



Skill: Proof by exhaustion

Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. Use the method of exhaustion to prove the following result:

“No 1 – digit or 2 – digit perfect square ends in 2,3,7 or 8”

State a generalisation of this result.

2. Suppose x and y are odd integers less than 7.

Prove that their sum is divisible by 2.

3. Use the method of exhaustion to prove the following result:

“ $2^p - 1$ is a prime number for all prime numbers p less than 11”

4. Use the method of exhaustion to prove the following result:

If n is even and $4 \leq n \leq 26$, then n can be written as the sum of two prime numbers.

5. Use the method of exhaustion to prove the following result:

All regular polygons with fewer than 7 sides have external angles which are an integer number of degrees.

6. Let $N(n)$ denote the number of factors of the integer n .

For example $N(12) = 6$, since 12 has the factors 1,2,3,4,6,12.

Prove, using the method of exhaustion, that for any single digit positive number integer n , $N(n) \leq n$.

7. Prove, using the method of exhaustion, that for the positive integers n , where $n \leq 4$, that:

$$(1 + n)^3 \geq 3^n$$

8. Use the method of exhaustion to prove the following result:

Every integer n on the interval $6 \leq n \leq 14$ can be written as the sum of three prime numbers.

9. Use the method of exhaustion to prove the following result:

Every odd integer n on the interval $7 \leq n \leq 21$ can be written as the sum of a prime number and the double of a prime number.