) = 16x - e^{2x}$, for all real x

The composite function fg is defined by $fg(x) = \dfrac{16}{x} - e^{\frac{2}{x}}$, $x \neq 0$

Evaluate $gf(0)$

MASTER

The functions f and g are defined as follows: $f(x) = 2 + \ln(x+3)$ for $x \geq 0$,
$g(x) = ax^2$, $x \in \mathbb{R}$, where a is a positive constant.

a. Given that $gf(e^4 - 3) = 9$, find the value of a.

b. Given that $f^2(e^N - 3) = \ln(53e^2)$, find the value of N.

A#6 INVERSE FUNCTIONS

TAILORED TUTORS
A LEVEL MATHS

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APPRENTICE

The function $f(x)$ is defined by $f(x) = 5 + e^{3x}$, for all real values of x

a. Find $f^{-1}(x)$ and state its domain.

b. Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same axes.

EXPERT

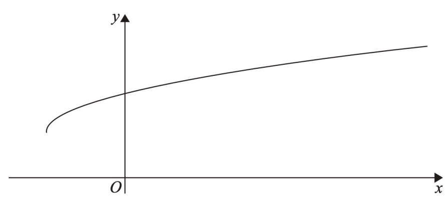
The function $f(x)$ is defined by $f(x) = \dfrac{x}{x-1}$ for $x < 1$.

Show that $f^{-1}(x) = f(x)$ and **hence** find $f^2(x)$.

MASTER

This sketch shows part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \quad x \geq -2$$



a. State the range of g.

b. Find $g^{-1}(x)$ and state its domain.

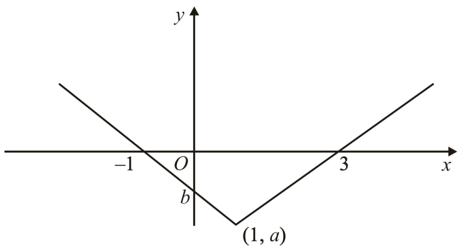
c. Find the exact value of x for which $g(x) = x$

d. Hence state the value of a for which $g(a) = g^{-1}(a)$

A#7 MODULUS GRAPHS

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE



This figure shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments which meet at the point $(1,\ a)$, $a < 0$. One line meets the x-axis at $(3,\ 0)$. The other line meets the x-axis at $(-1,\ 0)$ and the y-axis at $(0,\ b)$, $b < 0$.

a. Sketch the graph with equation $y = f(x + 1)$ showing the axis intercepts
b. Sketch the graph with equation $y = f\left(|x|\right)$ showing the axis intercepts

Given that $f(x) = |x - 1| - 2$,

c. Find the value of a and the value of b

EXPERT

Given that $f(x) = 2e^x - 5$, $x \in \mathbb{R}$

a. Sketch the curve with equation $y = f(x)$

b. Sketch the graph with equation $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes, as well as stating the equation of the asymptote.

c. Deduce the set of values for which $f(x) = |f(x)|$

MASTER

a. Sketch the curve with equation $y = 4 - |2x + 1|$, indicating the coordinates where the curve crosses the axes.

b. Describe the sequence of geometrical transformations which map the graph of $y = |x|$ onto the graph of $y = 4 - |2x + 1|$

A#8 MODULUS EQUATIONS

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APPRENTICE

a is a positive constant, and the functions f and g are defined for all real values of x by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a$$

Solve for x the equation $gf(x) = 31a$.

EXPERT

Solve the inequality $|5x - 3k| \geq 3|x + 4k|$ where k is a positive constant.

MASTER

Given that $f(x) = 2e^x - 5, \quad x \in \mathbb{R}$

a. sketch, on separate diagrams, the curve with equation

i) $y = f(x)$ ii) $y = |f(x)|$ iii) $y = f(|x|)$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.
On each diagram, state the equation of the asymptote.

b. Deduce the set of values of x for which $f(x) = |f(x)|$

c. Find the exact solutions of the equation $|f(x)| = 2$

A#9 PARAMETRIC EQUATIONS

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

A curve has parametric equations $x = 3\cos\theta$, $y = \dfrac{3}{2}\sin\theta$, where $-2\pi \leq \theta \leq 2\pi$

The Cartesian equation of the same curve is $x^2 + 4y^2 = 9$ which is an ellipse.

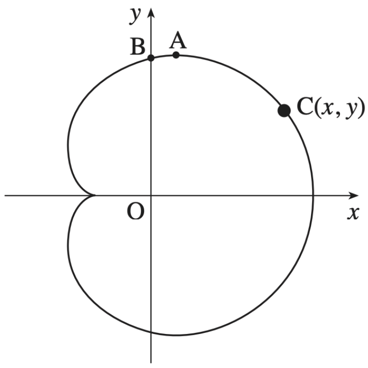
State the range of this equation.

EXPERT

The curve shows the path of a particle C in a vertical plane.
The path of C is modelled by the parametric equations

$$x = 10\cos\theta + 5\cos 2\theta, \quad y = 10\sin\theta + 5\sin 2\theta, \quad 0 \leq \theta < 2\pi$$

Find the coordinates of the x intercepts



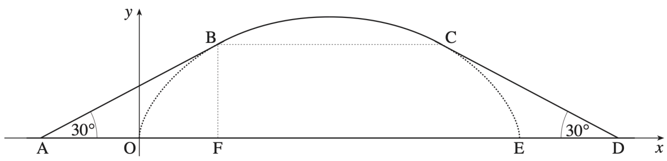
MASTER

The sketch shows the arch ABCD of a bridge.

The section from B to C is part of the curve OBCE with parametric equations

$$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta)$$

for $0 \leq \theta \leq 2\pi$ where a is a constant.



a. Find, in terms of a, the length of the straight line OE
b. Find, in terms of a, the maximum height of the arch.

The sections AB and CD are inclined at $30°$ to the horizontal and are tangents to the curve at B and C respectively. BC is parallel to the x axis. BF is parallel to the y axis.

The value of the parameter θ at B is $\theta = \dfrac{2\pi}{3}$

c. Show that BF $= \dfrac{3}{2}a$ and find OF in terms of a, giving your answer exactly.

d. Given that the straight line distance AD is 20 metres, calculate the value of a

A#10 PARAMETRIC CONVERSION

TAILORED TUTORS
A LEVEL MATHS

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APPRENTICE

A curve C has parametric equations $x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$

Show that the Cartesian equation of the curve C can be written in the form $y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$

where a and b are integers to be determined.

EXPERT

A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leq t \leq \pi$.

a. Find the coordinates of the points where the curve meets the x-axis.

b. Find the cartesian equation of the curve. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial.

c. Sketch the curve.

MASTER

A curve C is defined by the parametric equations $x = \frac{4 - e^{2-6t}}{4}, \quad y = \frac{e^{3t}}{3t}, \quad t \neq 0$

a. Show that $x = \frac{4 - e^{2-6t}}{4}$ can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{(1-x)}}$

b. Hence find the Cartesian equation of C, giving your answer in the form $y = \frac{e}{f(x)[1 - \ln(f(x))]}$