



**Skill:**      **Locating Roots Approximately**

### Questions

*Attempt these questions independently showing full and clear solutions. Check each answer as you go.*

1. By drawing suitable sketches, state the number of roots to the following equations. Furthermore, state the sign of each root.

a.  $x^2 = 2x + 1$

b.  $x^3 = 9 - x$

c.  $\sqrt{x+2} = x^2 - 3$

d.  $\sin x = e^{-x} \quad (|x| \leq \pi)$

e.  $x \ln(x-1) = 1$

f.  $2 \cos x = -3 \quad (|x| \leq \pi)$

g.  $|e^x - 4| - 1 = 0$

2. Use the change of sign method to show that the following equations have roots in the given interval:

a.  $\sin x - 2x + 1 = 0$   
in the interval  $(0.8, 0.9)$

b.  $e^x = 5 - x$   
in the interval  $(1, 2)$

c.  $x^{\frac{1}{3}} = 7 - x$   
in the interval  $(5, 6)$

d.  $\cos 2x + x = 0$   
in the interval  $(-1, 0)$

3. Consider  $f(x) = \frac{1}{x}$

a. Sketch  $y = f(x)$  for  $x \in \mathbb{R}$ .

b. Calculate  $f(1)$  and  $f(-1)$ .

c. Why does your answer to (b) NOT imply that a root lies in the interval  $(-1, 1)$ .

4. Prove that the equation  $x^2 + 8x - 5 = \sqrt{x}$  has root  $x = 0.671$  correct to 3 s.f.

5. Prove that the equation  $\sin x = \frac{1}{2}x$  has root  $x = 1.895$  correct to 4 s.f.

6. Prove that the equation  $e^{\frac{1}{5}x} = \frac{1}{x}$  has root  $0.845$  correct to 3 d.p.

7. Prove that the equation  $\sec x = \frac{1}{x^2}$  has root  $x = 0.82$  correct to 2 d.p.