## Distance between points in three dimensions

Given two points $A$ and $B$ in three-dimensional space,

$$
\begin{aligned}
& A\left(x_{1}, y_{1}, z_{1}\right) \\
& B\left(x_{2}, y_{2}, z_{2}\right)
\end{aligned}
$$

we can calculate the distance between them using the distance formula.

$$
D=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}}
$$

It doesn't matter which point is $A$ and which point is $B$. The fact that the square the differences inside the square root means that all of our values will be positive, which means we'll get a positive value for the distance between the points.

## Example

Use the distance formula to

1. Find the distance between $(0,1,3)$ and ( $-1,4,5$ ).
2. Say which of $(0,1,3)$ and $(-1,4,5)$ lies in the $y z$-plane.

3 . Say which of $(0,1,3)$ and $(-1,4,5)$ is closer to the $x y$-plane.

For the first part of the question, we'll use the distance formula to calculate the distance between the points.

$$
\begin{aligned}
& D=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}} \\
& D=\sqrt{(-1-0)^{2}+(4-1)^{2}+(5-3)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& D=\sqrt{1+9+4} \\
& D=\sqrt{14}
\end{aligned}
$$

For the second part of the question, we need to realize that in order for a point to be in the $y z$-plane, its $x$-coordinate must be 0 . With that in mind, we can say that $(0,1,3)$ lies on the $y z$-plane, and that $(-1,4,5)$ does not lie in the $y z$-plane.

For the third part of the question, we need to realize that the $z$-value of the coordinate point will tell us how far the point is from the $x y$-plane. So if we just take the absolute value of the $z$-coordinate for each of our points, we'll be able to say which one is closer.

Point $(0,1,3)$ has $|z|=|3|=3$
Point $(-1,4,5)$ has $|z|=|5|=5$
Since the absolute value of $z$ in the point $(0,1,3)$ is less than the absolute value of $z$ in the point ( $-1,4,5$ ), we can say that $(0,1,3)$ is closer to the $x y$-plane.

