AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. Please note - this topic is difficult and worth a lot of marks. I recommend you use the learning quiz regularly to develop your confidence and skills and/or practice the top 50 integrals activity in the Integration Revision Pack.

## APPRENTICE

Find

a. \( \int 1 + \frac{1}{3} \sin(1 - x) \, dx \)

b. \( \int 4 \cosec^2 \frac{x}{4} \, dx \)

c. \( \int \frac{u + 1}{2u \sqrt{u}} \, du \)

d. \( \int \sec^2 4x - 1 \, dx \)

## EXPERT

Find

a. \( \int \left(2\sqrt{u} - \frac{k}{u}\right)^2 \, du \)

b. \( \int \frac{4}{3x + 2} \, dx \)

c. \( \int \frac{4}{(3x + 2)^2} \, dx \)

d. \( \int \frac{4}{\sqrt{3x + 2}} \, dx \)

## MASTER

Find

a. \( \int x^2 \sqrt{1 - x^3} \, dx \)

b. \( \int kxe^{1-x^2} \, dx \)

c. \( \int 3^{2u} \, du \)

d. \( \int \frac{4x}{\sqrt{3x^2 + 2}} \, dx \)
A#30 INTEGRATING FRACTIONS & TRIG

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APPRENTICE

Find

a. \[ \int \frac{2 \cos x}{3 - \sin x} \, dx \]

b. \[ \int \tan^2 \left( \frac{x}{3} \right) \, dx \]

c. \[ \int \frac{2x^2}{3 - x^3} \, dx \]

EXPERT

Find

a. \[ \int \frac{2}{3} \tan 4x \, dx \]

b. \[ \int \frac{\frac{\pi}{8}}{\frac{\pi}{16}} 9 - 6 \cos^2 4x \, dx \]

c. \[ \int \frac{19x - 2}{1 + 6x} \, dx \]

MASTER

a. Find the exact value of \[ \int_{4}^{6} \frac{x + 7}{x^2 - x - 6} \, dx \]

b. Show that \[ \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sqrt{1 + \cos 2x}}{\sin x \sin 2x} \, dx = \frac{1}{2} (\sqrt{6} - \sqrt{2}) \]
AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you’re ready to move onto past papers for this topic.

**APPRENTICE**

Evaluate \( \int_{0}^{1} 16x e^{4x} \, dx \)

**EXPERT**

Find \( \int \frac{x}{\cos^2 x} \, dx \)

**MASTER**

Evaluate \( \int_{1}^{e} (\ln x)^2 \, dx \)
Use the substitution \( t = \sqrt{x + 1} \) to find \( \int e^{2\sqrt{x + 1}} \, dx \)

Use the substitution \( u = 6 - x^2 \) to find the value of \( \int_{1}^{2} \frac{x^3}{\sqrt{6-x^2}} \, dx \), giving your answer in the form \( p\sqrt{5} + q\sqrt{2} \), where \( p \) and \( q \) are rational numbers.

a. Use the substitution \( x = 4 \sin^2 \theta \) to show that \( \int_{0}^{3} \sqrt{\frac{x}{4-x}} \, dx = \lambda \int_{0}^{\frac{\pi}{2}} \sin^2 \theta \, d\theta \) where \( \lambda \) is a constant.

b. Hence find \( \int_{0}^{3} \sqrt{\frac{x}{4-x}} \, dx \) giving your answer in the form \( a\pi + b \), where \( a \) and \( b \) are exact constants.
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**APPRENTICE**

Figure 1 shows a sketch of the curve C with parametric equations

\[ x = 5t^2 - 4, \quad y = t(9 - t^2) \]

The curve C cuts the x-axis at the points A and B.

a. Find the x-coordinate at the point A and the x-coordinate at the point B.

The region \( R \) as shown shaded in Figure 2, is enclosed by the loop of the curve.

b. Use integration to find the area of \( R \).

**EXPERT**

The curve shown in Figure 2 has parametric equations

\[ x = t - 2 \sin t, \quad y = 1 - 2 \cos t, \quad 0 \leq t \leq 2\pi \]

a. Show that the curve crosses the x-axis where \( t = \frac{\pi}{3} \) and \( t = \frac{5\pi}{3} \).

The finite region \( R \) is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

b. Show that the area \( R \) is given by the integral

\[ \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 \, dt \]

c. Use this integral to find the exact value of the shaded area.

**MASTER**

Figure 3 shows a sketch of part of the curve C with parametric equations

\[ x = 3\theta \sin \theta, \quad y = \sec^3 \theta, \quad 0 \leq \theta < \frac{\pi}{2} \]

a. The point \( P(k,8) \) lies on \( C \), where \( k \) is a constant. Find the exact value of \( k \).

The finite region \( R \) shown shaded in Figure 4, is bounded by the curve \( C \), the y-axis, the x-axis and the line with equation \( x = k \).

b. Show that the area of \( R \) can be expressed in the form

\[ \lambda \int_{\alpha}^{\beta} \left( \theta \sec^2 \theta + \tan \theta \sec^2 \theta \right) \, d\theta \]

where \( \lambda, \alpha \) and \( \beta \) are constants to be determined.

c. Hence use integration to find the exact value of the area of \( R \).
APPRENTICE

The rate of decay of the mass of a particular substance is modelled by the differential equation \( \frac{dx}{dt} = -\frac{5}{2} x \), where \( x \) is the mass of the substance measured in grams and \( t \) is the time measured in days.

Given that \( x = 60 \) when \( t = 0 \), solve the differential equation, giving \( x \) in terms of \( t \).

EXPERT

Given that \( \frac{dy}{dx} = \frac{(16 + 5x - 2x^2)y}{(x + 1)^2(x + 4)} \) and that \( y = \frac{1}{256} \) when \( x = 0 \), find the exact value of \( y \) when \( x = 2 \).

Give your answer in the form \( A e^n \).

MASTER

a. By writing \( \theta = \tan^{-1}\left( \frac{3x}{2} \right) \) as \( 2 \tan \theta = 3x \), use implicit differentiation to show that

\[ \frac{d\theta}{dx} = \frac{k}{4 + 9x^2}, \]

where \( k \) is an integer.

b. Hence solve the differential equation \( 9y(4 + 9x^2) \frac{dy}{dx} = \cosec \ 3y \) given that \( x = 0 \) when \( y = \frac{\pi}{3} \).

Give your answer in the form \( g(y) = h(x) \).