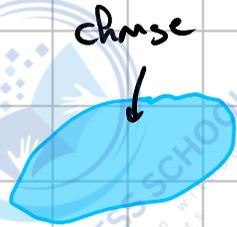




obsessed with perfection

E lectromagnetisme

Le but:



- Champ \vec{E} ; • ligne de champ, $\vec{E} \wedge d\vec{m} = \vec{0}$
- potentiel V ; • surface equipotentielle, $V = cte$
- $\vec{E} = -\text{grad } V$

Distribution

Discrete

$$Q = \sum_i q_i$$



Continue

$$Q = \int_D dq$$



$$dq = \underbrace{\rho}_{\text{volume}} d\tau = \underbrace{\sigma}_{\text{surface}} dS = \underbrace{\lambda}_{\text{fil}} dl$$

Loi de Coulomb

Discrete

$$\vec{E}(M) = \sum_i \frac{q_i \vec{P}_i M}{4\pi\epsilon_0 P_i M^3}$$

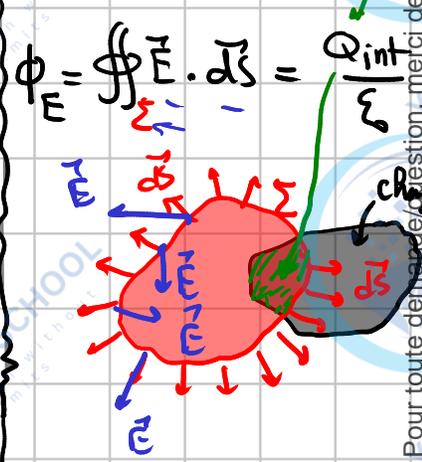
Continue

$$\vec{E}(M) = \int_V \frac{dq \vec{P} M}{4\pi\epsilon_0 P M^3}$$

Appli

- PP fini / infini
- Spire
- Disque

theo. de Gauss



$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$$

Appli

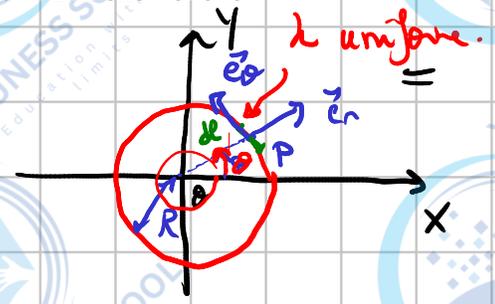
- fil infini
- cylindre infini
- sphere
- plan infini

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Exemple 1 spirale circulaire

Coulomb

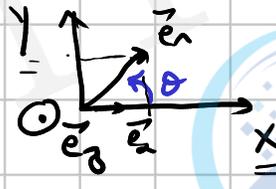


$$\vec{E}(M) = \int \frac{dq \vec{PM}}{4\pi\epsilon PM^3} = \int \frac{\lambda dl (\vec{PO} + \vec{OM})}{4\pi\epsilon PM^3}$$

$$\vec{PM} = \vec{PO} + \vec{OM} = -R\vec{e}_x + z\vec{e}_z \Rightarrow PM = \sqrt{R^2 + z^2}$$

$$d\vec{l} = dl \vec{e}_\theta = R d\theta \vec{e}_\theta = R d\theta (-\sin\theta \vec{e}_x + \cos\theta \vec{e}_y) \Rightarrow dl = R d\theta$$

$$\vec{E}(M) = \int_0^{2\pi} \frac{\lambda R d\theta (-R\vec{e}_x + z\vec{e}_z)}{4\pi\epsilon (R^2 + z^2)^{3/2}}$$

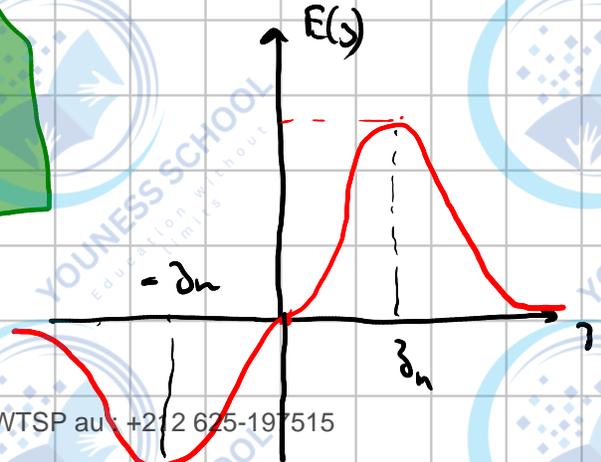


$$= \frac{\lambda R}{4\pi\epsilon (R^2 + z^2)^{3/2}} \left[-R \int_0^{2\pi} \vec{e}_x d\theta + z \vec{e}_z \int_0^{2\pi} d\theta \right]$$

$$\int_0^{2\pi} (\cos\theta \vec{e}_x + \sin\theta \vec{e}_y) d\theta = \left[\sin\theta \vec{e}_x - \cos\theta \vec{e}_y \right]_0^{2\pi} = (\sin(2\pi) - \sin(0)) \vec{e}_x - (\cos(2\pi) - \cos(0)) \vec{e}_y = (0-0)\vec{e}_x - (1-1)\vec{e}_y = \vec{0}$$

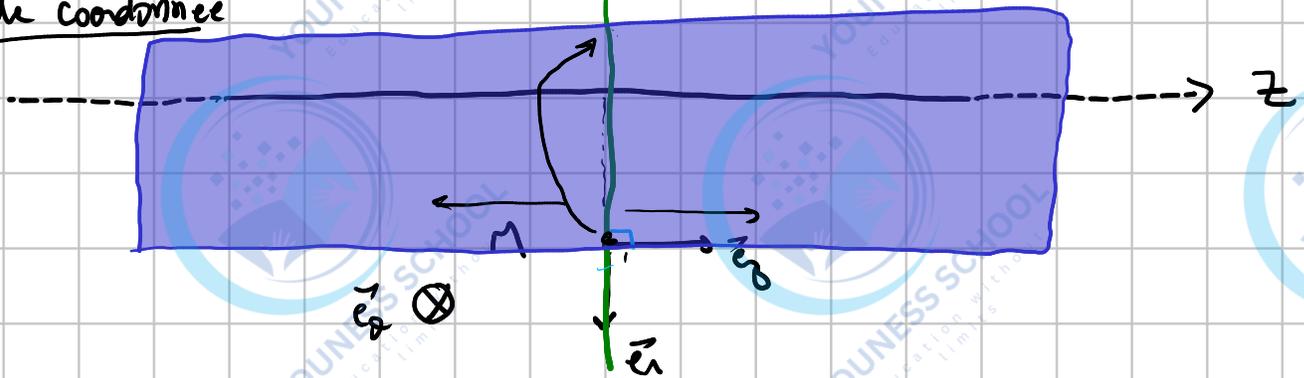
$$\vec{E}(M) = \frac{\lambda R z \vec{e}_z}{2\epsilon (R^2 + z^2)^{3/2}}$$

$$\frac{z}{(z^2)^{3/2}} = \frac{1}{z^2}$$

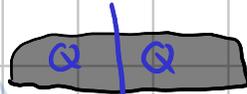


Exemple 1 fil infini λ uniforme.

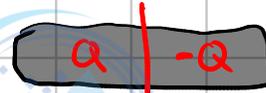
• sys. de coordonnées



• plans de symétrie



plan de sym



plan d'anti-sym

$(M, \vec{e}_r, \vec{e}_\phi)$ plan de symétrie. $\Rightarrow \vec{E} \in (M, \vec{e}_r, \vec{e}_\phi)$.

$(M, \vec{e}_r, \vec{e}_z)$ plan de symétrie $\Rightarrow \vec{E} \in (M, \vec{e}_r, \vec{e}_z)$

$\Rightarrow \vec{E}(M) = E(r) \vec{e}_r$
 champ radial

• Etude des invariances:

• symétrie cylindrique \Rightarrow invariance par rotation autour de (Oz) .

$\Rightarrow \vec{E}$ ne dépend pas de ϕ

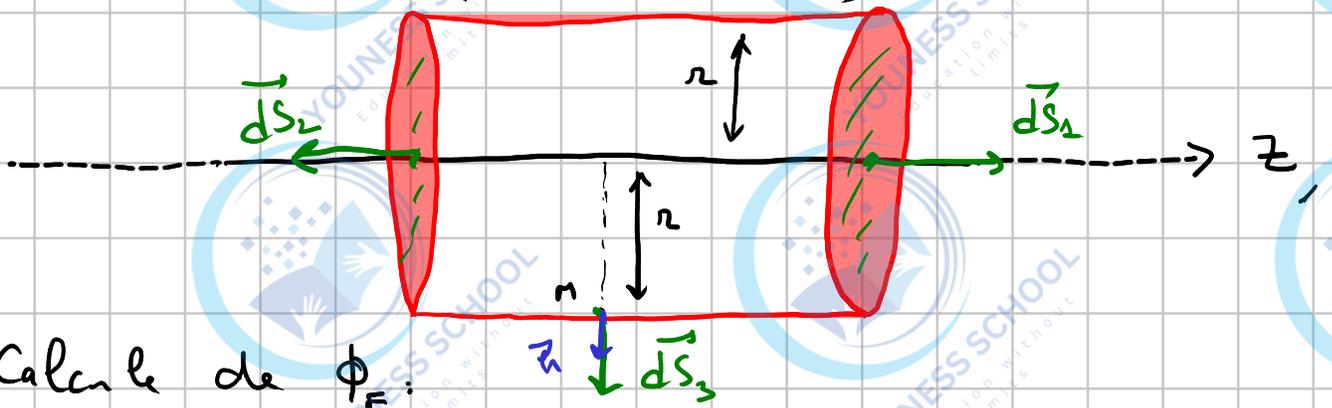
• Distribution infinie \Rightarrow invariance par translation le long de (Oz) .

$\Rightarrow \vec{E}$ ne dépend pas de z .

$\Rightarrow \vec{E}(M) = E(r) \vec{e}_r$

• Choix du surface de Gauss:

symétrie cylindrique \Rightarrow cylindre de rayon r , et de hauteur h



• Calcul de ϕ_E :

$$\begin{aligned} \phi_E &= \oint_{\Sigma} \vec{E} \cdot d\vec{S} = \oint_{\Sigma} E(r) \vec{e}_r \cdot d\vec{S} \\ &= \int_{\text{top}} E(r) \vec{e}_r \cdot dS_1 \vec{e}_3 + \int_{\text{bottom}} E(r) \vec{e}_r \cdot (-dS_2 \vec{e}_3) \\ &\quad + \int_{\text{side}} E(r) \vec{e}_r \cdot dS_3 \vec{e}_r \end{aligned}$$

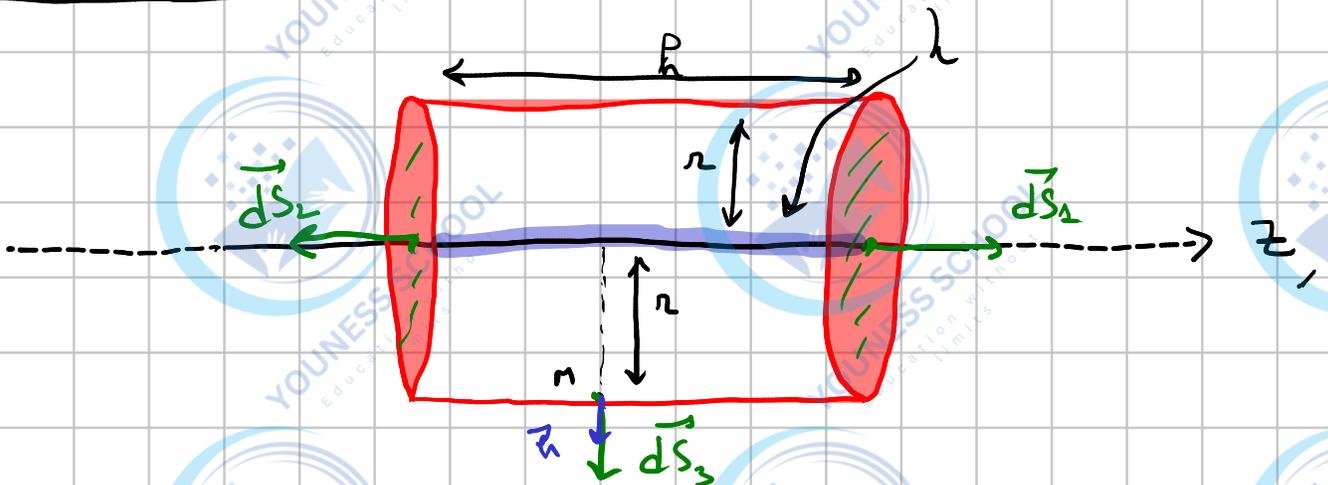
$$\phi_E = \int_{\text{side}} E(r) dS_3 = \int E(r) r d\theta dz$$

$$d\vec{r} = \cancel{dr} \vec{e}_r + r d\theta \vec{e}_\theta + dz \vec{e}_3$$

$$\phi_E = r E(r) \int_0^{2\pi} d\theta \int_0^h dz$$

$$\phi_E = \epsilon \pi r h E(r)$$

• Calcul de Q_{int} :



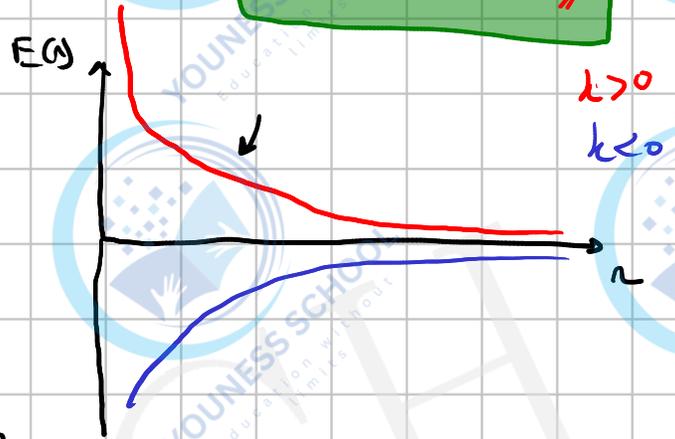
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$$Q_{int} = \int dq = \int \lambda dl = \lambda \int dl = \lambda R.$$

$$\Phi_E = \epsilon \pi R E(\omega) = \frac{\lambda R}{\epsilon} \Rightarrow E(\omega) = \frac{\lambda}{2\pi \epsilon_0 R}$$

$$\Rightarrow \vec{E}(r) = \frac{\lambda}{2\pi \epsilon_0 R} \vec{u}_r$$

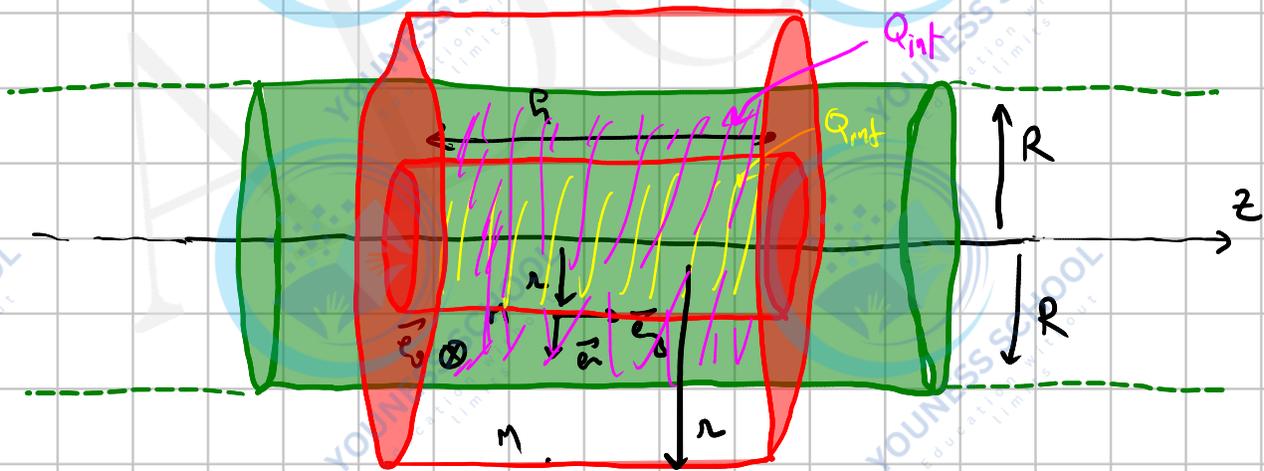


Commentaire:

$$r \rightarrow 0 \Rightarrow E \rightarrow +\infty$$



Exemple 3 Cylindre infini chargé par ρ uniforme.



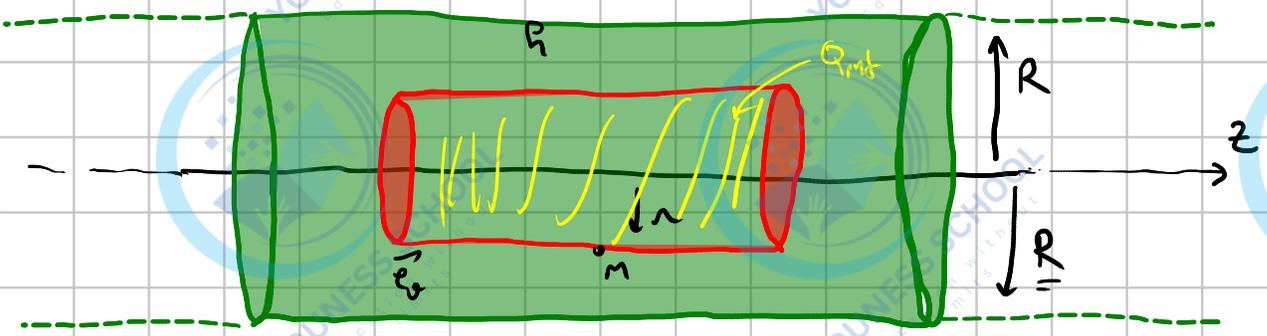
$$\Phi_E = \oint \vec{E} \cdot d\vec{s} = 2\pi r E(\omega).$$

$$\begin{aligned} \bullet \underline{Q_{int}} : \quad Q_{int} &= \int dq = \iiint \rho d\tau = \rho \iiint r dr d\theta dz \\ &= \rho \int_0^{2\pi} d\theta \int_0^R r dr \int_{-\infty}^{\infty} dz \end{aligned}$$

Cas ①

$r < R$

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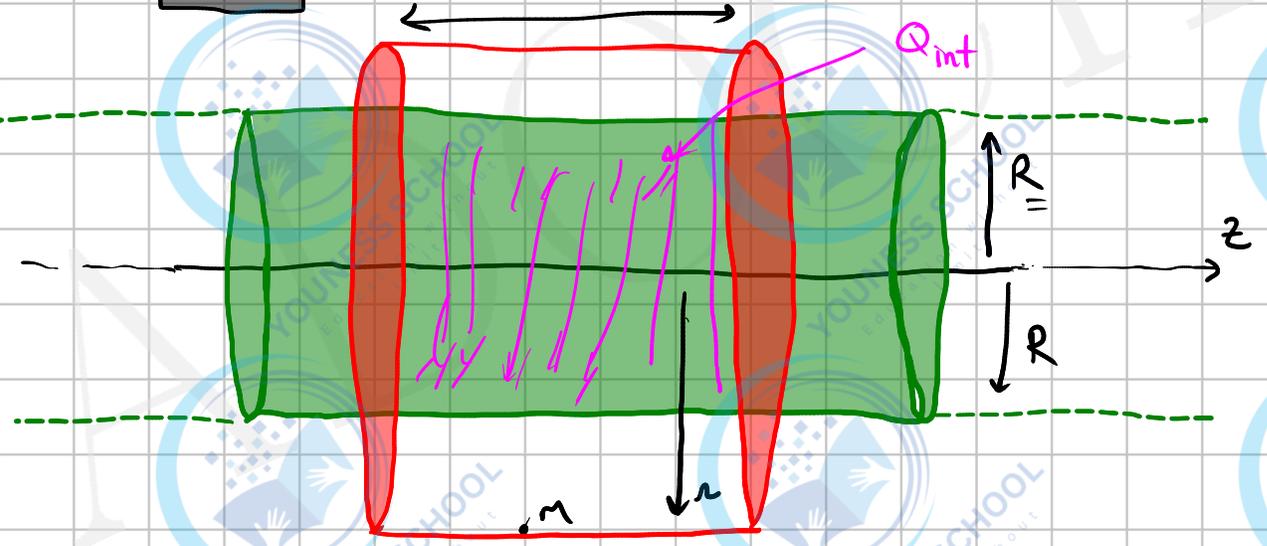


$$Q_{int} = \rho \int_0^r r' dr' \int_0^{2\pi} d\theta \int_0^h dz = \rho \frac{r^2}{2} \cdot 2\pi \cdot h$$

$$= \rho \pi r^2 h = \pi r^2 h E(r) \epsilon \Rightarrow E(r) = \frac{\rho r}{2 \epsilon}$$

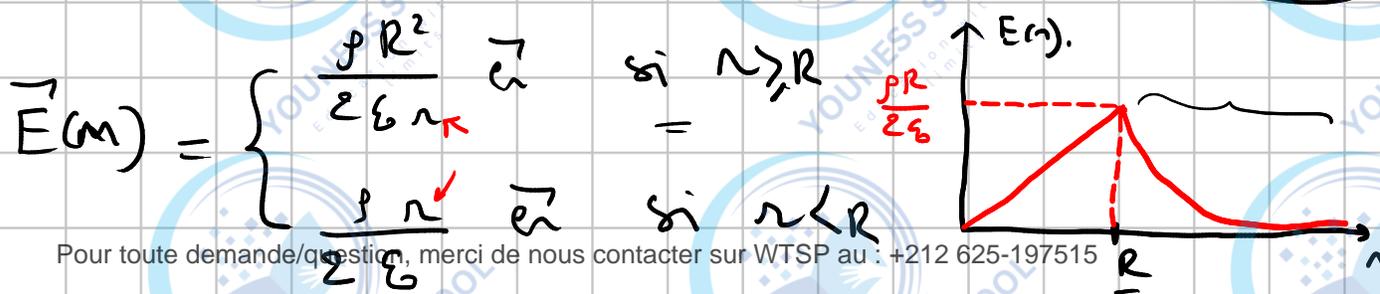
Cas ②

$r > R$



$$Q_{int} = \rho \int_0^R r' dr' \int_0^{2\pi} d\theta \int_0^h dz = \rho \frac{R^2}{2} \cdot 2\pi \cdot h$$

$$= \rho R^2 \pi h = 2\pi h E(r) \epsilon \Rightarrow E(r) = \frac{\rho R^2}{2 \epsilon r}$$



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$$R \rightarrow 0 \Rightarrow \text{cylindre} \rightarrow \text{fil} \Rightarrow \frac{\rho R^2}{2\epsilon_0} \rightarrow \frac{\lambda}{2\epsilon_0}$$

$$\Rightarrow \boxed{\lambda = \rho R^2}$$

$$\vec{E}(r) = \begin{cases} \frac{\rho r}{2\epsilon_0} \vec{e}_r & \text{si } r < R \\ \frac{\rho R^2}{2\epsilon_0 r} \vec{e}_r & \text{si } r \geq R \end{cases} \Rightarrow \vec{E} = E(r) \vec{e}_r$$

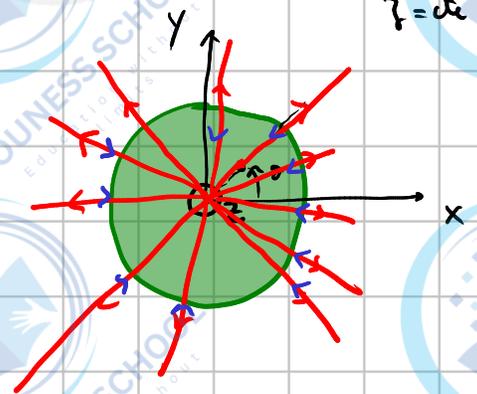
Lignes de champ

$$\vec{E} \wedge d\vec{O}M = E(r) \vec{e}_r \wedge (dr \vec{e}_r + r d\theta \vec{e}_\theta + dz \vec{e}_z) = 0$$

$$r d\theta \vec{e}_\theta - dz \vec{e}_z = \vec{0} \Rightarrow r d\theta = 0 \text{ et } dz = 0$$

$$\Rightarrow \theta = \text{cte (droite)} \text{ et } z = \text{cte (plan)}$$

lignes
de
champ
 $\rho > 0$
 $\rho < 0$



Le potentiel

$$E(r) \vec{e}_r = \vec{\nabla} V = - \text{grad } V = - \left(\frac{\partial V}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{e}_\theta + \frac{\partial V}{\partial z} \vec{e}_z \right)$$

$$E(r) = - \frac{dV}{dr} \Rightarrow \int dV = - \int E(r) dr$$

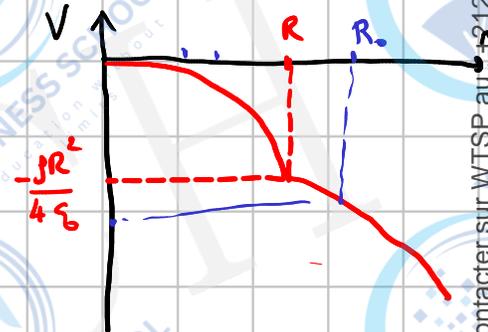
$$\Rightarrow \boxed{V(r) = - \int E(r) dr}$$

$$\vec{E}(r) = \begin{cases} \frac{\rho R^2}{2\epsilon_0} \vec{e}_r & \text{si } r < R \\ \frac{\rho R^2}{2\epsilon_0 r} \vec{e}_r & \text{si } r \geq R \end{cases} \Rightarrow V(r) = \begin{cases} -\frac{\rho R^2}{4\epsilon_0} + cte & \text{si } r \leq R \\ -\frac{\rho R^2}{2\epsilon_0} \ln(r) + cte' & \text{si } r \geq R \end{cases}$$

$V(r)$ est toujours continu $\Rightarrow -\frac{\rho R^2}{4\epsilon_0} + cte = -\frac{\rho R^2}{2\epsilon_0} \ln(R) + cte'$

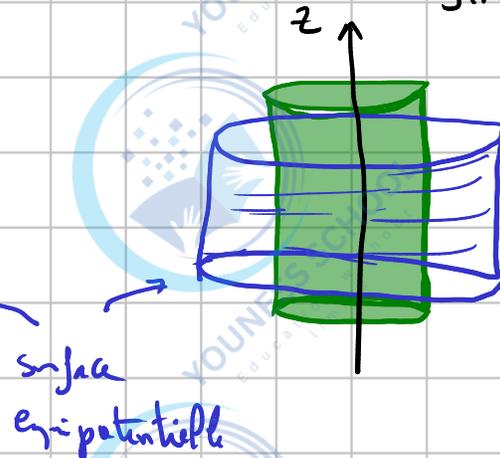
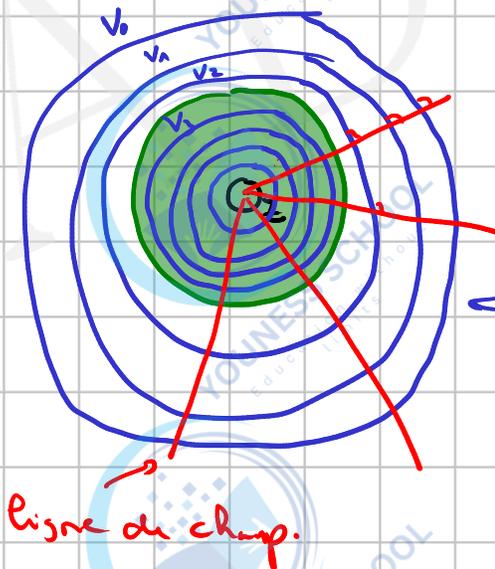
$V(r \rightarrow 0) = 0$ (choix) $\Rightarrow cte = 0 \Rightarrow cte' = -\frac{\rho R^2}{4\epsilon_0} + \frac{\rho R^2}{2\epsilon_0} \ln(R)$

$$V(r) = \begin{cases} -\frac{\rho R^2}{4\epsilon_0} & \text{si } r \leq R \\ \frac{\rho R^2}{2\epsilon_0} \ln\left(\frac{R}{r}\right) - \frac{\rho R^2}{4\epsilon_0} & \text{si } r \geq R \end{cases}$$



Surface equipotentielle

$V = cte \Rightarrow r = cte \Rightarrow$ surface latérale d'un cylindre.



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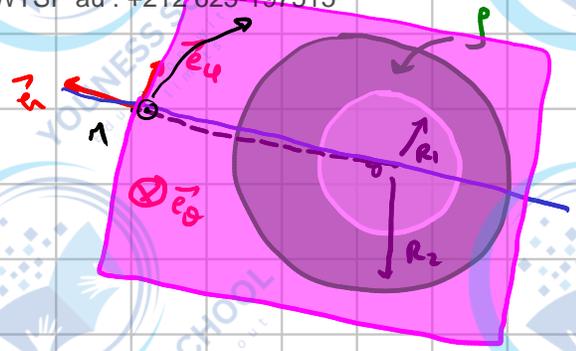
Exemple 4

symétrie

$(M, \vec{e}_n, \vec{e}_\varphi)$ plan de sym.

$(n, \vec{e}_r, \vec{e}_\varphi)$ " " "

$\Rightarrow \vec{E}(n) = E(n) \vec{e}_n$

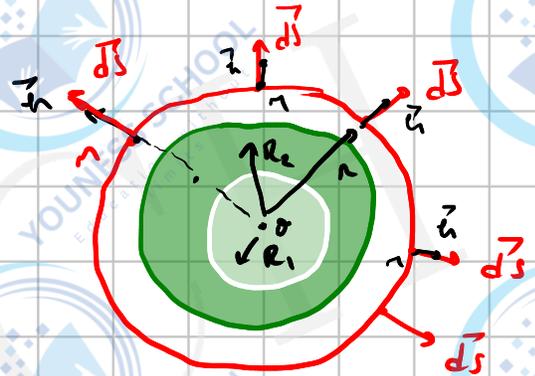


• invariance par rotation selon \vec{e}_φ et $\vec{e}_\varphi \Rightarrow \vec{E}$ ne dépend pas de θ et φ .

$\Rightarrow \vec{E}(n) = E(n) \vec{e}_n$

• Surface de Gauss:

sphère de centre O et de rayon n



• Le flux:

$\Phi_E = \oint \vec{E} \cdot d\vec{s} = \iint E(n) \vec{e}_n \cdot d\vec{s} \vec{e}_n$

$= \iint E(n) \cdot ds = \iint E(n) \cdot n d\varphi n \sin\varphi d\theta$

$d\vec{s} = \cancel{n} + \frac{n d\varphi}{1} \vec{e}_\varphi + \frac{n \sin\varphi d\theta}{1} \vec{e}_\theta$

$\Phi_E = n^2 E(n) \int_0^\pi \sin\varphi d\varphi \int_0^{2\pi} d\theta = 4\pi n^2 E(n)$

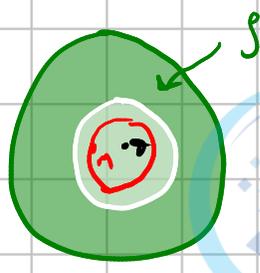
$\Phi_E = 4\pi n^2 E(n)$

• Q_{int}:

3 cas possibles pour n :

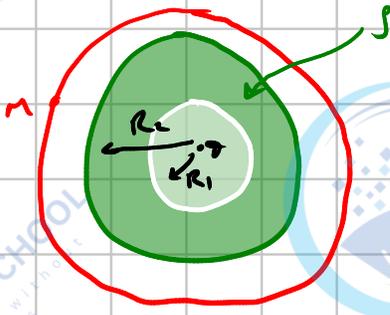
$4\pi n^2 E(n) = \frac{Q_{int}}{\epsilon_0}$

Cas ① : $r < R_1$



$Q_{int} = 0 \Rightarrow E(r) = 0$

Cas ② : $r > R_2$

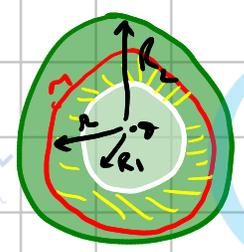


$$Q_{int} = \int dq = \iiint j \, d\tau$$

$$= j \iiint r^2 \, dr \, \sin\varphi \, d\varphi \, d\theta$$

$$= j \int_{R_1}^{R_2} r^2 \, dr \int_0^\pi \sin\varphi \, d\varphi \int_0^{2\pi} d\theta$$

Cas ③ : $R_1 < r < R_2$



$$Q_{int} = \int dq = \iiint j \, d\tau$$

$$= j \int_{R_1}^r r^2 \, dr \int_0^\pi \sin\varphi \, d\varphi \int_0^{2\pi} d\theta$$

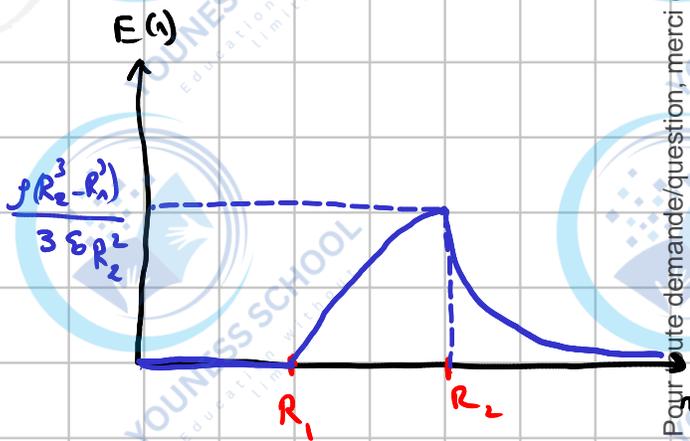
$$= \frac{4\pi}{3} j (r^3 - R_1^3)$$

$$\frac{1}{\epsilon} \nabla^2 E(r) \cdot \epsilon = Q_{int} = j \left(\frac{R_2^3}{3} - \frac{R_1^3}{3} \right) \cdot 4\pi = \frac{4\pi}{3} j (R_2^3 - R_1^3)$$

$E(r) = \frac{j(R_2^3 - R_1^3)}{3\epsilon r^2}$

$E(r) = \frac{j(r^3 - R_1^3)}{3\epsilon r^2}$

$$E(r) = \begin{cases} 0 & \text{si } r < R_1 \\ \frac{j(r^3 - R_1^3)}{3\epsilon r^2} & \text{si } R_1 < r < R_2 \\ \frac{j(R_2^3 - R_1^3)}{3\epsilon r^2} & \text{si } r > R_2 \end{cases}$$



Ligne de champ:

$$\vec{E} \wedge d\vec{or} = \vec{0} \Rightarrow E(r) \vec{e}_r \wedge (dr \vec{e}_r + r d\varphi \vec{e}_\varphi + r \sin\varphi d\theta \vec{e}_\theta) = \vec{0}$$

$$\Rightarrow r \, d\varphi \, \vec{e}_\theta - r \sin\varphi \, d\theta \, \vec{e}_\varphi = 0$$

$$\Downarrow \quad \Downarrow$$

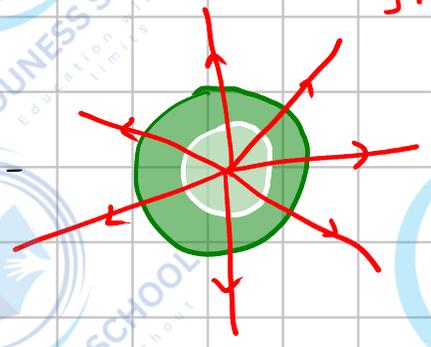
$$d\varphi = 0 \quad d\theta = 0$$

$$\Downarrow \quad \Downarrow$$

$$\varphi = \text{cte} \quad \theta = \text{cte}$$

potentiel

$$E(r) \vec{a} = - \text{grad } V = - \frac{dV}{dr} \vec{a}_r$$



$$V(r) = \int dV = - \int E(r) dr$$

$$E(r) = \begin{cases} 0 & \text{si } r < R_1 \\ \frac{\rho(r^3 - R_1^3)}{3\epsilon_0 r^2} \vec{a}_r & \text{si } R_1 < r < R_2 \\ \frac{\rho(R_2^3 - R_1^3)}{3\epsilon_0 r^2} \vec{a}_r & \text{si } r > R_2 \end{cases} \Rightarrow V(r) = \begin{cases} \text{cte} & \text{si } r \leq R_1 \\ -\frac{\rho r^2}{6\epsilon_0} - \frac{\rho R_1^3}{3\epsilon_0 R_1} + \text{cte}' & \text{si } R_1 < r < R_2 \\ + \frac{\rho(R_2^3 - R_1^3)}{3\epsilon_0 r} + \text{cte}'' & \text{si } r > R_2 \end{cases}$$

Continuité:

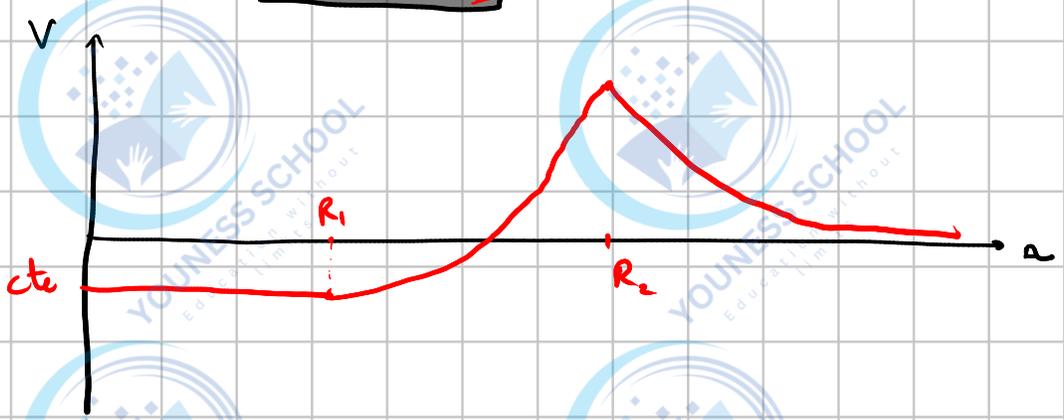
$$\text{cte} = -\frac{\rho R_1^2}{6\epsilon_0} - \frac{\rho R_1^3}{3\epsilon_0 R_1} + \text{cte}'$$

$$-\frac{\rho R_2^2}{6\epsilon_0} - \frac{\rho R_2^3}{3\epsilon_0 R_2} + \text{cte}' = \frac{\rho(R_2^3 - R_1^3)}{3\epsilon_0 R_2} + \text{cte}''$$

Condition:

$V(r \rightarrow +\infty) = 0$ en absence de charge à l'infini

$$\Rightarrow \text{cte}'' = 0$$



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Surface eq-potentelle

$$V = cte \Rightarrow r = cte \Rightarrow \text{sphère.}$$

ligne
de
charge

