Math Review

Why Math

All trades in construction use math for numerous tasks. Construction workers must be able to take measurements with measuring tools and make calculations based on information given on prints. Construction estimators must be able to use basic math skills to calculate material and labor costs. This section presents basic math principles and provides a review of information covered in Unit 2. All tradeworkers in construction should have a basic understanding of these principles.

Calculators

Calculators can be a great time saver, but you should not rely on a calculator to replace your knowledge of basic math. There will be times in the field when a calculator is not available and you will need to do some basic math. However, a calculator can be a handy tool when one is available. Knowing some basic math can help prevent errors when using a calculator.

It is important to do operations in the correct order when using a calculator. For example, the formula for the area of a circle requires multiplying the constant π (pi, or 3.1416) by the radius of the circle squared. If the radius of a circle is 5″, the area is found by first multiplying 5 × 5 (5 squared), and then multiplying the result by 3.1416. If you know that 5 × 5 (written 5^2) is 25, and 3 times 25 is 75, you will know the answer is slightly more than 75. Now you can compute the answer with your calculator. The correct answer is 78.54 in^2. Note in this example that you must know the order in which to multiply the numbers. Using this example, if you multiply 3.1416 × 5 and then multiply the result of that operation by itself (15.708 x 15.708), you'll get 246.7413—not even close to the correct answer. Knowing the correct order to do the operations on paper will help you enter numbers in the correct order on a calculator.

Several types of calculators are available. The most familiar type is a *general calculator*, which allows the user to perform basic math functions. See **Figure 1**. General calculators have a memory function that allows the user to store the results of a calculation so that number can be recalled and used in a subsequent calculation. The √ key is used for finding the square root of a number. The ± key changes the value of a displayed number from positive to negative or negative to positive. Scientific calculators are more advanced, having the ability to do trigonometric functions and other advanced operations. Scientific calculators are used by professionals such as engineers and surveyors.

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Figure 1.

Another type of calculator that can be very useful for construction is the *construction calculator*. See **Figure 2**. Construction calculators can be used to make calculations for stairs and roofs and to compute area and volume totals. Construction calculators can also be used to convert inches to feet and feet to inches. Most construction calculators also allow the user to convert between metric and US Customary measurements. Construction calculators vary from one manufacturer and model to the next, but all calculators come with instructions that explain their functions.

Dan Dorfmueller

Figure 2.

Whole Numbers

Whole numbers are simply numbers without fractions or decimal points—numbers such as 1, 2, 3, 4, etc. Whole numbers are added, subtracted, multiplied, and divided as follows.

Adding and Subtracting Whole Numbers

The key to addition is to line up the columns of digits correctly. The whole numbers should be aligned on the right.

$$\begin{array}{r} 3 \\ 5 \\ +\ \ 2 \\ \hline 10 \end{array}$$

The same alignment is used in subtraction. For example, the result of subtracting 12 from 37 is 25, because we know that 2 from 7 is 5 and 1 from 3 is 2.

$$\begin{array}{r} 37 \\ -\ \ 12 \\ \hline 25 \end{array}$$

In subtraction, if the number being subtracted (the number on the bottom) is larger than the number it is being subtracted from (the number on the top), borrow 10 from the next digit to the left and add it to the one on the right. Write small numerals above the column to help you keep track.

Example:

$$\begin{array}{r} {\scriptstyle 2\ 16} \\ \not{3}\not{6} \\ -\ \ 19 \\ \hline 17 \end{array}$$

Multiplying Whole Numbers

Multiplication of whole numbers becomes easier with memorization of a multiplication table. For example, knowing that $6 \times 5 = 30$ is easier than adding $6 + 6 + 6 + 6 + 6$. This becomes more important for larger multiplication problems. To multiply numbers whose values are 10 or more (those with more than one digit), align the digits representing 0 through 9 (the 1s) in the right-hand column. Then multiply the top row by the 1s digit in the second row:

Example:

10s
1s

$$\begin{array}{r} 31 \\ \times\ \ 15 \\ \hline 155 \end{array}$$

Next, multiply the top row by the 10s digit in the second row. Because you multiplied by the 10s digit, the *product* (the result of multiplication) is written with its right-most digit in the 10s column:

$$\begin{array}{r} 31 \\ \times\ \ 15 \\ \hline 155 \\ 31 \end{array}$$

If the problem has more digits in the second row, the above steps are repeated for each digit with the products being written in rows beneath one another, with the right-most digit in each row being written in the column for the place it represents: 100s, 1000s, etc.

When all of the multiplication is complete, add the products just as you would for an addition problem. The result of this addition is the product (answer) of the multiplication problem.

$$\begin{array}{r} 31 \\ \times\ \ 15 \\ \hline 155 \\ 31 \\ \hline 465 \end{array}$$

Dividing Whole Numbers

Division of whole numbers is the reverse of multiplication, but the problem must be set up differently. The *dividend* (the number being divided) is written inside the division symbol. The *divisor* (the number the dividend will be divided by) is written to the left of the symbol:

divisor ⟶ 7)‾28‾ ⟵ dividend

From a multiplication table, we know that $7 \times 4 = 28$. So, if 28 is divided into 7 parts, each part will have 4, or $28 \div 7 = 4$. The number 4 is the *quotient* (the answer to a division problem) and it is written above the division symbol and above the 1s place of the 28:

$$\begin{array}{r} 4 \leftarrow \text{Quotient} \\ 7 \,\overline{)\,28} \end{array}$$

For division problems where the dividend has more places under the division symbol, the process is divided into steps as follows:

$$4 \,\overline{)\,320}$$

4 goes into 32 8 times. Write the 8 above the 2 (the right column of the 32). Now multiply 4×8, which is 32. Write the 32 beneath the 32 in the division symbol.

$$\begin{array}{r} 8 \\ 4 \,\overline{)\,320} \\ \underline{32} \end{array}$$

Subtract the product of your multiplication (32) from the number above it in the symbol (32). Because 4 goes into 32 exactly 8 times, the numbers are the same, so the result of your subtraction is 0.

$$\begin{array}{r} 8 \\ 4 \,\overline{)\,320} \\ \underline{32} \\ 0 \end{array}$$

Drop the next digit to the right in the dividend (0 in this case) down beside the result of your subtraction. That makes the number at the bottom 00. 4 will not go into 0 (or 00), so the quotient of that step is 0. The quotient (answer) of the division problem is 80.

$$\begin{array}{r} 80 \\ 4 \,\overline{)\,320} \\ \underline{32} \\ 00 \end{array}$$

320 can be divided by 4 80 times. If there are more places under the division symbol, just keep doing the same division, multiplication, and subtraction steps, and drop down each digit moving to the right.

Example:

$$\begin{array}{r} 102 \\ 6 \,\overline{)\,616} \\ \underline{6} \\ 01 \\ \underline{0} \\ 016 \\ \underline{12} \\ 4 \leftarrow \text{remainder} \end{array}$$

If the last number produced by the drop-down cannot be divided evenly by the divisor, that number is called the *remainder*. In the previous example, the quotient is 102 with a remainder of 4.

Practice A-1

Test your skills with the following problems.

A. $\begin{array}{r} 342 \\ +\ \ 16 \\ \hline \end{array}$

E. $\begin{array}{r} 18 \\ \times\ \ 4 \\ \hline \end{array}$

B. $\begin{array}{r} 79 \\ +\ 29 \\ \hline \end{array}$

F. $\begin{array}{r} 213 \\ \times\ \ 24 \\ \hline \end{array}$

C. $\begin{array}{r} 68 \\ -\ 13 \\ \hline \end{array}$

G. $3 \,\overline{)\,36}$

D. $\begin{array}{r} 124 \\ -\ \ 35 \\ \hline \end{array}$

H. $7 \,\overline{)\,214}$

Equations

An *equation* is a mathematical statement that two things have the same or equal value. An equation can be thought of as a mathematical sentence. The words of the sentence are mathematical values called *terms*. An equation is always written with an equals sign (=). For example, $3 + 4 = 7$. In that statement, 3, 4, and 7 are terms. The statement says that 3 plus 4 has the same value as 7. Many useful formulas are stated as equations. Equations can be used to find the value of one unknown term when the other values in the equation are known. For example, assume a truck is loaded with 10 bundles of shingles weighing 80 lb each, and the truck is also loaded with an unknown weight of sheet metal. If the total load is 1000 lb, you can find the weight of the sheet metal with the following equation:

$$(80 \text{ lb} \times 10) + \text{weight of sheet metal} = 1000 \text{ lb}$$

The (80 lb × 10) represents the total weight of the shingles. It is one of the terms in the equation. It is enclosed in parentheses to indicate that it is a single term that should be computed before the rest of the equation. Whenever a mathematical term such as (80 lb × 10) is enclosed in parentheses, that computation should be done first. Now write the equation with the shingle weight computed:

$$800 \text{ lb} + \text{weight of sheet metal} = 1000 \text{ lb}$$

When a mathematical operation is done on one side of an equation, the equation remains a true statement if the same thing is done on the other side of the equals sign. If we subtract 800 lb from both sides of our equation, it is still a true equation:

$$800 \text{ lb} + \text{weight of sheet metal} - \mathbf{800\ lb} = 1000 \text{ lb} - \mathbf{800\ lb}$$

$$\text{Weight of sheet metal} = 200 \text{ lb}$$

Practice A-2

Find the unknown value in each equation.

A. cost = ($.60 – $.04) × 5

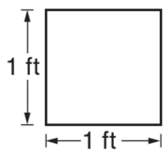
B. $X = \frac{3}{4} + 20$

C. 240 lb = 2 × weight of crate

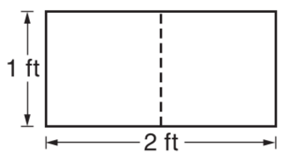
D. $\frac{1}{4} \div \frac{2}{3} = Y$

Area Measure

The area of a surface is always measured in square units, such as square inches, square feet, square meters, etc. When a number is squared, that means it is multiplied by itself. For example, 3 squared is 9. Square units are written with a superscript 2, indicating that the value is units × units.



This square is 1′ × 1′ or
1 square foot.
1 ft^2

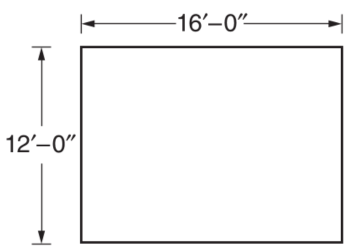


This rectangle is made up of 2 squares that are 1 square foot each. It is 1′ × 2′ or 2 square feet.
2 ft^2

Finding the Area of Squares and Rectangles

The area of a square or a rectangle is the number of units it is wide multiplied by the number of units it is long.

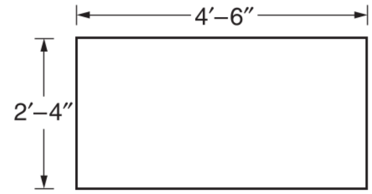
Example: Find the area of this rectangle.



$12 \text{ ft} \times 16 \text{ ft} = 192 \text{ ft}^2$

The width and length must be expressed in the same units.

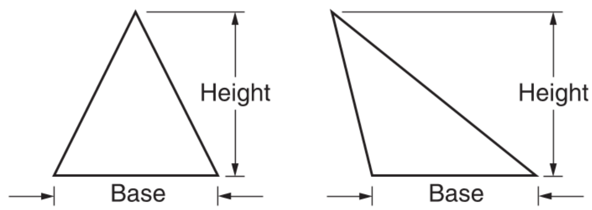
Example: To find the area of this rectangle, convert all measurements to decimal feet, then multiply.



2′–4″ = 2.33′
4′–6″ = 4.5′
$2.33' \times 4.5' = 10.5 \text{ ft}^2$

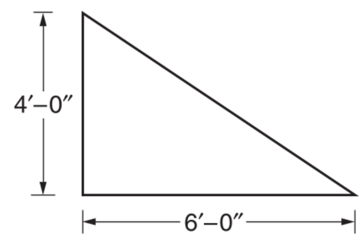
Finding the Area of Triangles

To find the area of a triangle, it is necessary to know the names of two parts of a triangle.



To find the area of any triangle, multiply the height by 1/2 the base.

Example: Find the area of this triangle.



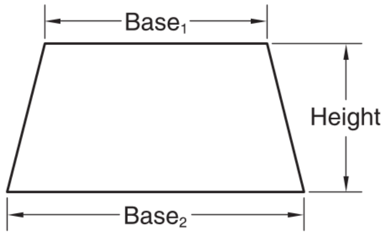
$$4 \text{ ft} \times \frac{6 \text{ ft}}{2} = 4 \text{ ft} \times 3 \text{ ft} = 12 \text{ ft}^2$$

Another way to achieve the same result is to multiply the base by the height, then divide that value by 2.

$$4 \text{ ft} \times 6 \text{ ft} = 24 \text{ ft}^2$$
$$24 \text{ ft}^2 \div 2 = 12 \text{ ft}^2$$

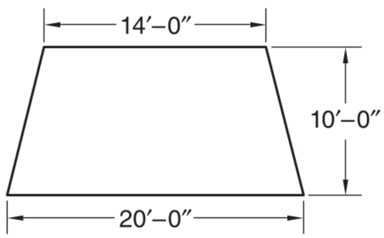
Finding the Area of a Trapezoid

To find the area of a trapezoid, first study its parts.



The two parallel sides are called the *bases*. The area is equal to half the sum of the two bases times the height.

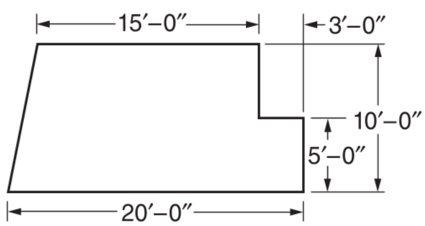
Example: Find the area of this trapezoid.

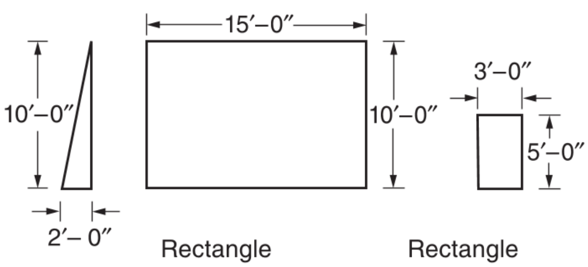


$$\frac{14 \text{ ft} + 20 \text{ ft}}{2} \times 10 \text{ ft} = 17 \text{ ft} \times 10 \text{ ft} = 170 \text{ ft}^2$$

Finding Multiple Areas

Some figures may be made up of squares, rectangles, and triangles of varying sizes. To find the area of such a figure, break it up into its various parts and find the area of each part, then add those areas.





Rectangle
$10 \text{ ft} \times 15 \text{ ft} = 150 \text{ ft}^2$

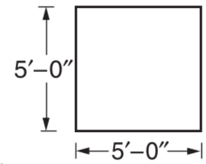
Rectangle
$3 \text{ ft} \times 5 \text{ ft} = 15 \text{ ft}^2$

Triangle
$10 \text{ ft} \times 2 \text{ ft} = 20 \text{ ft}^2$
$20 \text{ ft}^2 \div 2 = 10 \text{ ft}^2$

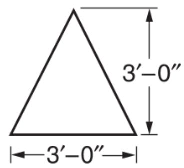
$$10 \text{ ft}^2 + 150 \text{ ft}^2 + 15 \text{ ft}^2 = 175 \text{ ft}^2$$

Practice A-3

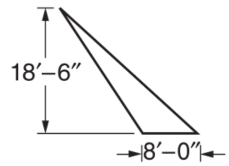
Find the areas of the figures.



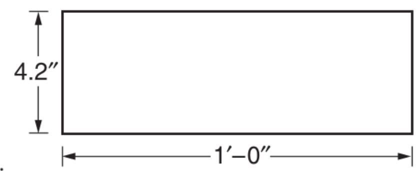
A.



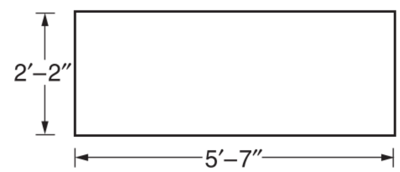
B.



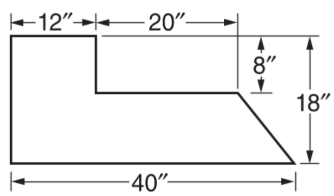
C.



D.



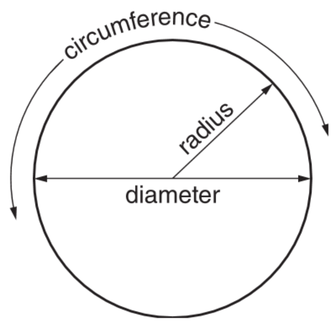
E.



F.

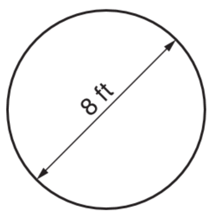
Finding the Circumference and Area of a Circle

The distance from a circle's center point to its outer edge is its *radius*. The total distance across a circle through its center point is its *diameter*.



The *circumference* of a circle is its perimeter. To find the circumference of a circle, multiply the diameter by π. This is the same as multiplying the radius by 2 and multiplying that product by π.

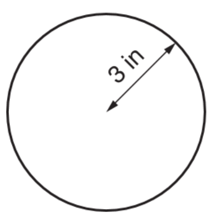
Example: Find the circumference of a circle with a diameter of 8′.



$$\text{Circumference} = \pi \times \text{diameter} = 3.1416 \times 8 \text{ ft}$$
$$3.1416 \times 8 \text{ ft} = 25.1328 \text{ ft}$$

The area of a circle is found by multiplying π by the radius squared (the radius times the radius).

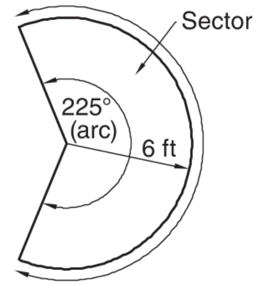
Example: Find the area of a circle with a radius of 3″.



$$\text{Area} = \pi \times \text{radius}^2 \text{ (or radius} \times \text{radius)}$$
$$\text{Area} = 3.1416 \times 3 \text{ in} \times 3 \text{ in} = 3.1416 \times 9 \text{ in}^2$$
$$\text{Area} = 28.2744 \text{ in}^2$$

The area of a portion of a circle can be calculated in similar fashion by determining the fractional part of the circle. Calculate the area of the circle and then multiply it by the portion of the complete circle.

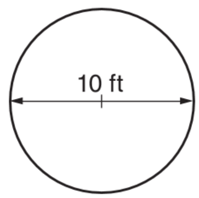
Example: Find the area of a portion of a circle with a radius of 6′.



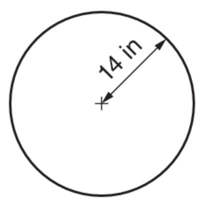
$$\text{Area of circle} = 3.1416 \times 6^2 = 113.0976 \text{ ft}^2$$
$$\text{Portion of circle} = \frac{225°}{360°} = .625$$
$$\text{Area of circle} = 113.0976 \text{ ft}^2 \times .625 = 70.686 \text{ ft}^2$$

Practice A-4

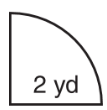
Find the area of each of these figures.



A.



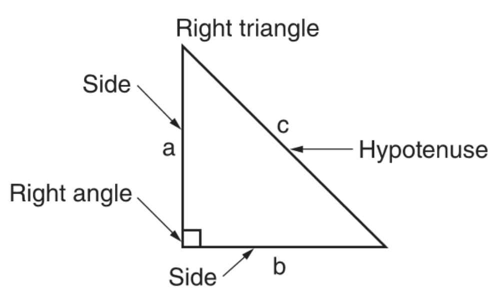
B.



C. Quarter circle

Working with Right Angles

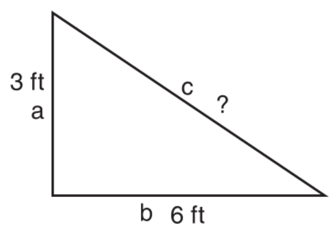
A *right angle* is a 90° angle. A triangle having a right angle is called a *right triangle*. It is helpful to know a few terms associated with right triangles.



A right triangle can only have one 90° corner angle. The total of all three corner angles is always 180°, so if one is 90°, the other two must add up to 90° together.

The *Pythagorean Theorem* is a mathematical formula that makes right triangles convenient to work with. Named after the Greek mathematician Pythagoras, the Pythagorean Theorem states that the sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse. To help keep track of the Pythagorean Theorem, it is common to label the two sides *a* and *b* and the hypotenuse *c*. Then the theorem can be stated as an equation: $a^2 + b^2 = c^2$. If the lengths of the two sides of a right triangle are known, the Pythagorean Theorem can be used to find the length of the hypotenuse.

Example:



$$a^2 + b^2 = c^2$$
$$3\text{ ft} \times 3\text{ ft} + 6\text{ ft} \times 6\text{ ft} = c^2$$
$$9\text{ ft}^2 + 36\text{ ft}^2 = c^2$$
$$45\text{ ft}^2 = c^2$$

The square root of $45\text{ ft}^2 = c$

$6.7082\text{ ft} = c$

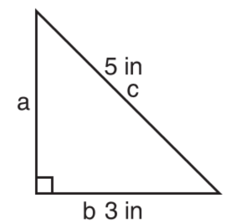
Rounded to 1 decimal place, the hypotenuse is 6.7 ft.

The Pythagorean Theorem can be used to find the length of any side of a right triangle if the other two sides are known. For example, if side *b* and the hypotenuse are known, $a^2 + b^2 = c^2$ can be rearranged to $a^2 = c^2 - b^2$.

Explanation:

- $a^2 + b^2 = c^2$
- The equation stays in balance if you do the same thing on both sides of the equals sign.
- Subtract b^2 from both sides of the equation.
- $a^2 + \cancel{b^2} - \cancel{b^2} = c^2 - b^2$
- $a^2 = c^2 - b^2$

Example:



$$a^2 = c^2 - b^2$$
$$a^2 = 25\text{ in}^2 - 9\text{ in}^2$$
$$a^2 = 16\text{ in}^2$$
$$a = 4\text{ in}$$

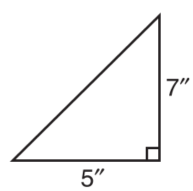
The same can be done to find side *b* when side *a* and the hypotenuse are known.

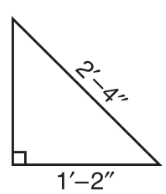
Special Right Triangles

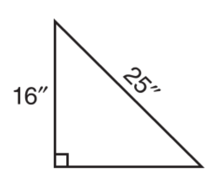
The Pythagorean Theorem can be used to verify that a corner of a triangle is 90° by measuring along the two sides, then checking the length of the hypotenuse between them. This is usually simplified by using 3 and 4 units as the sides. If 3 and 4 units are used, the hypotenuse of a triangle with a square corner is 5 units. This is called a 3-4-5 triangle. If 6 and 8 units are used, the hypotenuse is 10 units. This is a multiple of the 3-4-5 triangle. These right triangles are preferred for easy calculations. Note that all sides are whole numbers in a 3-4-5 triangle or any of its multiples.

Practice A-5

Find the length of the unknown side.

A.


B.


C.


Computing Averages

An average is a typical value of one unit in a group of units. For example, if four windows have areas of 12.0 ft^2, 11.2 ft^2, 11.2 ft^2, and 14.5 ft^2, the average size is 12.2 ft^2. The average is computed by adding all of the units and dividing that sum by the number of units in the group.

Example:

$$\begin{array}{r} 12.0\ ft^2 \\ 11.2\ ft^2 \\ 11.2\ ft^2 \\ \underline{14.5\ ft^2} \\ 48.9\ ft^2 \end{array}$$

$$\begin{array}{r|l} & 12.22 \\ 4 & 48.90 \\ & \underline{4} \\ & 08 \\ & \underline{8} \\ & 0\,9 \\ & \underline{8} \\ & 10 \end{array}$$

The quotient should be rounded off to the same number of decimal places as is used in the problem.

Average window size is 12.2 ft^2

Practice A-6

Compute the averages of these groups.

A. 14, 14.4, 14.5, 15

B. 80 lb, 83 lb, 88 lb, 79.5 lb, 81.6 lb, 84 lb

C. 11 cubic yards, 13 cubic yards, 11.5 cubic yards, 12 cubic yards, 12.8 cubic yards

Percent and Percentage

A *percent* is one part in a hundred. One penny is 1% of a dollar. Twenty-five cents is 25% of a dollar. On the other hand, 25 cents is 50% of a half-dollar, because if the half dollar were divided into 100 parts of 1/2 cent each and the quarter were also divided into 1/2-cent increments, the quarter would equal 50 of those 1/2-cent increments.

Think of percent as hundredths. To find a given percentage of an amount, multiply the amount by the desired number of hundredths.

Example:

Find 12% of $4.40.

12% is 0.12 times the whole.

$4.40 × .12 = $0.528, or 53 cents.

Using what was covered in the section on equations and solving for an unknown, it is possible to calculate the whole if you know the percentage.

Example:

What was the total spent on tools if $22.00 was spent on sales tax and the tax rate is 8%?

1. Write an equation with the facts you know: $22.00 = 8% × total.
2. Write percent as hundredths: $22.00 = 0.08 × total.
3. Divide each side of the equation by 0.08: $275 = total.

Practice A-7

A. What is 10 percent of 225?

B. What is 65 percent of $350.50?

C. If merchandise cost $22.25 and the bill comes to $23.14, what percentage was added for sales tax?

D. If a 500-gallon tank is 60 percent full of water, how much water does it contain?