Test 2 – Polynomial Functions, Equations & Inequalities

> Syllabus content on test: • polynomial functions • quadratic formula & discriminant • factor & remainder theorems • polynomial division • sum and product of roots of a polynomial equation • rational functions • solving inequalities

total marks on test: **60**

Part I: **No** calculator – questions 1-6 **[34 marks]**

1. When the polynomial $2x^3 + ax^2 + b$ is divided by $(x-2)$, the remainder is 2, and when divided by $(x+1)$, the remainder is -1. Find the value of a and the value of b. **[5 marks]**

2. Find a cubic polynomial with integer coefficients that has zeros of $x = 2$ and $x = 1 + 3i$. **[4 marks]**

3. Given that $m > 0$, find the value(s) of m that solve the inequality $mx^2 + mx + 3 > 0$. **[5 marks]**

4. If α and β are the roots of the quadratic equation $2x^2 - 6x + 1 = 0$, find a quadratic equation whose roots are:

    (a) $2\alpha,\ 2\beta$ (b) $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$ **[8 marks]**

5. $(x^2 - 1)$ is a factor of the cubic polynomial $x^3 + px^2 + qx + r$, and the polynomial leaves a remainder of 12 when divided by $(x - 2)$. Find the value of p, the value of q and the value of r.

    [6 marks]

6. Consider the quartic equation $2x^4 - 11x^3 + 20x^2 - 7x - 10 = 0$. Given that one of the zeros of the equation is $r_1 = 2 - i$, find the other three zeros r_2, r_3 and r_4. **[6 marks]**

Part II: calculator allowed – questions 7-11 **[26 marks]**

7. Sketch the graph of $y = \dfrac{x - 10}{5x - 2}$. Clearly label any x- or y-intercepts and any asymptotes. **[5 marks]**

8. The cubic polynomial $x^3 + mx^2 + n$ has a double root of $x = c$ and a single root of $x = 2$. Given that $n \neq 0$, find the value of c. **[6 marks]**

9. Solve for x: $\dfrac{3x - 3}{4 - x} \leq 3$ **[4 marks]**

10. Find the range of values of k such that the equation $kx^2 - 2x + k - 1 = 0$ has no real solutions. Express your answer **exactly**. **[6 marks]**

11. Consider the rational function $g(x) = \dfrac{x + a}{bx + c}$, $x \neq -\dfrac{c}{b}$. The graph of g has asymptotes $x = -6$ and $y = 3$, and the point $\left(6, \dfrac{5}{2}\right)$ lies on the graph. Find the values of a, b and c. **[5 marks]**