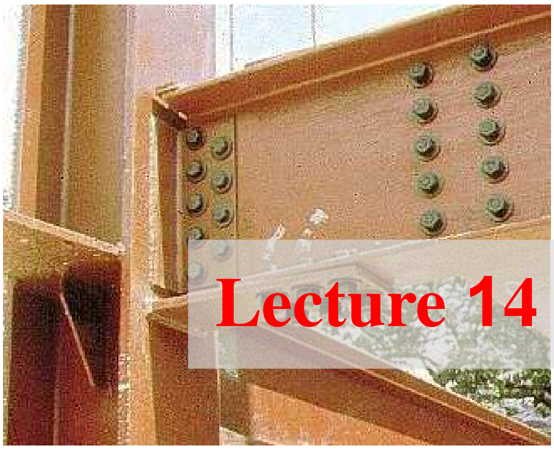


Timber and Steel Design

Lecture 14 Bolted Connections II

- Gusset Plate
- Eccentric Shear
- Axial Tension
- Shear and Tension
- End Plate Connections

Mongkol JIRAVACHARADET

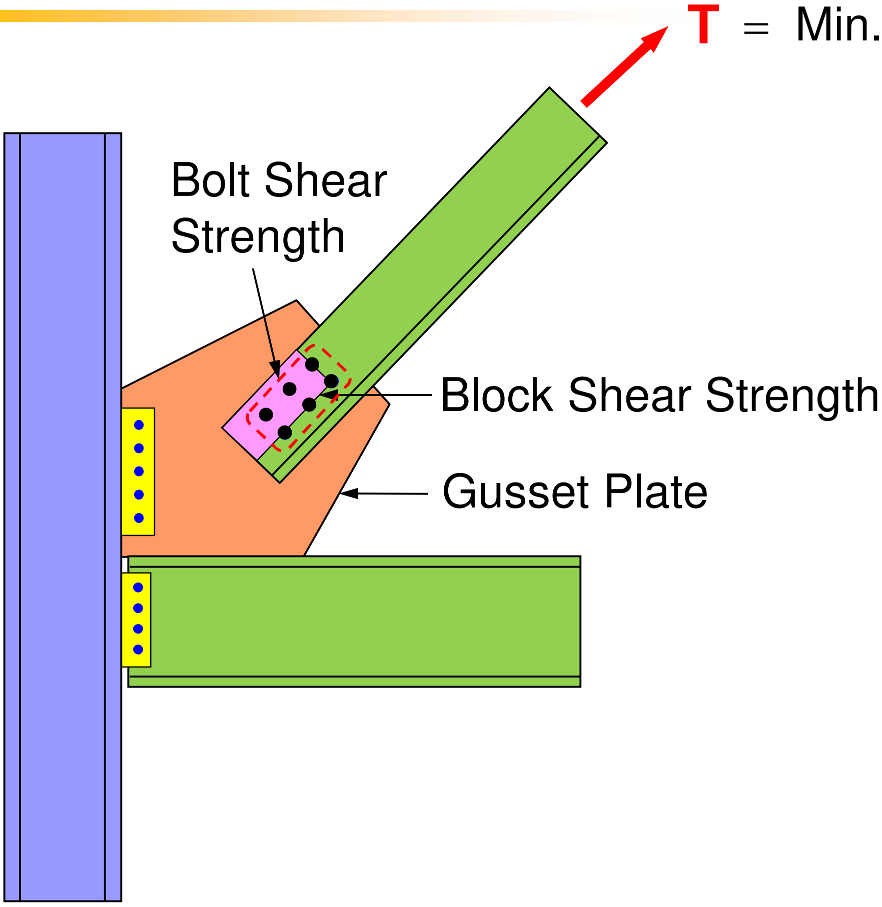
SURANAREE UNIVERSITY OF TECHNOLOGY

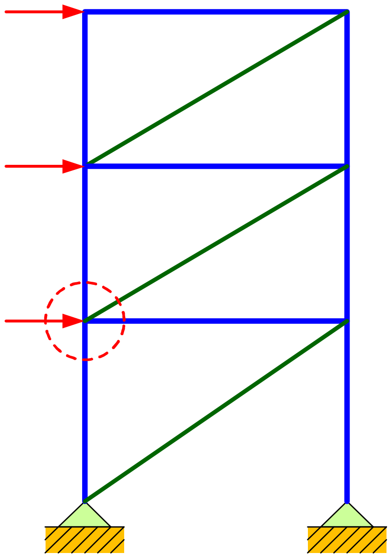
INSTITUTE OF ENGINEERING

SCHOOL OF CIVIL ENGINEERING

Bracing Connections

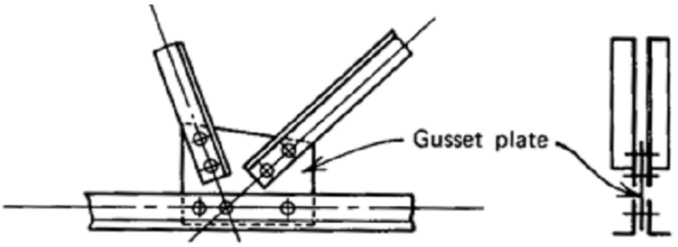
$$T = \text{Min. of} \begin{cases} 0.6\, F_y\, A_g \\ 0.5\, F_u\, Ae \end{cases}$$

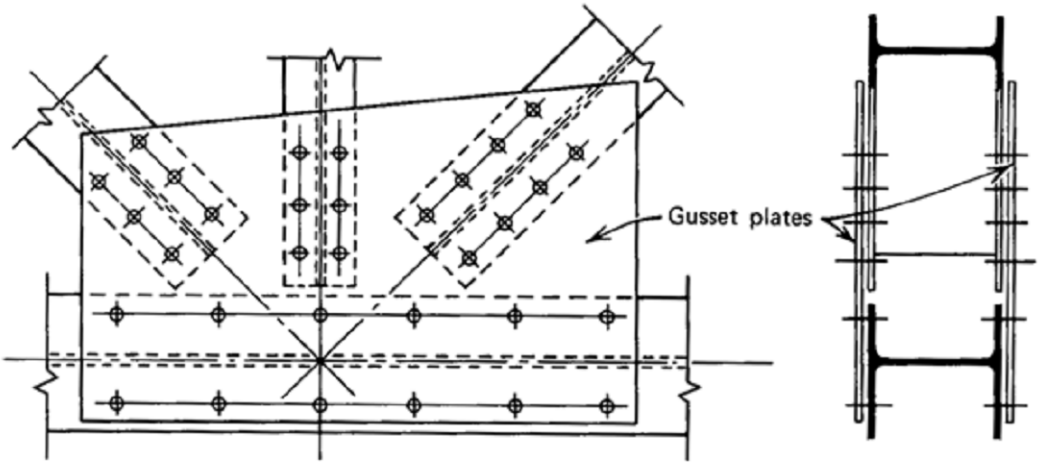




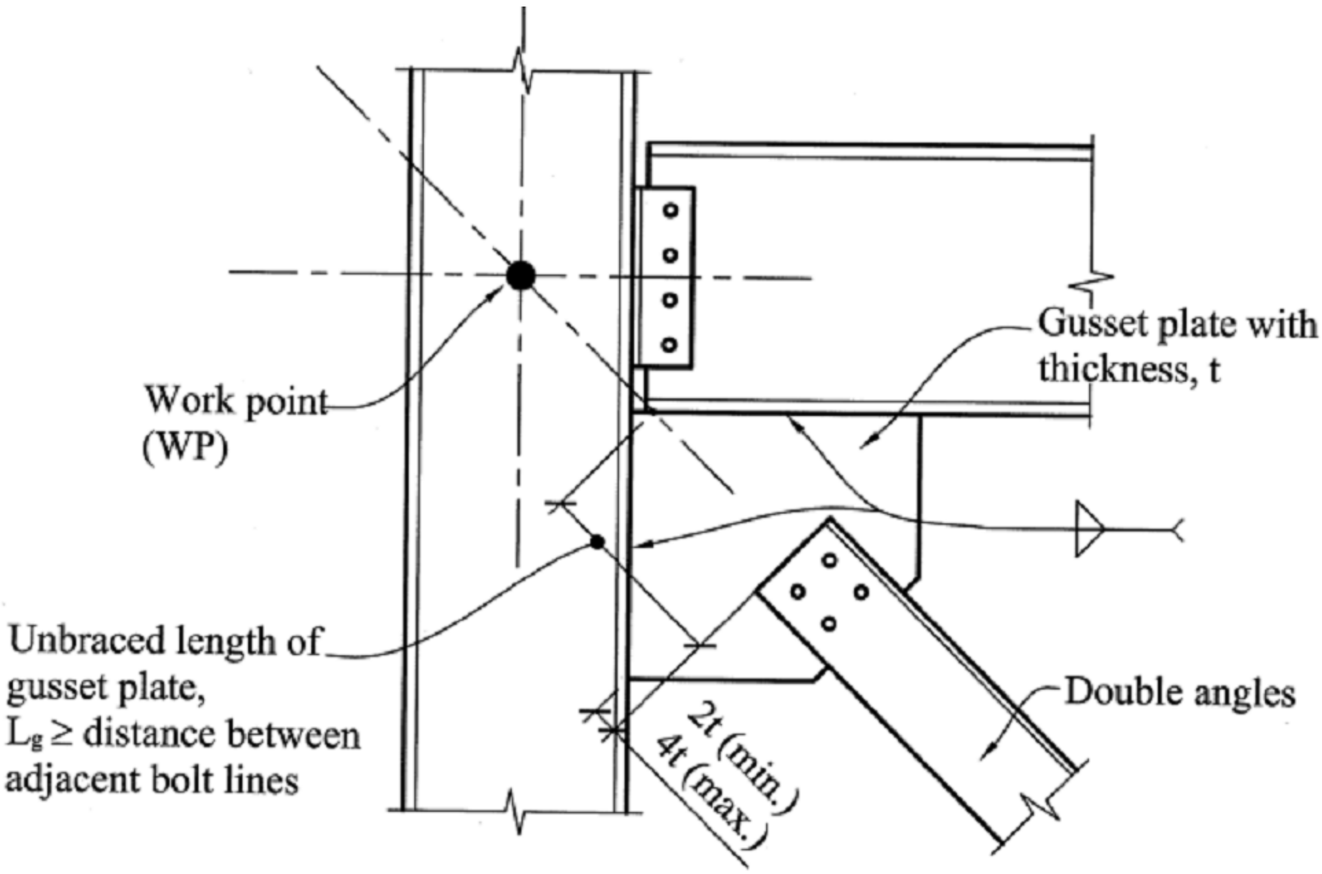
Gusset Plate

Gusset plate is used to transfer load between members that can not be joined directly.





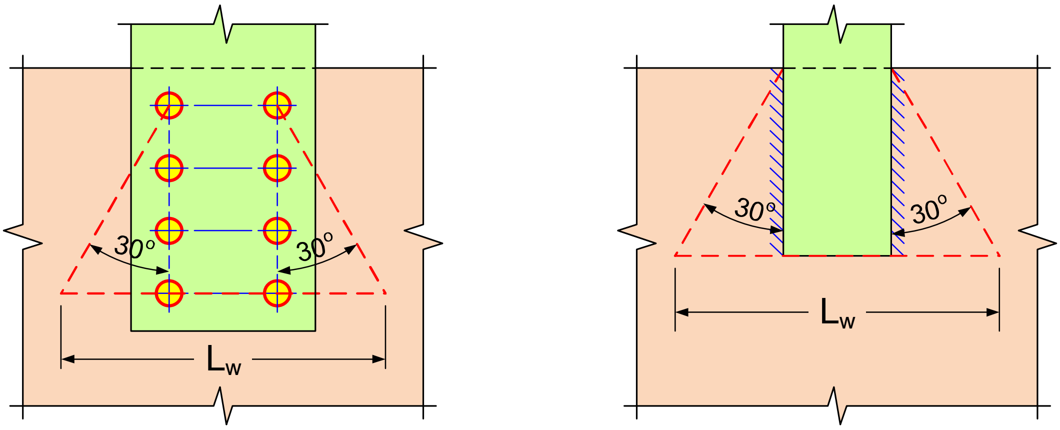
Gusset Plate of a Diagonal Brace



Gusset Plate

Whitmore (1952) study stress distribution in gusset plate and found that the stress can be assumed uniformly distributed over an effective area.

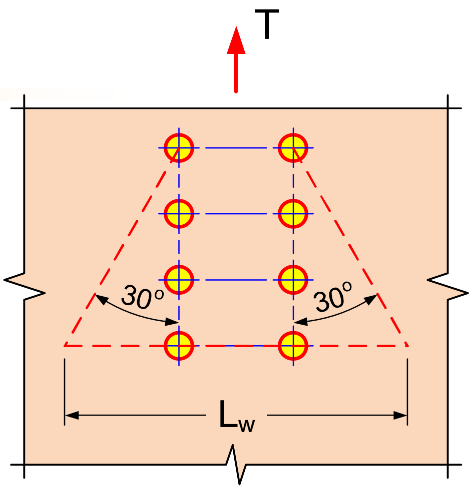
- Projecting 30º lines on both side from first to last row of bolts
- For welded connection, projecting 30º lines on both side of the welds



(a) Bolted Joint (b) Welded Joint

Effective area = Effective width (L_w) × Thickness (t)

Gusset Plate in Tension

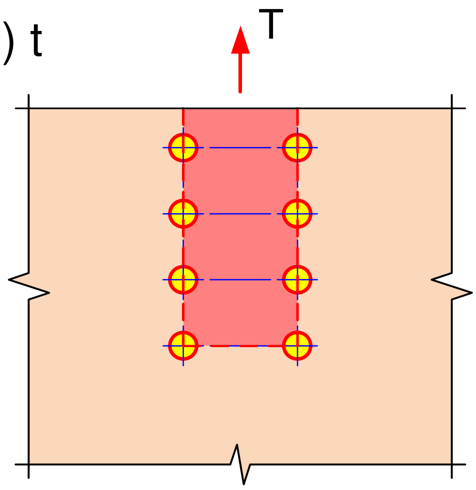


Gross Section Yielding Strength

$$T = 0.6\,F_y\,A_g = 0.6\,F_y\,L_w\,t$$

Net Section Rupture Strength

$$T = 0.5\,F_u\,A_e = 0.5\,F_u\,(L_w - n\,d_{hole})\,t$$



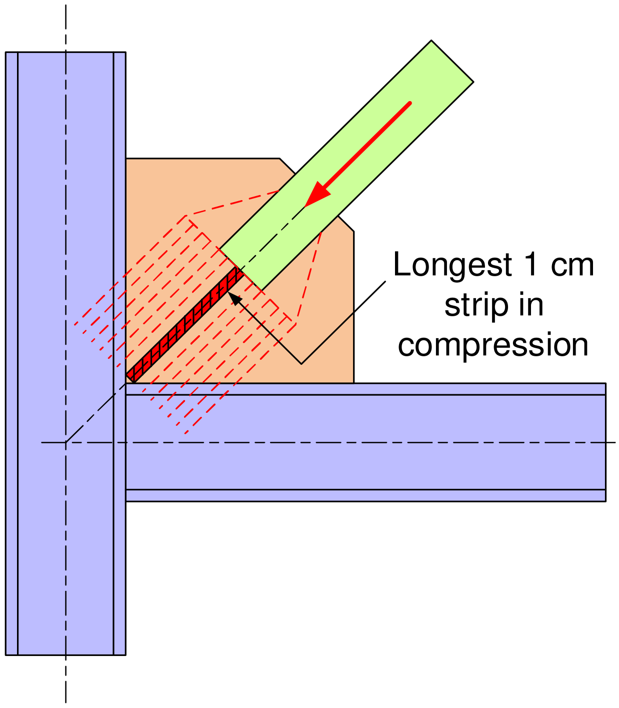
Block Shear Strength

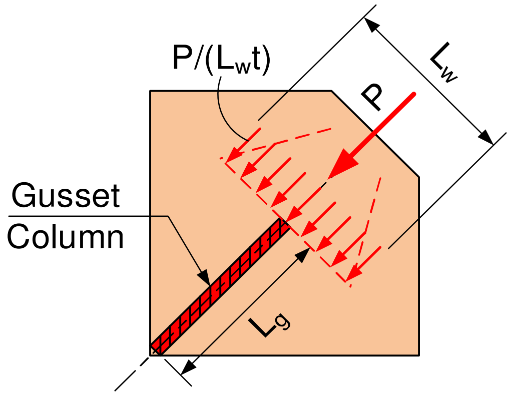
$$T = 0.3\,F_u\,A_v + 0.5\,F_u\,A_t$$

Gusset Plate in Compression

Under compression, a gusset plate can buckle in the area beyond the end of bracing.

To compute buckling strength, 1-cm longest strip within the Whitmore's effective width is treated as a column with an effective length factor K = 1.2.





L_w = Whitmore's width

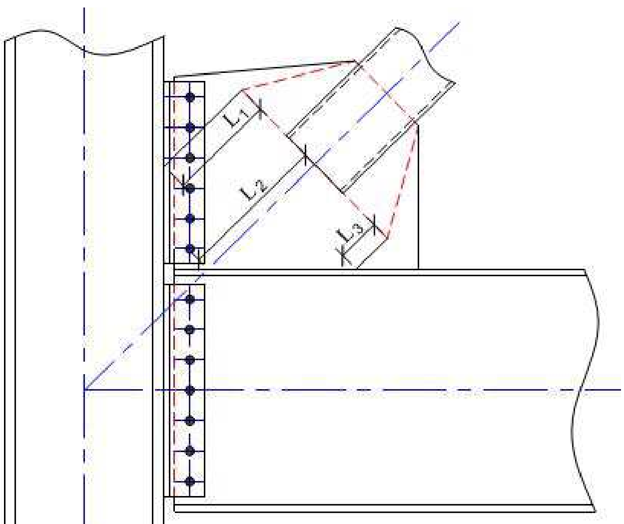
t = Gusset thickness

L_g = Length of gusset column

Gusset Plate Buckling Length

Two methods are permitted to determine an unbraced length L_g of gusset plate.

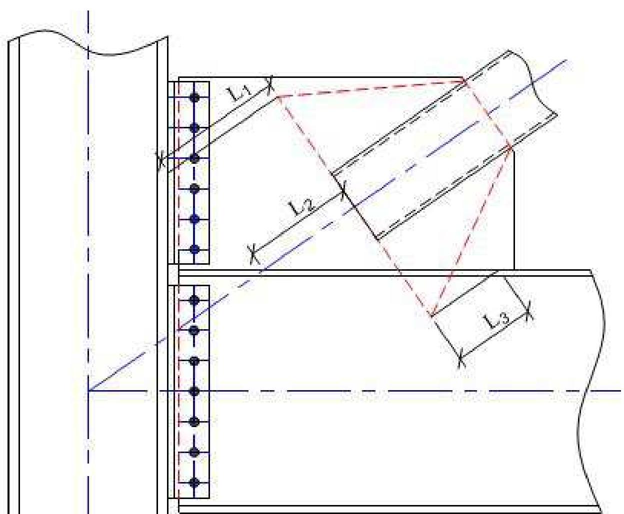
A largest distance (L_2) or an average of the three lengths (L_1, L_2 and L_3).



$$L_g = L_2$$

or

$$L_g = \frac{L_1 + L_2 + L_3}{3}$$



$$L_g = L_2$$

or

$$L_g = \frac{L_1 + L_2 - L_3}{3}$$

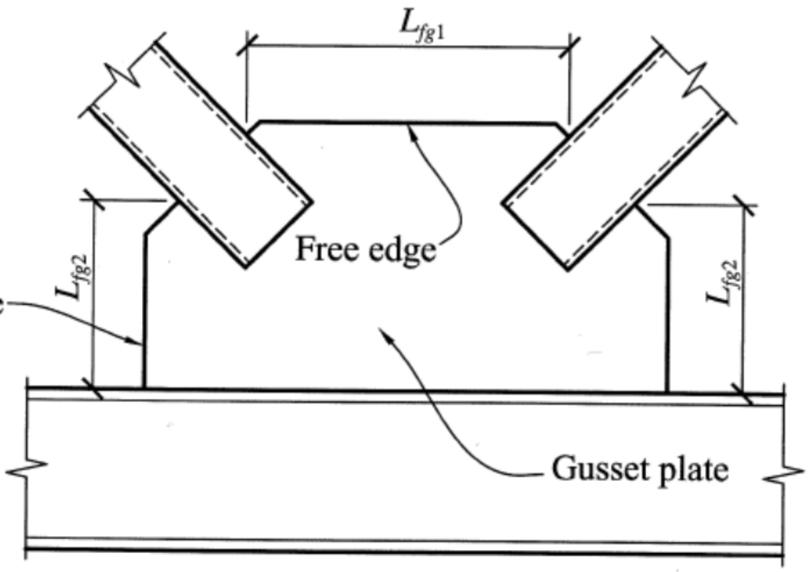
Buckling of Free Edge

For monotonic or static loading

$$t \geq 0.5 L_{fg} \sqrt{\frac{f_y}{E}}$$

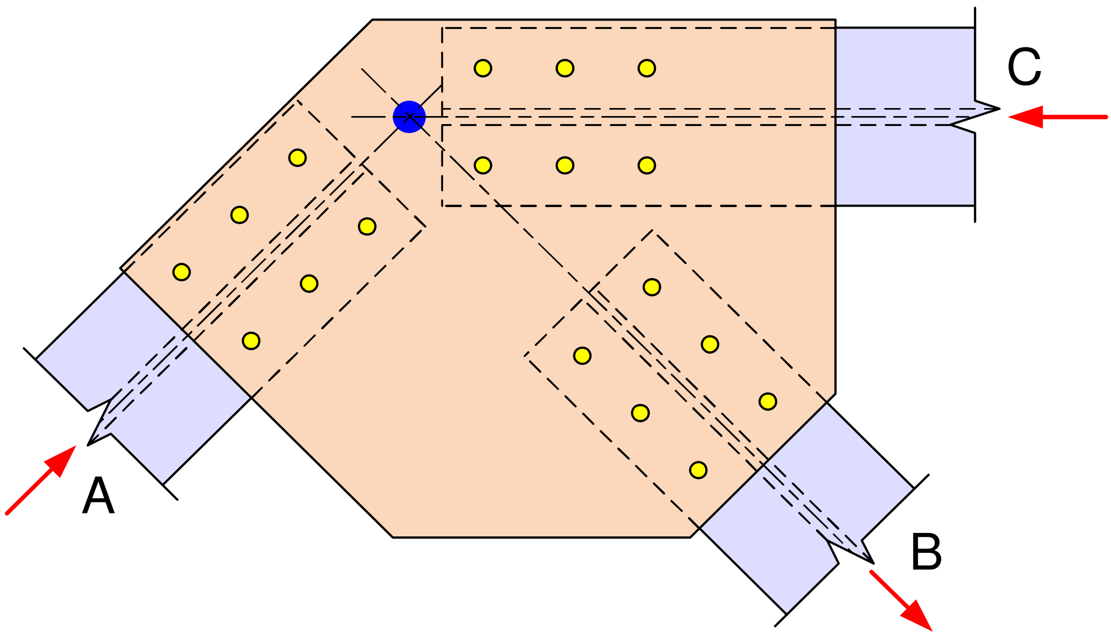
For cyclic or seismic loading

$$t \geq 1.33 L_{fg} \sqrt{\frac{f_y}{E}}$$

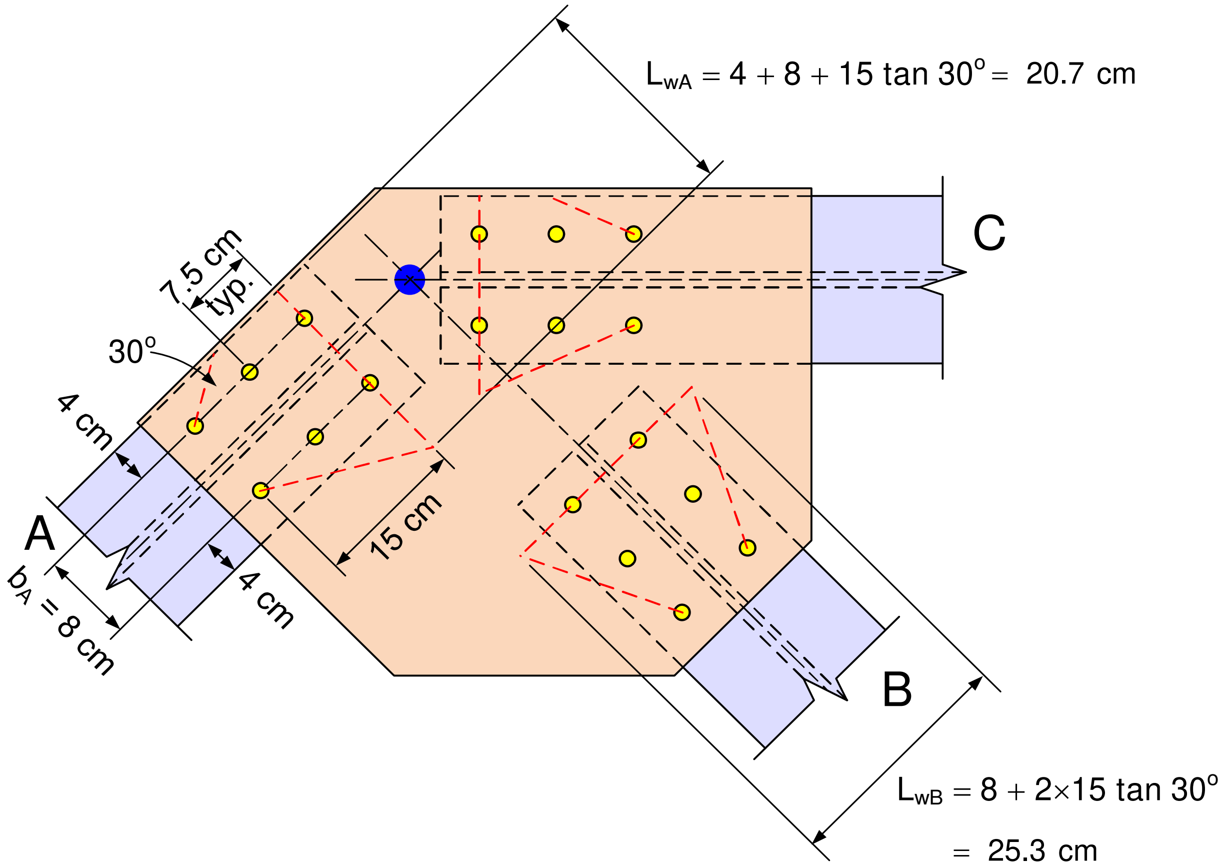


Use the largest free unbraced length, L_{fg} = larger of L_{fg1} or L_{fg2}.

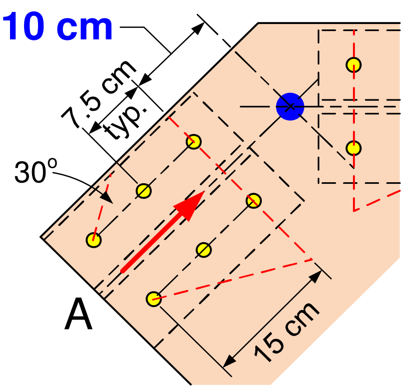
ตัวอย่างที่ 14-1 สำหรับจุดต่อโครงถักดังในรูป แผ่นประกับมีความหนา 16 ม.ม. เป็นเหล็ก A36 กำลังคราก F_y = 2,500 กก./ซม.^{2} และกำลังดึง F_u = 4,000 กก./ซม.^{2} ใช้สลักเกลียว M20 จงพิจารณากำลังของแผ่นประกับที่ท่อน A และ B



1) Whitmore effective width



2) Compression buckling at A



$$I_A = \frac{1}{12} L_{wA} t^3 = \frac{1}{12}(20.7)(1.6)^3 = 7.07 \text{ cm}^4$$

$$r_A = \sqrt{\frac{I_A}{A}} = \sqrt{\frac{7.07}{20.7 \times 1.6}} = 0.46 \text{ cm}$$

$$\frac{KL_g}{r_A} = \frac{1.2 \times 10}{0.46} = 26.1 \xrightarrow{\text{ตาราง ข.1}} F_a = 1{,}406 \text{ kg/cm}^2$$

$$P_A = F_a A = 1.406 \times 20.7 \times 1.6 = 46.6 \text{ ton}$$

3) Tension Strength at B

Yielding Strength :

$$T = 0.6\, F_y\, L_w\, t = 0.6\times2.5\times25.3\times1.6 = 60.7 \text{ ton}$$ **CONTROL**

Rupture Strength :

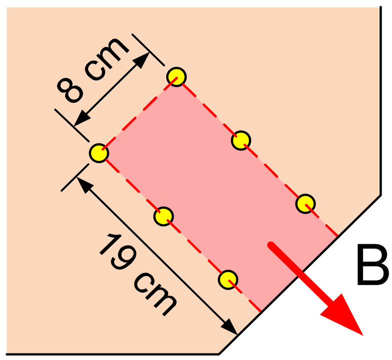
$$T = 0.5\, F_u\, (L_w - nd)\, t = 0.5\times4.0(25.3\text{-}2\times2.2)\times1.6 = 66.9 \text{ ton}$$

Block Shear Strength :

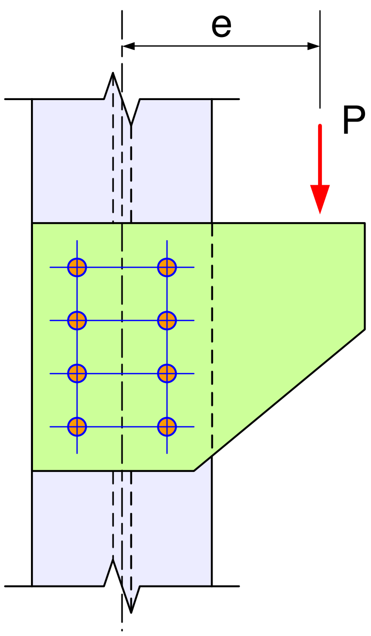
$$T = 0.3\, F_u\, A_v + 0.5\, F_u\, A_t$$

$$= 0.3\times4.0\times2(19\text{-}2.5\times2.2)\times1.6 + 0.5\times4.0(8\text{-}2.2)\times1.6$$

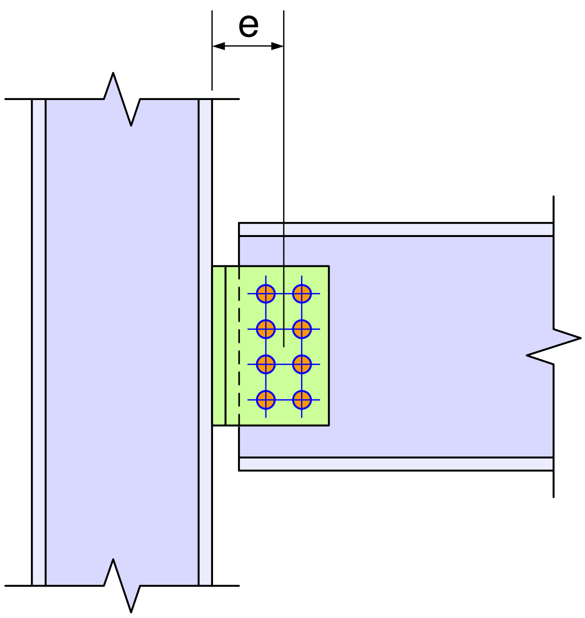
$$= 70.4 \text{ ton}$$



Eccentric Shear Connections



Bracket Connection

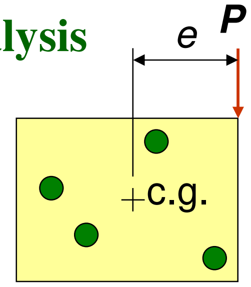


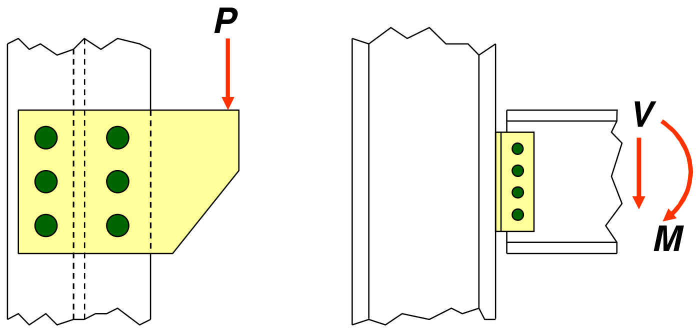
Beam-Column Connection

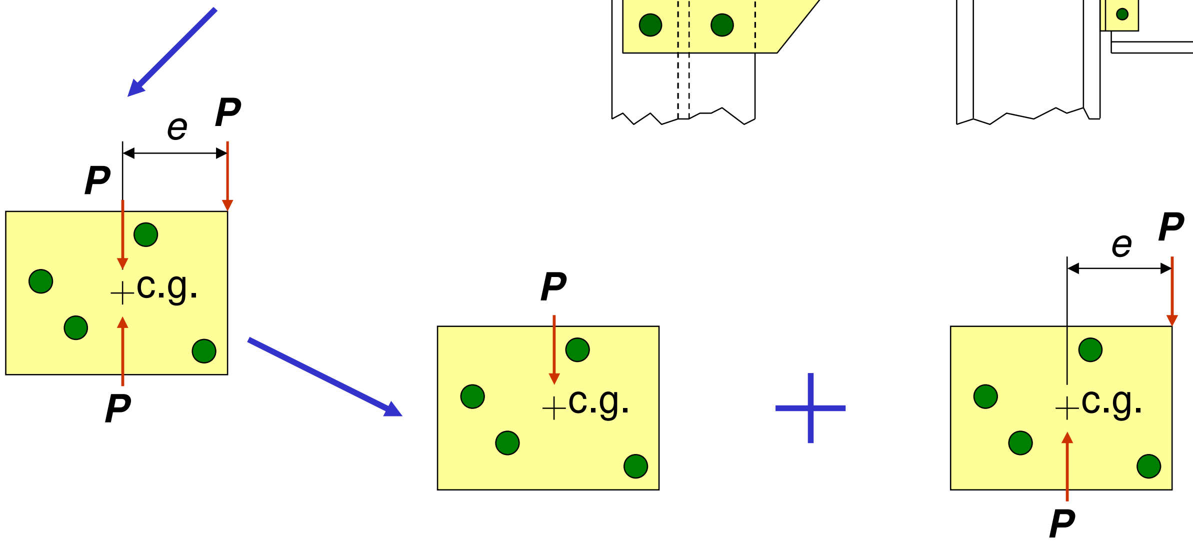
Bolts Subjected to Eccentric Shear

Elastic Analysis

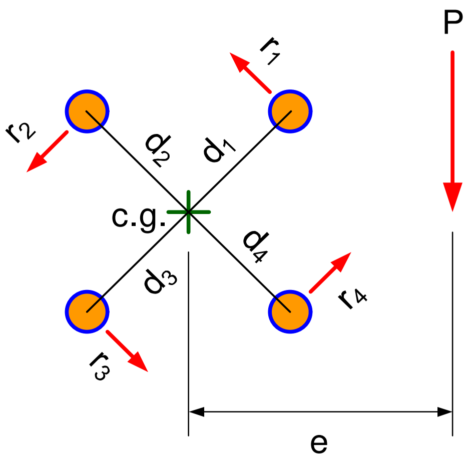
When the load ***P*** does not pass through the center of gravity of a bolt group.



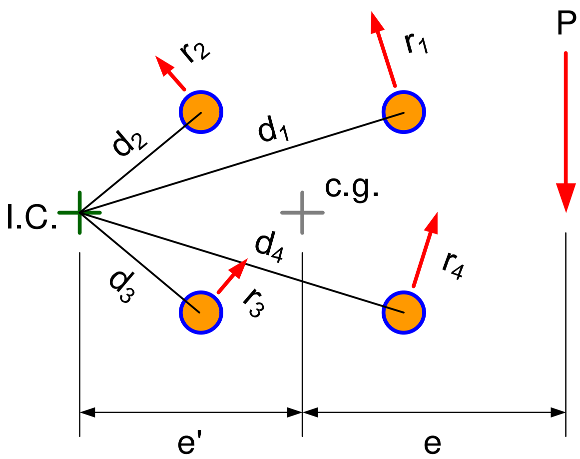




Analysis Methods

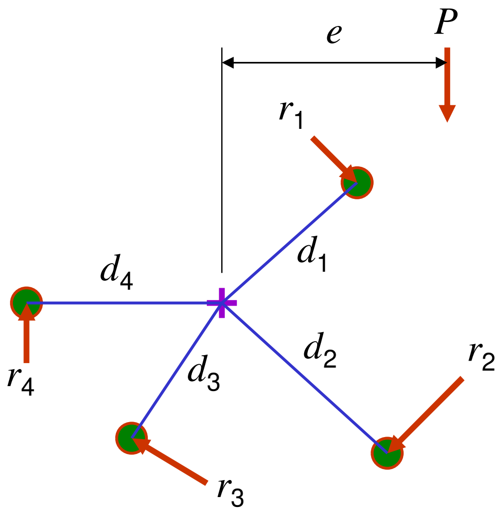


Elastic Method



Instantaneous Center (IC) Method

Moment Resistant of Bolt Group



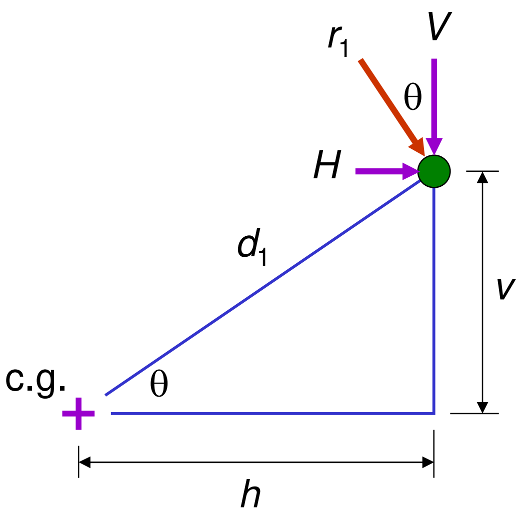
$$M_{\text{c.g.}} = Pe = r_1 d_1 + r_2 d_2 + r_3 d_3 + r_4 d_4$$

Forces on bolts proportions to C.G. distance

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \frac{r_3}{d_3} = \frac{r_4}{d_4}$$

$$r_1 = \frac{Md_1}{\sum d^2}, \quad r_2 = \frac{Md_2}{\sum d^2}, \quad r_3 = \frac{Md_3}{\sum d^2}, \quad r_4 = \frac{Md_4}{\sum d^2}$$

Vertical and Horizontal Components

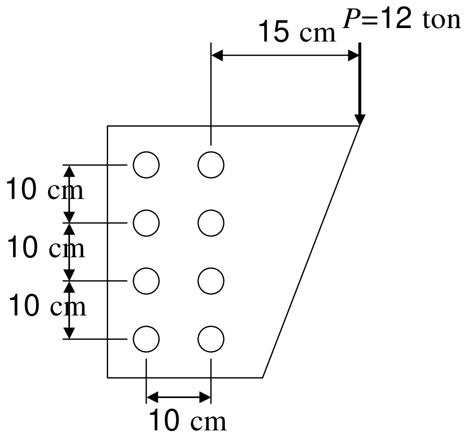


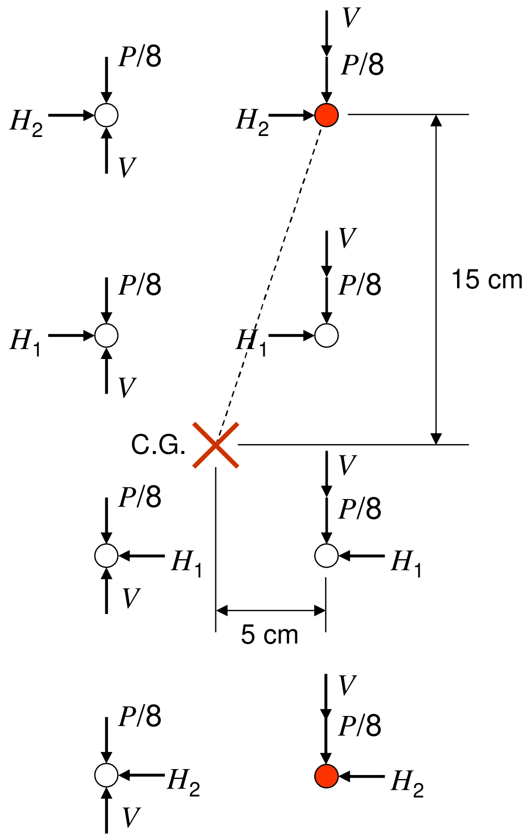
$$\frac{r_1}{d_1} = \frac{r_1 \sin\theta}{d_1 \sin\theta} = \frac{H}{v}$$

$$H = \frac{r_1 v}{d_1} = \left(\frac{Md_1}{\sum d^2}\right)\left(\frac{v}{d_1}\right) = \frac{Mv}{\sum d^2}$$

$$V = \frac{Mh}{\sum d^2}$$

ตัวอย่างที่ 12-1 จงพิจารณาแรงในสลักเกลียวที่มากที่สุดในกลุ่มที่แสดงอยู่ในรูป โดยใช้วิธีการวิเคราะห์แบบอีลาสติก





วิธีทำ สลักเกลียวแต่ละตัวและแรงที่กระทำต่อมันโดยตรงและจากโมเมนต์ถูกแสดงไว้ในรูป ซึ่งจะเห็นได้ว่าสลักเกลียวตัวขวาบนสุดและตัวซ้ายล่างสุดจะมีหน่วยแรงมากที่สุดและเท่ากัน

$$e = 15 + 5 = 20 \text{ cm}$$

$$M = Pe = (12)(20) = 240 \text{ t-cm}$$

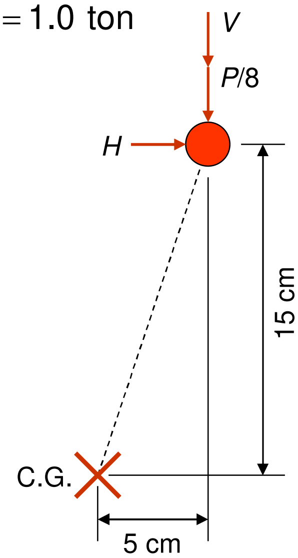
$$\sum d^2 = \sum h^2 + \sum v^2$$

$$\sum d^2 = 8(5)^2 + 4(5^2 + 15^2) = 1{,}200$$

$$H = \frac{Mv}{\sum d^2} = \frac{240(15)}{1200} = 3.0 \text{ ton}$$

$$V = \frac{Mh}{\sum d^2} = \frac{240(5)}{1200} = 1.0 \text{ ton}$$

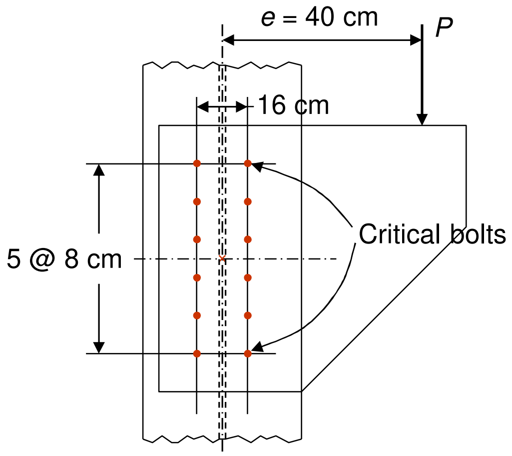
$$P/8 = 12/8 = 1.5 \text{ ton}$$



แรงลัพท์ในสลักเกลียวตัวขวาล่างสุด(เท่ากับตัวขวาบนสุด)จะเท่ากับ

$$r = \sqrt{3.0^2 + 2.5^2} = \underline{\underline{3.9 \text{ ton}}}$$

ตัวอย่างที่ 12-2 จงพิจารณาแรง **P** มากที่สุดที่แป้นหูช้างดังในรูปสามารถรองรับได้ เสาและแป้นหูช้างทำด้วยเหล็ก A36 ใช้สลักเกลียว A325 ขนาด 22 ม.ม. ในรูเจาะมาตรฐาน เกลียวอยู่ในระนาบเฉือน โดยใช้วิธีการวิเคราะห์แบบอิลาสติก



วิธีทำ

1. โมเมนต์: **M** = **Pe** = 40**P** ตัน-ซม.

2. ผลรวมระยะกำลังสองจากจุดศูนย์ถ่วง

$$\Sigma h^2 = 12(8)^2 = 768 \text{ cm}^2$$

$$\Sigma v^2 = 4(4)^2 + 4(12)^2 + 4(20)^2 = 2,240 \text{ cm}^2$$

$$\Sigma d^2 = \Sigma h^2 + \Sigma v^2 = 768 + 2,240 = 3,008 \text{ cm}^2$$

3. องค์ประกอบแรงที่สลักเกลียววิกฤต พิจารณาตัวล่างสุดและบนสุดทางด้านขวา

$$H = \frac{Mv}{\Sigma d^2} = \frac{40P(20)}{3,008} = 0.266P \qquad V = \frac{Mh}{\Sigma d^2} = \frac{40P(8)}{3,008} = 0.106P$$

P/12 = 0.083**P** ตัน

4. แรงลัพธ์ในสลักเกลียววิกฤต

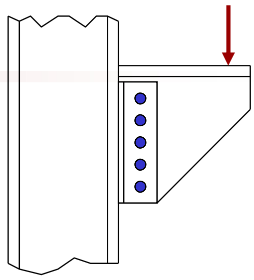
$$r = \sqrt{H^2 + (V + P/12)^2} = P\sqrt{0.266^2 + (0.106 + 0.083)^2} = 0.326P$$

จากตาราง ง.1 สลักเกลียว A325 ขนาด 22 ม.ม. รูเจาะมาตรฐาน เกลียวอยู่ในระนาบเฉือน รับการเฉือนเดี่ยวได้ตัวละ 5.63 ตัน ดังนั้น

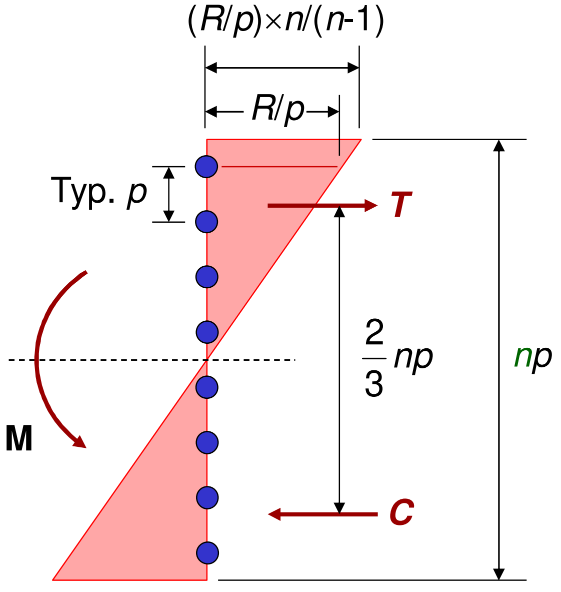
0.326**P** = 5.63 ตัน

น้ำหนักมากที่สุด **P** = 17.3 ตัน ■

Design of Single Line Fasteners under Moment



How many bolts required (n=?)



Assuming $\boldsymbol{R}$ is the force in the outermost fastener

$$R = \frac{M\,d}{\Sigma\,d^2}$$

Average load/length = R/p @ outermost fastener

$$\text{Load/length @ extreme fiber} = \frac{R}{p}\left(\frac{n}{n-1}\right)$$

$$T = \frac{1}{2}\left(\frac{np}{2}\right)\frac{R}{p}\left(\frac{n}{n-1}\right)$$

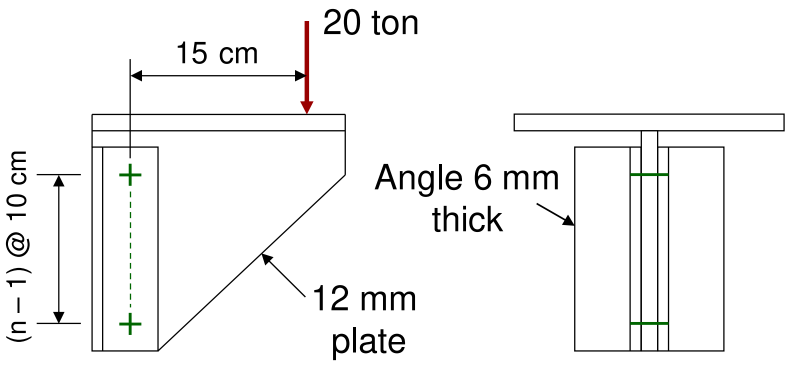
$$M = T\left(\frac{2}{3}np\right) = \frac{R\,n^3 p}{6(n-1)}$$

Solving equation for $\boldsymbol{n}$:

$$n = \sqrt{\frac{6M}{Rp}\left(\frac{n-1}{n}\right)} \cong \sqrt{\frac{6\,M}{R\,p}}$$

From **Steel Structures Design and Behavior 4th Ed. (1996)** by Charles G. Salmon pp.150-156

Example 4.12.6 : Determine the required number of 22-mm-diam A325 bolts for one vertical line of fasteners shown in figures below. Assume it to be a bearing type connection with threads included in the shear planes (A325-N)



Solution:

Design strength of A325-N 22 mm

Double shear: **(control)**

R = 2(π/4)(2.2)^2(1.48) = 11.25 ton

Bearing:

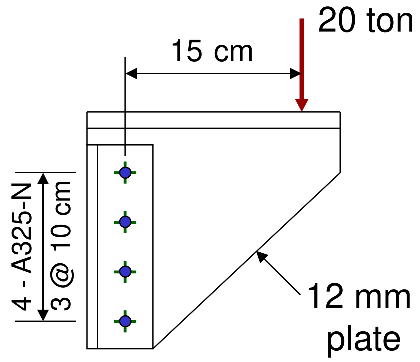
R = 1.2(2.2)(1.2)(4.0) = 12.67 ton

Estimate the number of bolts required, $n = \sqrt{\frac{6M}{Rp}} = \sqrt{\frac{6\times20\times15}{11.25\times10}} = 4.0$

The ***R*** value has ***not*** been adjusted for the direct shear; **try 4 fasteners**

Next Step : Check !

Check the adequacy using an elastic analysis



Moment: $M = 20 \times 15 = 300$ t-cm

$\Sigma d^2 = \Sigma v^2 = 2 \times 5^2 + 2 \times 15^2 = 500$ cm^2

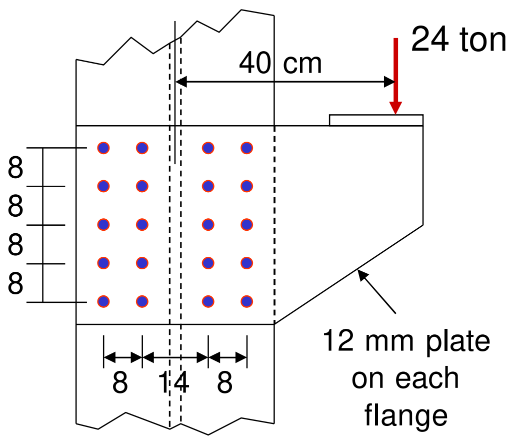
Moment component:

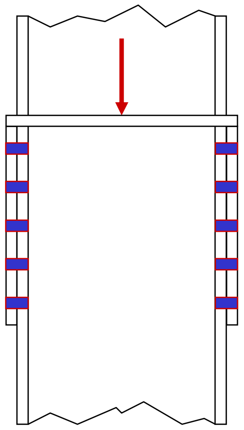
$$R_x = \frac{Mv}{\Sigma d^2} = \frac{300 \times 15}{500} = 9.0 \text{ ton} \leftarrow$$

Direct shear component: $R_s = 20/4 = 5.0$ ton $\downarrow$

Then, the resultant: $R = \sqrt{9^2 + 5^2} = 10.3 \text{ ton} < [\, 11.25 \text{ ton} \,]$ **OK**

Example 4.12.7 : Determine the required number of 19-mm-diam A325 bolts in standard holes for the bracket plate, assuming 4 vertical rows. Assume it to be a bearing type connection with threads included in the shear planes (A325-N)





Solution:

Design strength of A325-N 19 mm

Single shear: **(control)**

$R = (\pi/4)(1.9)^2(2.1) = \underline{5.95 \text{ ton}}$

Bearing:

$R = 1.2(1.9)(1.2)(4.0) = 10.94$ ton

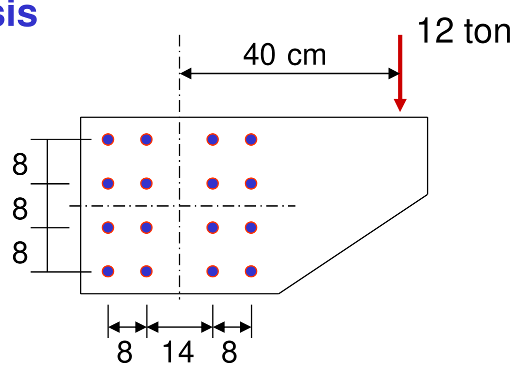
Half load carried by each plate: $P = 24/2 = 12$ ton

Load per line of fasteners: $P/4 = 12/4 = 3$ ton per line

Estimate number of fasteners: $n = \sqrt{\frac{6M}{Rp}} = \sqrt{\frac{6 \times 3 \times 40}{5.95 \times 8}} = 3.89$

Try 4 bolts per row

Check the adequacy using an elastic analysis



Moment: $M = 12 \times 40 = 480$ t-cm

$\Sigma h^2 = 8 \times 7^2 + 8 \times 15^2 = 2{,}192$ cm^2

$\Sigma v^2 = 8 \times 4^2 + 8 \times 12^2 = 1{,}280$ cm^2

$\Sigma d^2 = \Sigma h^2 + \Sigma v^2 = 2{,}192 + 1{,}280 = 3{,}472$ cm^2

Moment component:

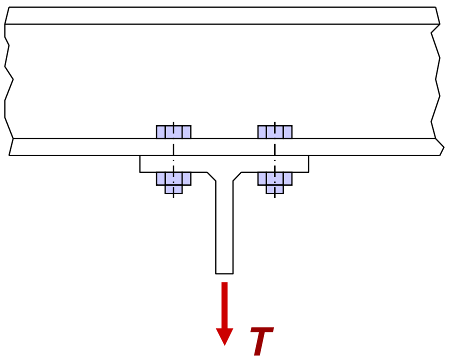
$$R_x = \frac{Mv}{\Sigma d^2} = \frac{480 \times 12}{3{,}472} = 1.66 \text{ ton} \leftarrow$$

$$R_y = \frac{Mh}{\Sigma d^2} = \frac{480 \times 15}{3{,}472} = 2.07 \text{ ton} \downarrow$$

Direct shear component: $R_s = 12/16 = 0.75$ ton $\downarrow$

Then, the resultant: $R = \sqrt{1.66^2 + (2.07 + 0.75)^2} = 3.27 \text{ ton} < [\,5.95 \text{ ton}\,]$ **OK**

Fasteners Acting in Axial Tension



Tensile strength: $T = F_t A_b$

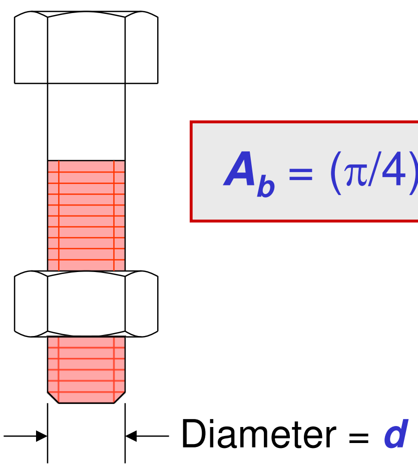
where A_b = fastener gross cross-sectional area

F_t = allowable tensile stress

= **1,400** kg/cm^2 for **A307**

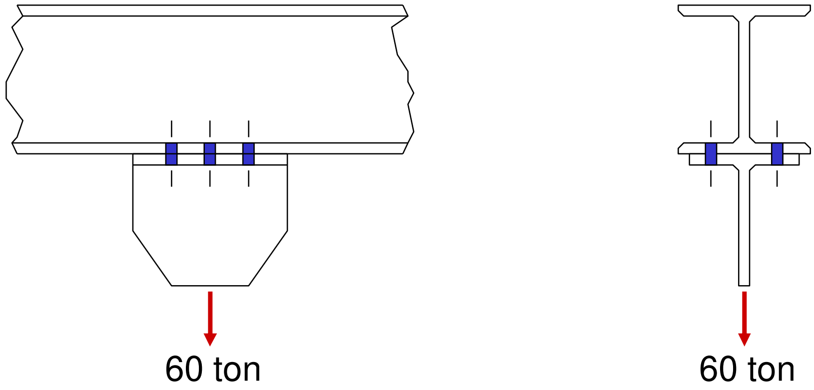
= **3,100** kg/cm^2 for **A325**

= **3,800** kg/cm^2 for **A490**



$A_b = (\pi/4)\,d^2$

Example 4.13.2 : Determine the required number of 19-mm.-diam A490 bolts for the connection.



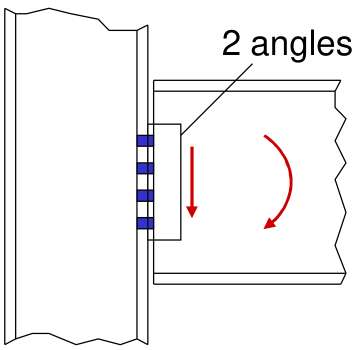
Solution:

Tension strength per bolts, $T = (\pi/4) \times 1.9^2 \times 3.8 = 10.77$ ton

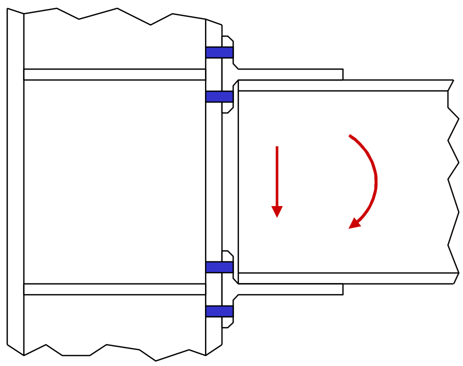
Number ***n*** of bolts required, $n = 60/10.77 = 5.57$, **Say 6**

USE 6 – 19 mm.-diam. A490 bolts ■ ■

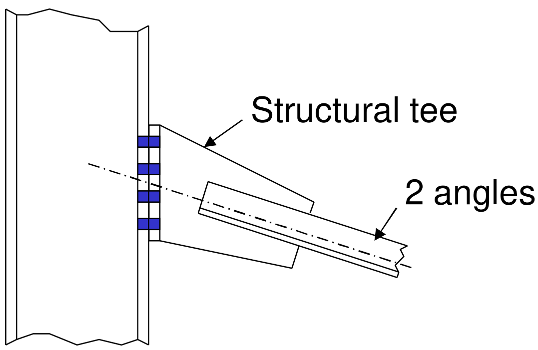
Combined Shear and Tension



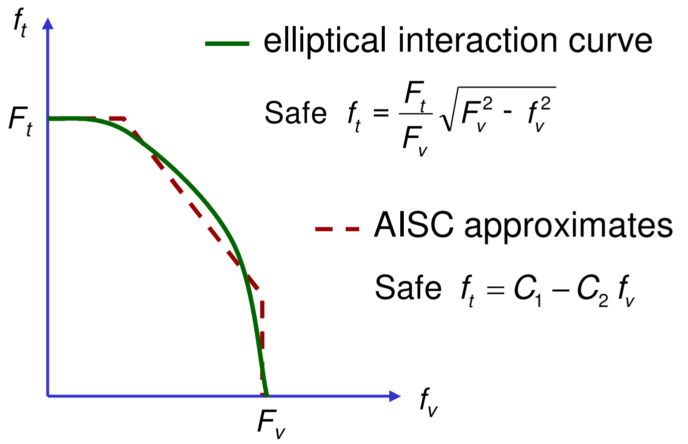
2 angles join the beam web
to the column flange

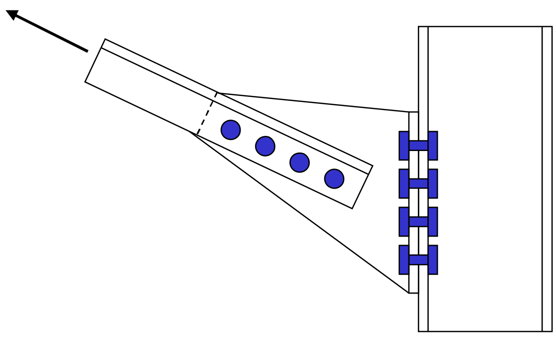


large moment transmitted
through the flanges of beam



Allowable Shear and Tensile Stresses

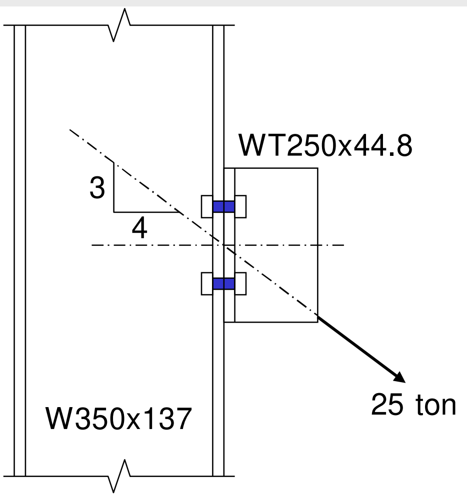


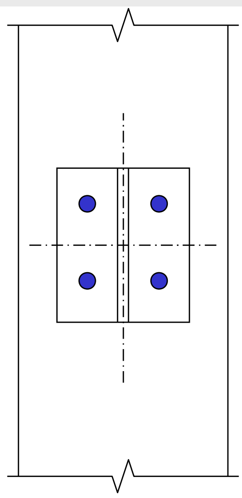


Allowable Tensile Stress F_t for Bolts
Subject to Combined Shear and Tension

Type of Bolt	เกลียวอยู่ในระนาบเฉือน	เกลียวไม่อยู่ในระนาบเฉือน
A307	$1{,}820 - 1.8 f_v \le 1{,}400$	
A325	$\sqrt{(3{,}080)^2 - 4.39 f_v^2}$	$\sqrt{(3{,}080)^2 - 2.15 f_v^2}$
A490	$\sqrt{(3{,}780)^2 - 3.75 f_v^2}$	$\sqrt{(3{,}780)^2 - 1.82 f_v^2}$

Example 7.9 : A WT250x44.8 is used as a bracket to transmit a 25 ton load to a W350x137 column as shown. Four 22-mm. diameter A325 bolts with thread in shear are used. Both the column and bracket are of A36 steel. Determine the adequacy of the connection.





Solution:

For shearing stress,

Total shear force = (3/5)25 = 15 tons

$A_b = (\pi/4)(2.2)^2 = 3.80 \text{ cm}^2$

$$f_v = \frac{15 \times 1{,}000}{4 \times 3.80} = 987 \text{ ksc}$$

$F_v = 0.4(2{,}500) = 1{,}000$ ksc
> 987 ksc **OK**

For bearing stress, $A = (2.2)(1.6) = 3.52 \text{ cm}^2$ (controlled by flange of tee)

$$f_p = \frac{15 \times 1{,}000}{4 \times 3.52} = 1{,}065 \text{ ksc}$$

$F_p = 1.2(2{,}500) = 3{,}000 \text{ ksc} > 1{,}065 \text{ ksc}$ **OK**

From **Steel Design 4th Ed. (2007)** by William T. Segui pp. 401

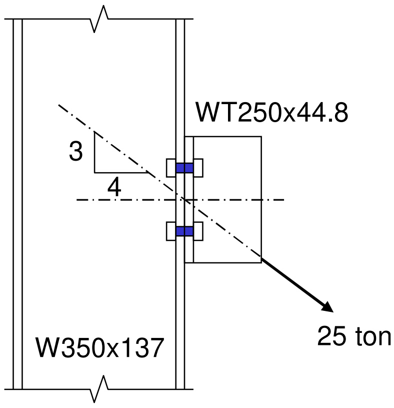
For tensile stress,

Total tensile force = (4/5)25 = 20 tons

$A_b = (\pi/4)(2.2)^2 = 3.80$ cm^2

$$f_t = \frac{20 \times 1{,}000}{4 \times 3.80} = 1{,}316 \text{ ksc}$$

A325 bolts with thread in shear plane:

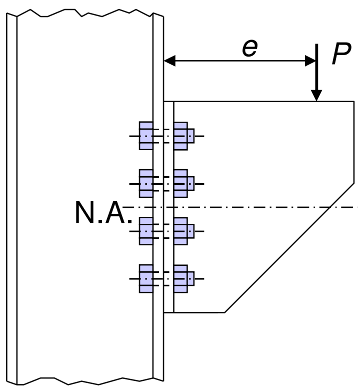


$$F_t = \sqrt{3{,}080^2 - 4.39 f_v^2} = \sqrt{3{,}080^2 - 4.39 \times 987^2} = 2{,}282 \text{ ksc} > [f_t = 1{,}316 \text{ ksc}] \quad \textbf{OK}$$

The connection is adequate as a bearing connection.

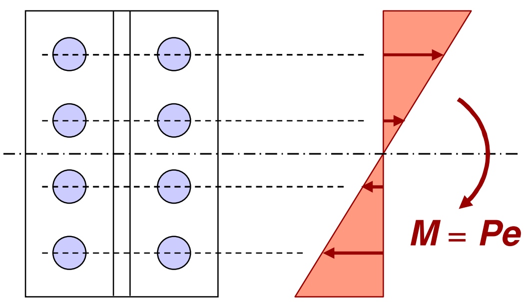
Eccentric Bolts under Shear + Tensile

สลักเกลียวภายใต้แรงเฉือนและแรงดึงเยื้องศูนย์



Shear stress in each bolt:

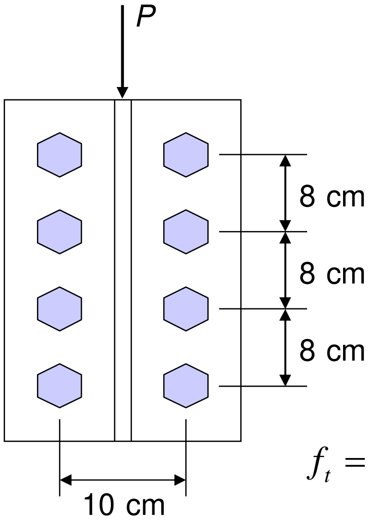
$$f_v = \frac{P}{nA}$$



Tensile stress from moment:

$$f_t = \frac{Mc}{I} = \frac{Pec}{I}$$

ตัวอย่างที่ 12-3 ตรวจสอบจุดต่อหูช้างที่ทำจากเหล็ก A36 ถูกต่อโดยใช้สลักเกลียว A325-N ขนาด 22 ม.ม. ติดกับปีกเสา หูช้างรับน้ำหนักบรรทุก 12 ตัน ห่างจากปีกเสา 30 ซม. ระยะห่างในแนวดิ่งระหว่างสลักเกลียว 8 ซม. และระยะห่างในแนวราบ 10 ซม.



วิธีทำ 1. โมเมนต์อินเนอร์เชียของสลักเกลียว

$$I = 3.8\,[4(4)^2 + 4(12)^2] = 2{,}432 \text{ ซม.}^4$$

หน่วยแรงดึงที่ยอมของ A325-N = 3,100 กก./ซม.^{2}

2. หน่วยแรงดึงที่เกิดขึ้นในสลักเกลียวตัวบนสุด

$$f_t = \frac{Pec}{I} = \frac{(12,000)(30)(12)}{2,432} = 1,776 \text{ ksc} < 3,100 \text{ ksc} \qquad \textbf{OK}$$

3. หน่วยแรงเฉือนที่ยอมให้ A325-N: $\mathbf{F_v}$ = 1,480 กก./ซม.^{2} หน่วยแรงเฉือนในสลักเกลียวคือ

$$f_v = \frac{12,000}{(8)(3.8)} = 395 \text{ ksc} < 1,480 \text{ ksc} \qquad \textbf{OK}$$

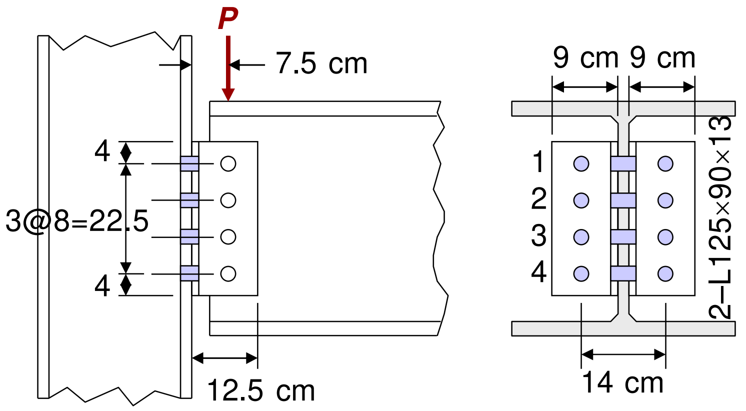
ตรวจสอบหน่วยแรงดึงที่ยอมให้โดยใช้สมการปฏิสัมพันธ์ (จากตารางที่ 11.5):

Type of Bolt	เกลียวอยู่ในระนาบเฉือน	เกลียวไม่อยู่ในระนาบเฉือน
A307	$1820 - 1.8 f_v \le 1400$	
A325	$\sqrt{(3080)^2 - 4.39 f_v^2}$	$\sqrt{(3080)^2 - 2.15 f_v^2}$
A490	$\sqrt{(3780)^2 - 3.75 f_v^2}$	$\sqrt{(3780)^2 - 1.82 f_v^2}$

$$F_t = \sqrt{3,080^2 - 4.39 f_v^2}$$

$$= \sqrt{3,080^2 - 4.39 \times 395^2} = 2,967 \text{ ksc} > 1,776 \text{ ksc} \qquad \textbf{OK}$$

Example 4.15.1 : Determine the service load capacity P for the connection in the figure below, if the fasteners are 19-mm.-diam. A325-X bolts subject to shear and tension in a bearing-type connection with no threads in the shear plane.



Solution:

Bolts moment of inertia:

$$I = 2.84(4\times4^2+4\times12^2) = 1{,}818 \text{ cm}^4$$

Tensile stress on top bolts:

$$f_t = \frac{Pec}{I} = \frac{P\times7.5\times12}{1{,}818} = 0.0495P$$

Shear stress on each bolt: $$f_v = \frac{P}{\Sigma A} = \frac{P}{8\times2.84} = 0.0440P$$

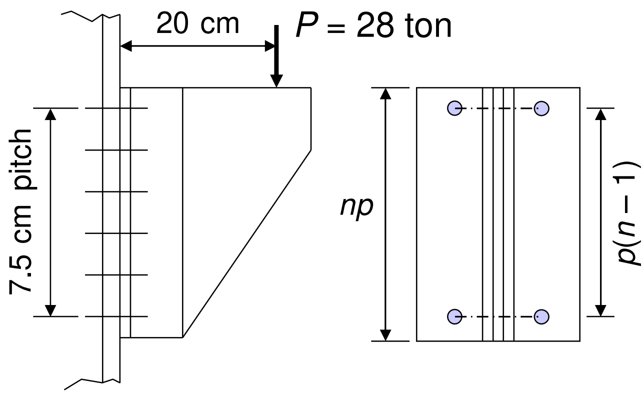
$$\text{A325 - X}:\ F_t = \sqrt{3.08^2 - 2.15f_v^2} = f_{t,\max} = 0.0495P$$

Solve equation for $P = 37.9$ **ton**

Therefore, the service load capacity P is 37.9 ton ■

From **Steel Structures: Design & Behavior 4th Ed. (1996)** by Charles G. Salmon pp. 174

Example 4.15.2 : For the connection of bracket in figure below, determine the number of 22-mm.-diam. A325-N. Use 7.5-cm vertical pitch.



Solution:

Shear strength of one bolt (single shear):

$$A_b = \frac{\pi}{4}(2.2)^2 = 3.8 \text{ cm}^2$$

$$R = 3.8(1.48) = 5.63 \text{ ton}$$

Tensile strength of one bolt:

$$R = 3.8(3.1) = 11.8 \text{ ton}$$

Load per vertical line of bolt:

$$P = 28/2 = 14 \text{ ton/line}$$

$$M = (28\text{x}20)/2 = 280 \text{ ton-cm/line}$$

Approximate number of bolt per line:

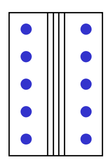
$$n = \sqrt{\frac{6M}{Rp}} = \sqrt{\frac{6\times280}{11.8\times7.5}}$$

$= 4.4$ required for M alone

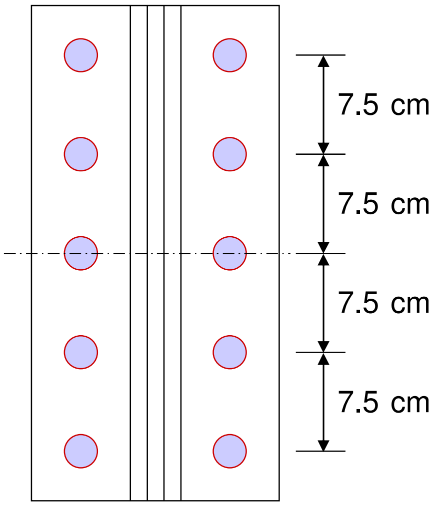
$$n = \frac{P}{R} = \frac{14}{5.63}$$

$= 2.5$ required for shear alone

Try 10 bolts (5 per line)



A325-N: ∅ 22 MM. 10 bolts (5 per line)



Moment of inertia (one line):

$$I = 3.8[2(7.5)^2 + 2(15)^2] = 2{,}137.5 \text{ cm}^4$$

Tensile stress:

$$f_t = \frac{Mc}{I} = \frac{280 \times 10^3 (15)}{2{,}137.5} = 1{,}965 \text{ ksc} < 3{,}100 \text{ ksc}$$ **OK**

Shear stress:

$$f_v = \frac{P}{\Sigma A_b} = \frac{14 \times 10^3}{5(3.8)} = 737 \text{ ksc} < 1{,}480 \text{ ksc}$$ **OK**

Check Shear + Tension Interaction:

$$F_t = \sqrt{3{,}080^2 - 4.39 f_v^2} = \sqrt{3{,}080^2 - 4.39(737)^2}$$

$$= 2{,}665 \text{ ksc} > [\, f_t = 1{,}965 \text{ ksc} \,]$$ **OK**

End of Lecture