The Average & Range Method for MSA

The **average and range** method is capable of separating **repeatability** from **reproducibility** when analyzing the total measurement system variation but requires more effort and calculation than the range method.

This method is also more accurate in estimating the error or variation within a measurement system.

In this method, 2 or 3 operators (appraisers) measure a minimum of 10 parts. More parts can be measured, which only increases the accuracy of the analysis.

Then, each sample part is measured by each operator 2 or 3 times (replicate measurements). Remember, these measurements should be taken randomly to avoid any bias in the data.

Below is the typical data collection form for the **average & range method**.

Appraiser	Replicate Measurements	Samples										
		1	2	3	4	5	6	7	8	9	10	Averages
A	1											
	2											
	3											
	Average											
	Range											
B	1											
	2											
	3											
	Average											
	Range											
C	1											
	2											
	3											
	Average											
	Range											

In the next section I'm going to review the calculations for each major aspect of the gauge R&R, which includes:

- **Repeatability**
- **Reproducibility**
- **Measurement System Variation (GR&R)**
- **Part to Part Variation**
- **Total Variation**

Then we will jump into an example so you can see these calculations at work. Let's start with the calculations for repeatability and reproducibility.

Repeatability Calculation (Equipment Variation – EV)

Repeatability (Equipment Variation) is estimated using **R-double bar**, which is the average value of the range measured within each operator and within each part.

$$Equipment\ Variation\ (EV) = \sigma_{Repeatability} = \frac{\bar{\bar{R}}}{d_2}$$

In this calculation $\mathbf{d_2}$ depends on m = (# of replicates, usually 2 or 3), and n = (# of parts) * (# of operators). The value of n should equal the number of range values used to calculate the average range (R-bar) value.

When the value of n is greater than 16, you can use d^*_2.

Reproducibility Calculation (Appraiser Variation – AV)

Reproducibility (Appraiser Variation) is determined by analyzing the variation between the average of the individual technicians for all parts.

You'll notice below that to calculate the reproducibility (appraiser variation), we must subtract the repeatability as the equipment variation would otherwise overinflate the estimate of appraiser variation.

Using the data collection form above, you will:

1. Calculate the average measurement for each operator across all samples and their replicate measurements
2. Find the range between these average values (largest operator average – smallest operator average), this is R_O, the **Operator Range**.
3. Calculate Appraiser Variation using the following equation, where:
4. **n** is the number of sample parts measured.
5. **r** is the number of replicate measurements for each sample part.

$$Appraiser\ Variation\ (AV) = \sigma_{Reproducibility} = \sqrt{\left(\frac{R_o}{d_2}\right)^2 - \frac{(EV)^2}{n * r}}$$

In this calculation d_2 depends on the number of operators (appraisers) involved in the experiment. When looking for this value, use n = 1 and m = (# of operators); thus for 2 operators $\mathbf{d_2 = 1.414}$ and for 3 operators, $\mathbf{d_2 = 1.912}$.

Measurement System Variation (GR&R) Calculation

Once we've estimated repeatability (equipment variation) and reproducibility (appraiser variation), we can now calculate the **measurement system variation (GRR)**.

$$Measurement\ System\ Variation\ (GRR) = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2}$$

Next, we must calculate the **part-to-part variation**, and then finally the **total variation**.

Part to Part Variation Calculation

Part to Part Variation is determined by analyzing the variation between the average of the individual technicians for all parts.

Using the data collection form above, you will:

1. Calculate the average measured value for each individual part, including all replicate measurements for each part.
2. Calculate the range across all of the sample part average values, this is the **part range** is $\mathbf{R_p}$.
3. Translate the part range into standard deviation using the following equation:

$$Part\ to\ Part\ Variation\ (PV) = \sigma_p = \frac{R_p}{d_2}$$

In this calculation d_2 depends on the parts used to calculate the range value. When looking for this value, use n = 1 and m = (# of sample parts), thus when the GRR includes 5 samples then $\mathbf{d_2 = 2.4812}$ and for 10 samples then $\mathbf{d_2 = 3.1791.}$

Total Variation Calculation

The **total variation** within a gauge R&R study can be calculated as the combination of **measurement system variation** (**GRR)** and the **part to part variation** (**PV)**:

$$Total\ Variation = \sqrt{(GRR)^2 + (PV)^2}$$

Once you've calculated the total variation, you can begin calculating the percent variation associated with **repeatability** and **reproducibility** and the **GRR**.

Now let's do an example of a gauge R&R using the average and range method.

Example of the Average & Range Method

Let's do an example where 3 operators are measuring 10 parts each, 3 times (3 replicate measurements) using a pair of calipers measuring in millimeters.

Below are the measurements from each operator and for each sample. These measurements were taken in a random order in order to minimize any bias.

Appraiser	Replicate Measurements	Samples										Averages
		1	2	3	4	5	6	7	8	9	10	
A	1	47.5	41.6	49.4	46.6	47.0	49.7	43.6	50.7	44.4	50.7	
	2	46.5	42.3	50.7	46.3	46.9	48.9	44.2	50.8	43.6	49.5	
	3	47.2	41.8	50.8	46.1	46.9	49.7	43.4	51.1	43.7	50.1	
	Average											
	Range											
B	1	47.0	41.4	49.4	47.0	45.7	48.9	44.5	51.6	42.7	49.0	
	2	45.9	41.8	49.8	45.8	45.9	48.1	43.3	50.8	43.0	49.2	
	3	46.7	40.9	49.4	46.4	45.7	48.7	44.0	51.2	43.3	49.5	
	Average											
	Range											
C	1	45.9	40.8	49.4	46.1	45.9	48.2	43.2	50.7	42.6	48.9	
	2	46.0	40.7	49.5	46.1	45.9	48.5	43.8	50.7	43.2	49.2	
	3	46.3	40.5	49.5	46.3	45.9	47.8	43.7	50.9	43.2	49.2	
	Average											
	Range											
Total	Part Averages											
	Average Range											
	Range Between Part Averages											
	Range Amongst Operators (R_o)											

Notice, we haven't calculated any average or range values yet for this gauge R&R, this is only the raw, measured data.

Now let's calculate the 5 elements of gauge R&R needed to assess the capability of our measurement system.

- **Repeatability**
- **Reproducibility**

- **Measurement System Variation (GRR)**
- **Part to Part Variation**
- **Total Variation**

Let's start with **Repeatability**.

Repeatability Calculation (Equipment Variation – EV)

Repeatability (Equipment Variation) is estimated using the average range the within part, and within operator for each sample measured.

$$Equipment\ Variation\ (EV) = \sigma_{Repeatability} = \frac{\bar{\bar{R}}}{d_2}$$

Appraiser	Replicate Measurements	Samples										Averages
		1	2	3	4	5	6	7	8	9	10	
A	1	47.5	41.6	49.4	46.6	47.0	49.7	43.6	50.7	44.4	50.7	47.12
	2	46.5	42.3	50.7	46.3	46.9	48.9	44.2	50.8	43.6	49.5	46.98
	3	47.2	41.8	50.8	46.1	46.9	49.7	43.4	51.1	43.7	50.1	47.09
	Average	47.06	41.92	50.31	46.36	46.96	49.42	43.70	50.88	43.93	50.09	47.06
	Range	1.00	0.76	1.43	0.46	0.14	0.83	0.76	0.49	0.79	1.21	0.79
B	1	47.0	41.4	49.4	47.0	45.7	48.9	44.5	51.6	42.7	49.0	46.72
	2	45.9	41.8	49.8	45.8	45.9	48.1	43.3	50.8	43.0	49.2	46.35
	3	46.7	40.9	49.4	46.4	45.7	48.7	44.0	51.2	43.3	49.5	46.58
	Average	46.53	41.36	49.50	46.40	45.76	48.58	43.93	51.20	42.99	49.27	46.55
	Range	1.12	0.86	0.41	1.16	0.17	0.72	1.23	0.79	0.53	0.50	0.75
C	1	45.9	40.8	49.4	46.1	45.9	48.2	43.2	50.7	42.6	48.9	46.16
	2	46.0	40.7	49.5	46.1	45.9	48.5	43.8	50.7	43.2	49.2	46.35
	3	46.3	40.5	49.5	46.3	45.9	47.8	43.7	50.9	43.2	49.2	46.32
	Average	46.04	40.66	49.45	46.17	45.89	48.15	43.56	50.76	42.98	49.09	46.28
	Range	0.38	0.29	0.15	0.25	0.06	0.62	0.54	0.18	0.58	0.30	0.34
Total	Part Averages	46.54	41.31	49.75	46.31	46.20	48.72	43.73	50.95	43.30	49.49	46.630
	Average Range											0.624
	Range Between Part Averages											9.635
	Range Amongst Operators (R_o)											0.788

R-double bar is the average value of the range measured within each part and within each operator, which can be see below.

First though we must find the range value of each operator for each sample part measured. Then we take the average of those 30 range values shown above to find **R-double bar** of 0.624.

d_2 depends on:

- m = # of replicate measurements for each sample = 3
- n = (# of parts) * (# of operators) = 10 * 3 = 30

Because the value of n is greater than 16, we can use $\mathbf{d^*_2}$, which is 1.693.

$$Equipment\ Variation\ (EV) = \sigma_{Repeatability} = \frac{\bar{\bar{R}}}{d_2} = \frac{0.624}{1.693} = 0.3688$$

Reproducibility Calculation (Appraiser Variation – AV)

Reproducibility (Appraiser Variation) is determined by analyzing the variation between the average of the individual technicians for all parts.

$$Appraiser\ Variation\ (AV) = \sigma_{Reproducibility} = \sqrt{\left(\frac{R_o}{d_2}\right)^2 - \frac{(EV)^2}{n * r}}$$

Appraiser	Replicate Measurements	Samples										Averages
		1	2	3	4	5	6	7	8	9	10	
A	1	47.5	41.6	49.4	46.6	47.0	49.7	43.6	50.7	44.4	50.7	47.12
	2	46.5	42.3	50.7	46.3	46.9	48.9	44.2	50.8	43.6	49.5	46.98
	3	47.2	41.8	50.8	46.1	46.9	49.7	43.4	51.1	43.7	50.1	47.09
	Average	47.06	41.92	50.31	46.36	46.96	49.42	43.70	50.88	43.93	50.09	47.06
	Range	1.00	0.76	1.43	0.46	0.14	0.83	0.76	0.49	0.79	1.21	0.79
B	1	47.0	41.4	49.4	47.0	45.7	48.9	44.5	51.6	42.7	49.0	46.72
	2	45.9	41.8	49.8	45.8	45.9	48.1	43.3	50.8	43.0	49.2	46.35
	3	46.7	40.9	49.4	46.4	45.7	48.7	44.0	51.2	43.3	49.5	46.58
	Average	46.53	41.36	49.50	46.40	45.76	48.58	43.93	51.20	42.99	49.27	46.55
	Range	1.12	0.86	0.41	1.16	0.17	0.72	1.23	0.79	0.53	0.50	0.75
C	1	45.9	40.8	49.4	46.1	45.9	48.2	43.2	50.7	42.6	48.9	46.16
	2	46.0	40.7	49.5	46.1	45.9	48.5	43.8	50.7	43.2	49.2	46.35
	3	46.3	40.5	49.5	46.3	45.9	47.8	43.7	50.9	43.2	49.2	46.32
	Average	46.04	40.66	49.45	46.17	45.89	48.15	43.56	50.76	42.98	49.09	46.28
	Range	0.38	0.29	0.15	0.25	0.06	0.62	0.54	0.18	0.58	0.30	0.34
Total	Part Averages	46.54	41.31	49.75	46.31	46.20	48.72	43.73	50.95	43.30	49.49	46.630
	Average Range											0.624
	Range Between Part Averages											9.635
	Range Amongst Operators (R_o)											0.788

This starts by calculating the Operator Range, which is the difference between the largest operator average and the smallest operator average. This is R_O, the **Operator Range**.

$$Operator\ Range = R_o = Maximum\ Operator\ Average - Minimum\ Operator\ Average$$

$$Operator\ Range = R_o = 47.06 - 46.28 = 0.788$$

Now we can translate the operator range into the reproducibility calculation:

$$\sigma_{Reproducibility} = \sqrt{\left(\frac{0.788}{1.912}\right)^2 - \frac{(0.3688)^2}{10 * 3}} = 0.4074$$

In this calculation $\mathbf{d_2}$ depends on the number of operators (3 in this instance). When looking for this value, use n = 1 and m = (# of operators); thus for 3 operators, $\mathbf{d_2 = 1.912}$.

Measurement System Variation (GRR) Calculation

Once we've estimated repeatability (equipment variation) and reproducibility (appraiser variation), we can now calculate the **measurement system variation (GRR)**.

$$Measurement\ System\ Variation\ (GRR) = \sigma_{GRR} = \sqrt{(EV)^2 + (AV)^2}$$

$$Measurement\ System\ Variation\ (GRR) = \sigma_{GRR} = \sqrt{(0.3688)^2 + (0.4074)^2} = 0.5496$$

Next, we must calculate the **part-to-part variation**, and then finally the **total variation**.

Part to Part Variation Calculation

Part to Part Variation is determined by analyzing the variation between the average of the individual technicians for all parts.

$$Part\ to\ Part\ Variation\ (PV) = \sigma_p = \frac{R_p}{d_2}$$

Appraiser	Replicate Measurements	Samples										Averages
		1	2	3	4	5	6	7	8	9	10	
A	1	47.5	41.6	49.4	46.6	47.0	49.7	43.6	50.7	44.4	50.7	47.12
	2	46.5	42.3	50.7	46.3	46.9	48.9	44.2	50.8	43.6	49.5	46.98
	3	47.2	41.8	50.8	46.1	46.9	49.7	43.4	51.1	43.7	50.1	47.09
	Average	47.06	41.92	50.31	46.36	46.96	49.42	43.70	50.88	43.93	50.09	47.06
	Range	1.00	0.76	1.43	0.46	0.14	0.83	0.76	0.49	0.79	1.21	0.79
B	1	47.0	41.4	49.4	47.0	45.7	48.9	44.5	51.6	42.7	49.0	46.72
	2	45.9	41.8	49.8	45.8	45.9	48.1	43.3	50.8	43.0	49.2	46.35
	3	46.7	40.9	49.4	46.4	45.7	48.7	44.0	51.2	43.3	49.5	46.58
	Average	46.53	41.36	49.50	46.40	45.76	48.58	43.93	51.20	42.99	49.27	46.55
	Range	1.12	0.86	0.41	1.16	0.17	0.72	1.23	0.79	0.53	0.50	0.75
C	1	45.9	40.8	49.4	46.1	45.9	48.2	43.2	50.7	42.6	48.9	46.16
	2	46.0	40.7	49.5	46.1	45.9	48.5	43.8	50.7	43.2	49.2	46.35
	3	46.3	40.5	49.5	46.3	45.9	47.8	43.7	50.9	43.2	49.2	46.32
	Average	46.04	40.66	49.45	46.17	45.89	48.15	43.56	50.76	42.98	49.09	46.28
	Range	0.38	0.29	0.15	0.25	0.06	0.62	0.54	0.18	0.58	0.30	0.34
Total	Part Averages	46.54	41.31	49.75	46.31	46.20	48.72	43.73	50.95	43.30	49.49	46.630
	Average Range											0.624
	Range Between Part Averages											9.635
	Range Amongst Operators (R_O)											0.788

This starts by calculating the **Part Range** (**R_P**), which is the range between the part averages. This can be seen in the purple boxes and arrows below where we calculate the average value for each part measured.

Now we must calculate the range between these part averages:

$$Operator\ Range = R_P = Maximum\ Part\ Average - Minimum\ Part\ Average$$

$$Operator\ Range = R_O = 50.95 - 41.31 = 9.635$$

Now we can convert this range value into standard deviation using the d_2 conversion factor.

$$Part\ to\ Part\ Variation\ (PV) = \sigma_p = \frac{R_p}{d_2} = \frac{9.635}{3.1791} = 3.0307$$

In this calculation d_2 depends on the parts used to calculate the range value. When looking for this value, use n = 1 and m = (# of sample parts) = 10, thus when the GRR includes 10 samples then **d_2 = 3.1791.**

Total Variation Calculation

The **total variation** within a gauge R&R study can be calculated as the combination of **measurement system variation** (**GRR)** and the **part to part variation** (**PV)**:

$$Total\ Variation = \sqrt{(GRR)^2 + (PV)^2}$$

$$Total\ Variation = \sqrt{(0.5496)^2 + (3.0307)^2} = 3.0802$$

Interpreting the Gauge R&R Results

Let's now interpret the results of our gauge R&R study using the Precision / Tolerance Ratio.

Let's review the results of our Gauge R&R.

$$\textbf{Equipment Variation}\ (EV) - Repeatability = 0.3688$$

$$\textbf{Appraiser Variation}\ (AV) - Reproducibility = 0.4074$$

$$\textbf{Total Measurement System Variation}\ (GR\&R) = 0.5496$$

$$\textbf{Part to Part Variation}\ (PV) = 3.0307$$

$$\textbf{Total Variation}\ (TV) = 3.0802$$

Now let's convert each of these categories into their percentage **of the total variation**:

$$\%\ \boldsymbol{Equipment\ Variation\ (EV)} = \frac{EV}{TV} * 100 = \frac{0.3688}{3.0802} = 12.0\%$$

$$\%\ \boldsymbol{Appraiser\ Variation\ (AV)} = \frac{AV}{TV} * 100 = \frac{0.4074}{3.0802} = 13.2\%$$

$$\%\ \boldsymbol{Measurement\ System\ Variation} = \frac{GRR}{TV} * 100 = \frac{0.5496}{3.0802} = 17.8\%$$

$$\%\ \boldsymbol{Part\ to\ Part\ Variation} = \frac{PV}{TV} * 100 = \frac{3.0307}{3.0802} = 98.4\%$$

It should be noted that these percentages **WILL NOT add up to 100%.**

Because our GR&R value **(17.8%)** is **between 10% - 30%**, then the measurement system is generally considered acceptable. We can also confirm that repeatability and reproducibility are both contributing equally to the **total GR&R**.

Now let's move on to the 3rd and final method for assessing **gauge R&R** for a measurement system, **the ANOVA Method**.