



Work Done by an External Force

Key Idea

Consider a simple example: lifting a ball and throwing it upward with a certain velocity. Here, the force you apply to the ball is an *external, non-conservative force*. This force does work on the ball, transferring energy to it.

Work Done by External Force & Energy Transfer

The work done by the applied force results in a change in the kinetic energy of the ball. Additionally, as the ball moves away from Earth, its gravitational potential energy changes. We denote this change in potential energy as ΔU . Therefore, the work done by the external force (W_{NC}) and the conservative force (W_C) of gravity can be expressed as:

W_{NC} = Work done by applied force F
 W_C = Work done by force of gravity

$$W_{NET} = W_{NC} + W_C \quad (1)$$

$$W_{NET} = \Delta K \quad (2)$$

$$W_C = -\Delta U \quad (3)$$

$$\Delta K = W_{NC} - \Delta U$$

$$W_{NC} = \Delta K + \Delta U$$

$$W_{NC} = \Delta E_{mech}$$

$$W_{NET} = W_{NC} + W_C \quad (1)$$

According to the work-energy theorem, the net work done on an object result in a change in its kinetic energy (ΔK):

$$W_{NET} = \Delta K \quad (2)$$

Also, the relationship between the work done by a conservative force and PE is given by:



$$W_c = -\Delta U \quad (3)$$

By substituting equations (2) and (3) into equation (1), we derive:

$$\Delta K = W_{NC} - \Delta U \text{ or}$$

$$W_{NC} = \Delta K + \Delta U$$

This implies that the work done by the external force equals the change in the mechanical energy of the system:

$$W_{NC} = \Delta E$$

These relationships are often referred to as "energy statements" for work done by external forces.

Inclusion of Friction: Extending the Concept of Work

Imagine pulling a block horizontally with a constant force (F) over a distance (d), resulting in a change in the block's velocity from v_0 to v . During this process, a kinetic frictional force (f_k) from the floor acts against the movement.

$$F - f_k = ma$$

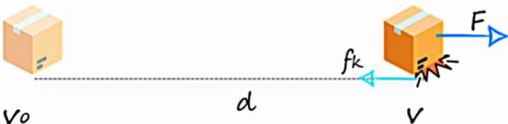
$$v^2 = v_0^2 + 2ad$$

$$a = (v^2 - v_0^2) / 2d$$

$$Fd = \Delta K + f_k d$$

$$Fd = \Delta K + \Delta E_{th}$$

$$W_{NC} = \Delta K + \Delta E_{th}$$



$$K = K_0 + W_{NC} - f_k d$$

$$W_{NC} = \Delta K + \Delta U + \Delta E_{th}$$

$$W_{NC} = \Delta E_{mech} + \Delta E_{th}$$

$$Fd = 1/2(mv^2) - 1/2(mv_0^2) + f_k d$$

Applying Newton's second law, we get:

$$F - f_k = ma \quad (4)$$

Where a is the constant acceleration of the block. The final velocity can then be expressed as:

$$v^2 = v_0^2 + 2ad$$



From which acceleration (a) can be solved:

$$a = (v^2 - v_0^2) / 2d$$

Rearranging and substituting "a" into (4) gives us:

$$Fd = 1/2 mv^2 - 1/2 mv_0^2 + f_k d$$

But, $1/2 mv^2 - 1/2 mv_0^2$ is the change in kinetic energy (ΔK), the equation becomes:

$$Fd = \Delta K + f_k d$$

Here, the term $f_k d$ represents the increase in thermal energy due to friction. Thus, the work done by the external force can be summarized as:

$$W_{NC} = \Delta K + \Delta E_{th}$$

Where ΔE_{th} is the change in thermal energy. This equation gives energy transformations when friction is present.

Energy Transfer and the Law of Conservation of Energy

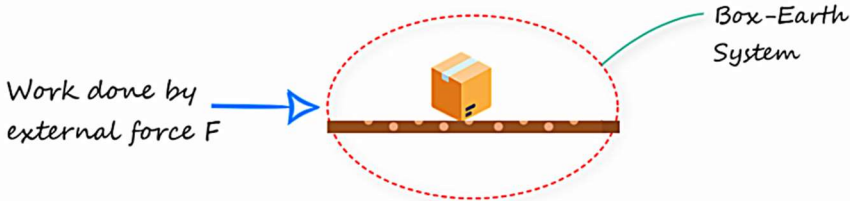
The law of conservation of energy states that the total change in a system's energy is equal to the energy transferred *to or from* the system. This principle can be applied to understand how work done by external forces alters the energy of a system, which can be broken down into mechanical, thermal, and possibly other forms of internal energy:

$$W_{NC} = \Delta E = \Delta E_{MEC} + \Delta E_{TH} + \Delta E_{INT}$$

LAW OF CONSERVATION OF ENERGY

The change in total energy E of a system = Magnitudes of energy transfer to or from the system

$$W_{NC} = \Delta E$$

$$W_{NC} = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$$


Work done by external force F

Box-Earth System



Summary of Formulas and Equations

S.N.	Formula/Equation	When to Use	Cautions While Using
1	$W_{NET} = W_C + W_{NC}$	Use to calculate the total work done on a system involving both non-conservative (W_{NC}) and conservative forces (W_C).	Ensure correct identification of forces as conservative or non-conservative to avoid incorrect results.
2	$W_{NET} = \Delta K$	Apply the work-energy theorem to relate the net work done on an object to the change in its kinetic energy (ΔK).	Verify that all forces doing work are accounted for; missing forces can lead to errors in calculating kinetic energy changes.
3	$W_C = -\Delta U$	Use to connect the work done by conservative forces to changes in potential energy (ΔU).	Ensure that ΔU represents the change in potential energy accurately; signs are critical as they indicate the direction of the change.
4	$W_{NC} = \Delta K + \Delta E_{TH}$	To determine the total work done by non-conservative forces considering both changes in kinetic energy and thermal energy due to friction (ΔE_{TH}).	Ensure that all forms of energy changes including thermal are accounted for to avoid discrepancies in energy calculations.
5.	$W_{NC} = \Delta E = \Delta E_{MEC} + \Delta E_{TH}$	If the potential energy is also changing then add ΔU in equation 4 above to get $\Delta K + \Delta U$ that is ΔE_{MEC}	Equations (4) and (5) are of the same form. Equation (5) just factors in change in PE as well

