Kinematics

Physics A level (The Chancellor, Masters, and Scholars of the University of Cambridge)

2. Kinematics

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CONTENTS

2.1 EQUATIONS OF MOTION

2.1.1 DISPLACEMENT, VELOCITY & ACCELERATION

Defining Displacement, Velocity & Acceleration

Scalar quantities

- Remember scalar quantities only have a magnitude (size)
  - **Distance:** the total length between two points
  - **Speed:** the total distance travelled per unit of time

Vector quantities

- Remember vector quantities have both magnitude and direction
  - **Displacement:** the distance of an object from a fixed point in a specified direction
  - **Velocity:** the rate of change of displacement of an object
  - **Acceleration:** the rate of change of velocity of an object

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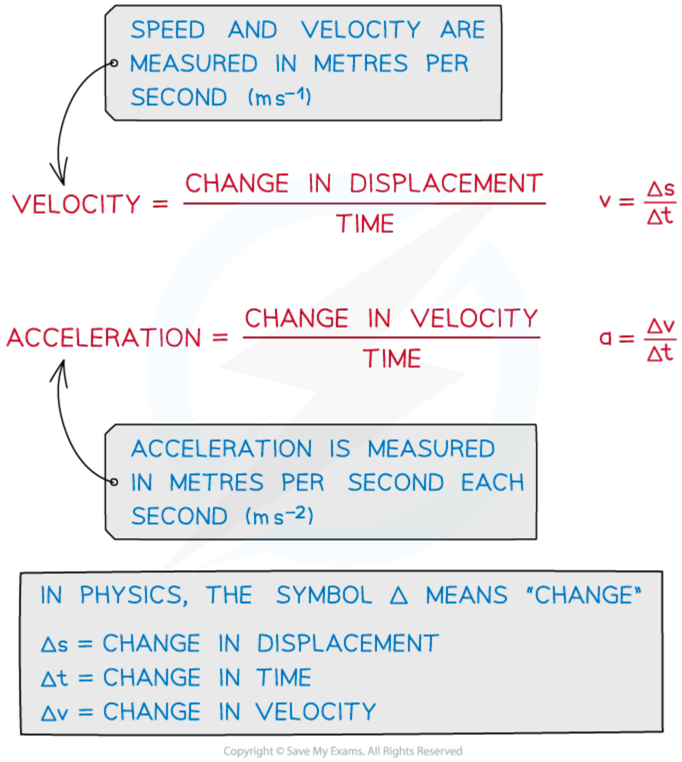
Equations



Equations linking displacement, velocity and acceleration

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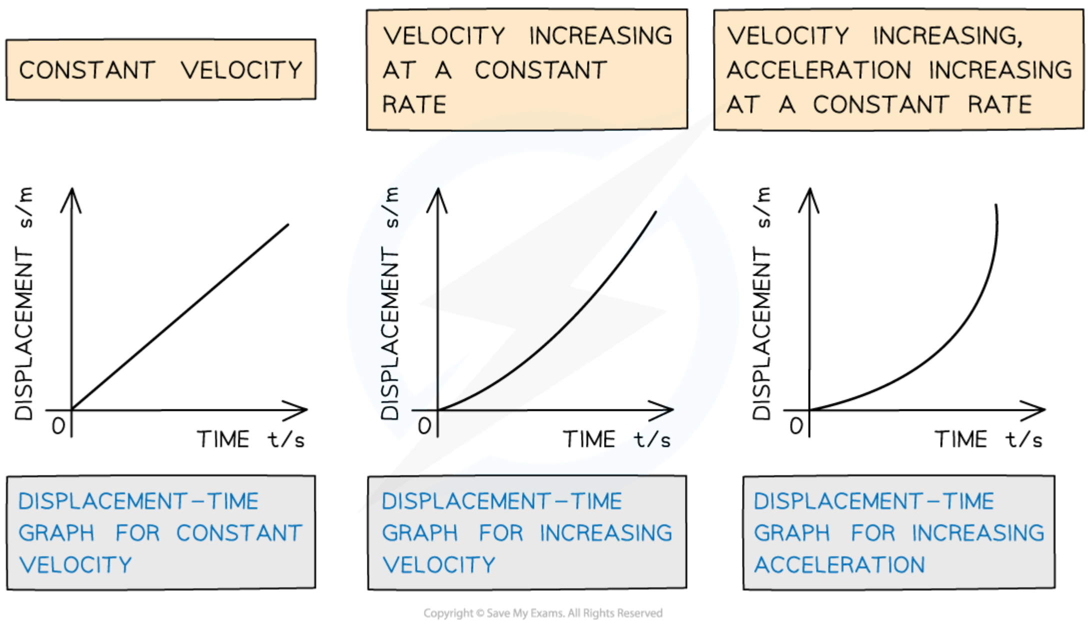
2.1.2 MOTION GRAPHS

Motion Graphs

- Three types of graph that can represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- On a **displacement-time graph…**
  - **slope** equals **velocity**
  - the **y-intercept** equals the **initial displacement**
  - a **straight** line represents a **constant** velocity
  - a **curved** line represents an **acceleration**
  - a **positive slope** represents motion in the **positive direction**
  - a **negative slope** represents motion in the **negative direction**
  - a **zero** slope (horizontal line) represents a state of **rest**
  - the area under the curve is meaningless



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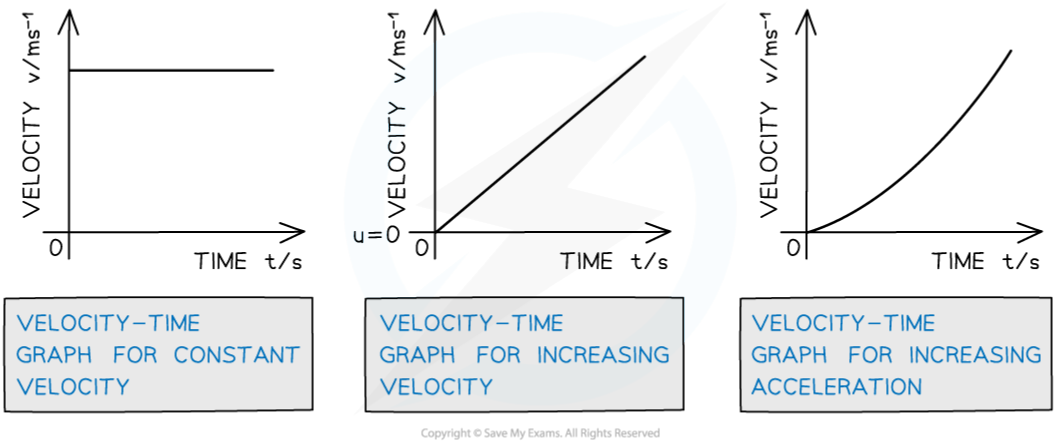
- On a **velocity-time graph...**
  - **slope** equals **acceleration**
  - the **y-intercept** equals the **initial velocity**
  - a **straight** line represents **uniform** acceleration
  - a **curved** line represents **non-uniform acceleration**
  - a **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - a **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - a **zero** slope (horizontal line) represents motion with **constant velocity**
  - the **area** under the curve equals the **change** in **displacement**



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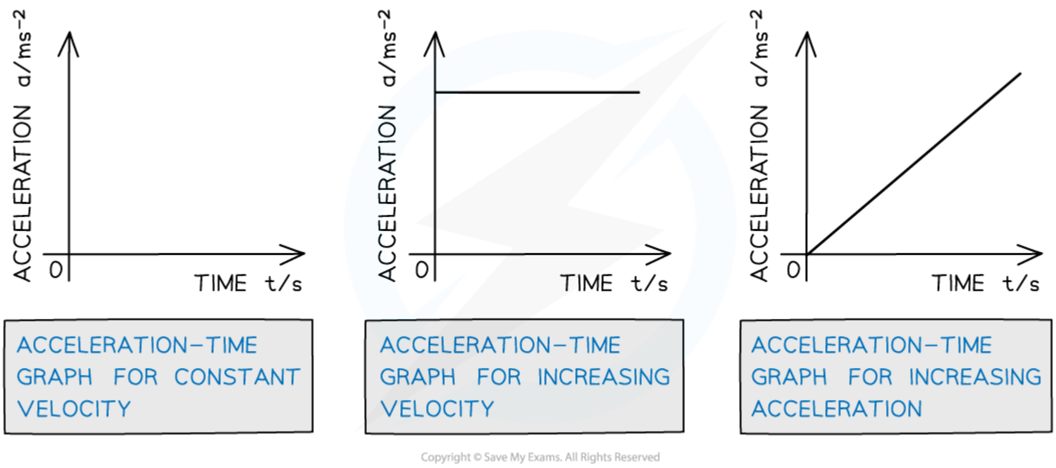
- On an **acceleration-time graph...**
  - slope is meaningless
  - the y-intercept equals the initial acceleration
  - a zero slope (horizontal line) represents an object undergoing constant acceleration
  - the area under the curve equals the change in velocity



How displacement, velocity and acceleration graphs relate to each other

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2.1.3 AREA UNDER A VELOCITY-TIME GRAPH

Area under a Velocity-Time Graph

- Velocity-time graphs show the speed and direction of an object in motion over a specific period of **time**
- The area under a velocity-time graph is equal to the **displacement** of a moving object

displacement = area under a velocity-time graph

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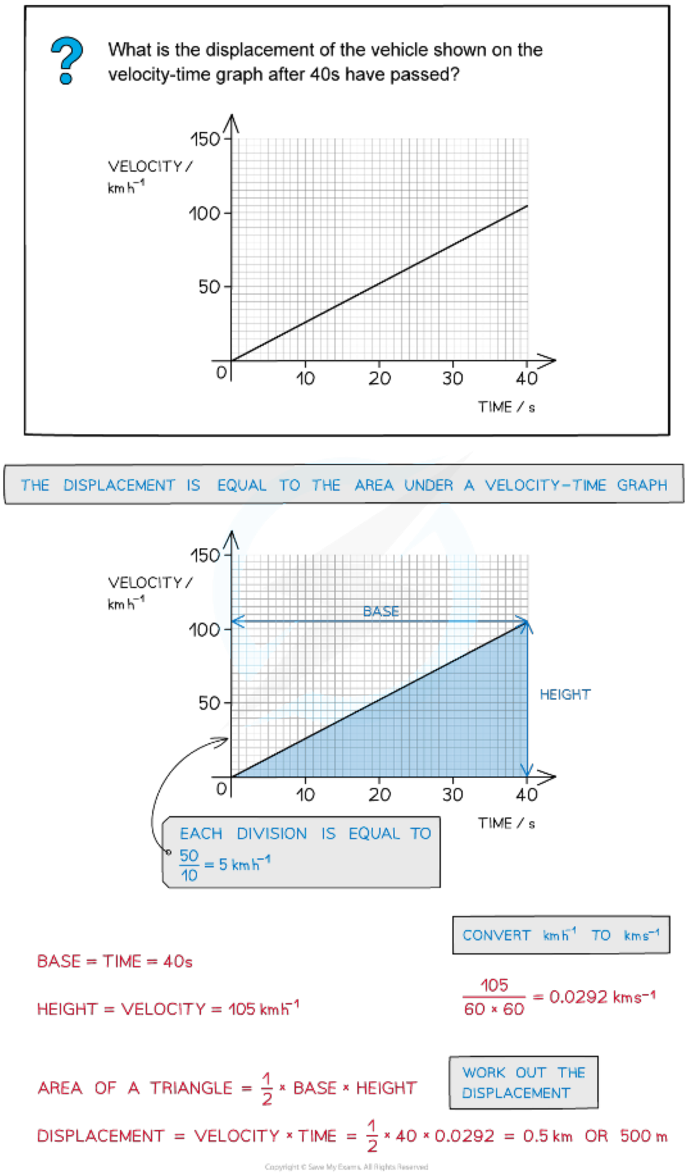

How to determine the area under a velocity-time graph

Exam Tip

Always check the values given on the y-axis of a motion graph – students often confuse displacement-time graphs and velocity-time graphs.

The area under the graph can often be broken down into triangles, squares and rectangles, so make sure you are comfortable with calculating area!

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2.1.4 GRADIENT OF A DISPLACEMENT-TIME GRAPH

Gradient of a Displacement-Time Graph

- Displacement-time graphs show the changing position of an object in motion
- They also show whether an object is moving forwards (positive displacement) or backwards (negative displacement)
  - A negative gradient = a negative velocity (the object is moving backwards)
- The gradient (slope) of a displacement-time graph is equal to **velocity**
  - The greater the slope, the greater the velocity

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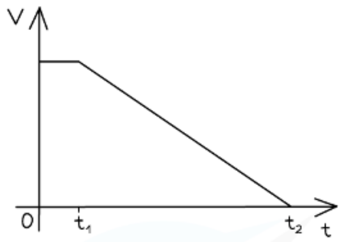
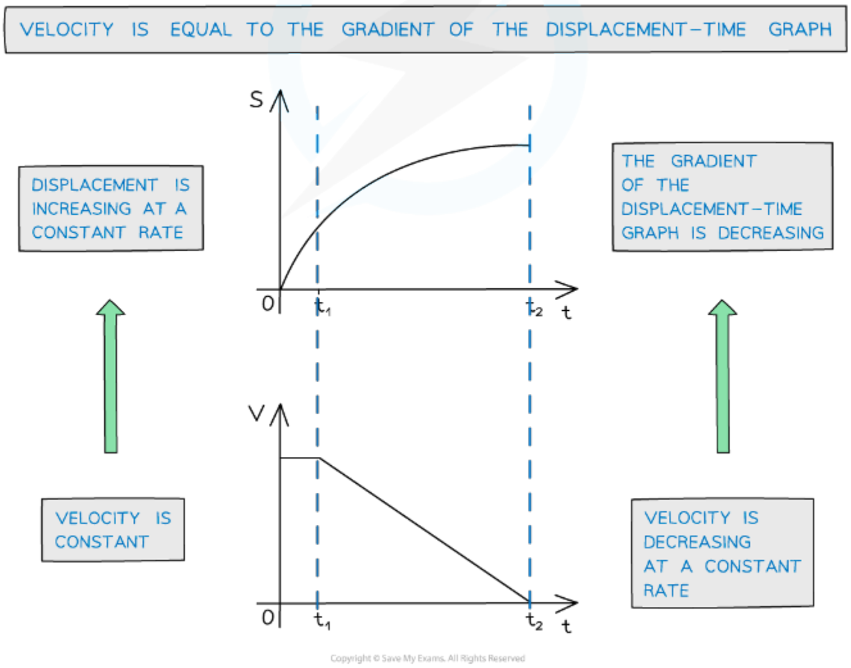
A car driver sees a hazard ahead and applies the brakes to bring the car to rest.

What does the displacement-time graph look like?





How to determine the slope of a displacement-time graph

Exam Tip

Don't forget that velocity is a vector quantity; it has a size and a direction. If velocity is initially positive and then becomes negative, then the object has changed direction.

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2.1.5 GRADIENT OF A VELOCITY-TIME GRAPH

Gradient of a Velocity-Time Graph

- **Acceleration** is any change in the velocity of an object in a given time

$$acceleration = \frac{change\ in\ velocity}{time} = \frac{(v-u)}{t}$$

- As velocity is a vector quantity, this means that if the **speed** of an object **changes**, or its **direction changes**, then it is accelerating
  - An object that slows down tends to be described as 'decelerating'
- The gradient of a velocity-time graph is equal to **acceleration**

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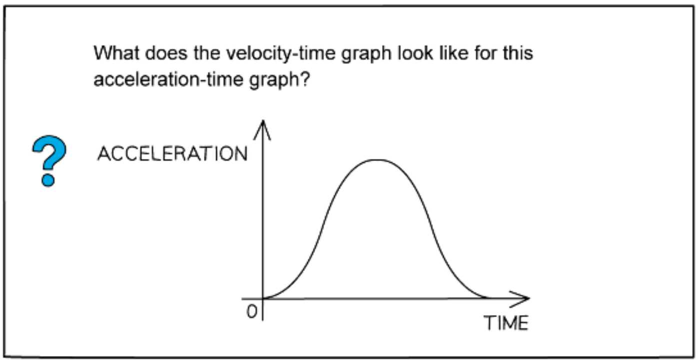

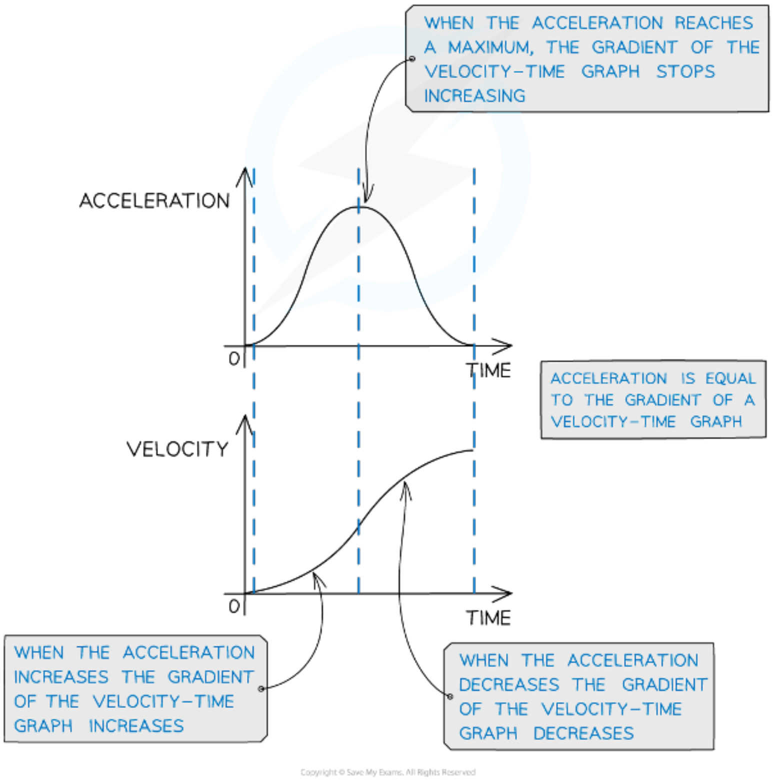

How to determine the slope of a velocity-time graph

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2.1.6 DERIVING KINEMATIC EQUATIONS

Deriving Kinematic Equations of Motion

- The kinematic equations of motion are a set of four equations which can describe any object moving with **constant** acceleration
- They relate the five variables:
    - s = **displacement**
    - u = **initial velocity**
    - v = **final velocity**
    - a = **acceleration**
    - t = **time interval**
- It's important to know where these equations come from and how they are derived:

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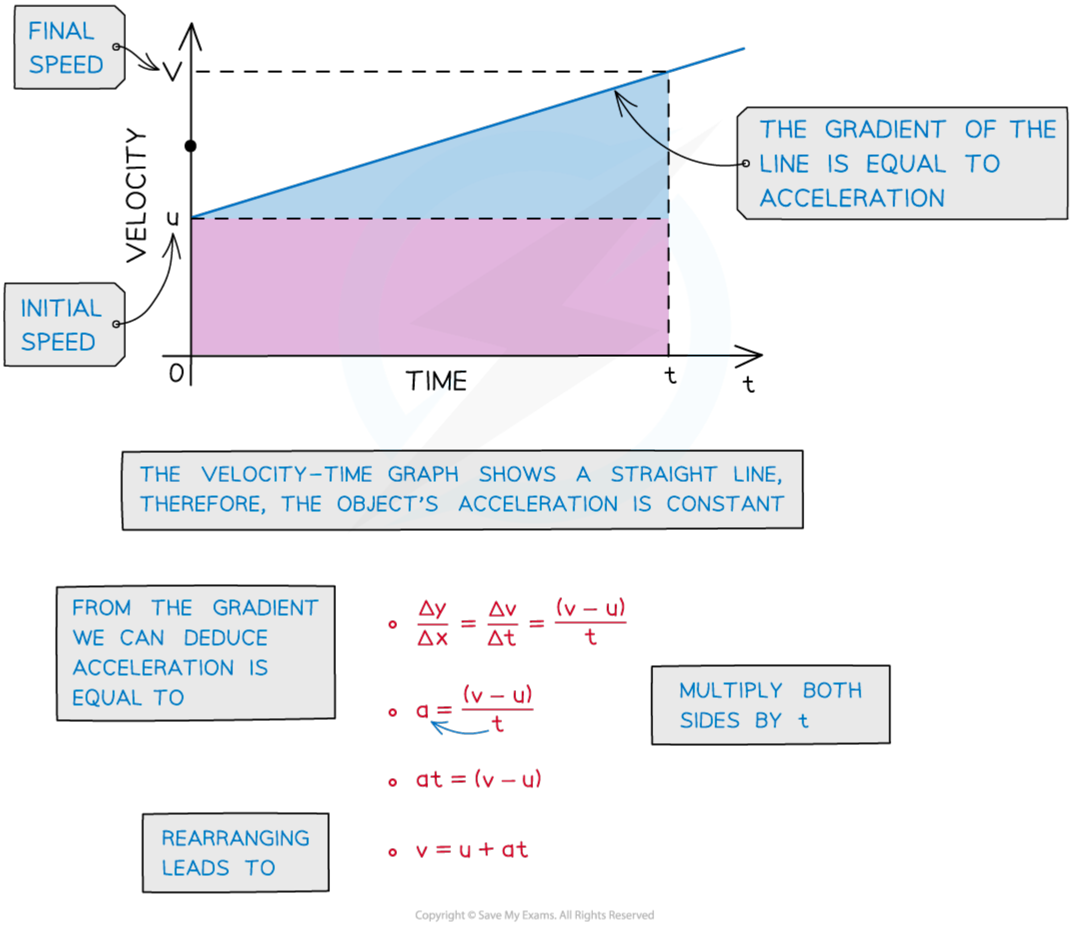
Deriving v = u + at



A graph showing how the velocity of an object varies with time

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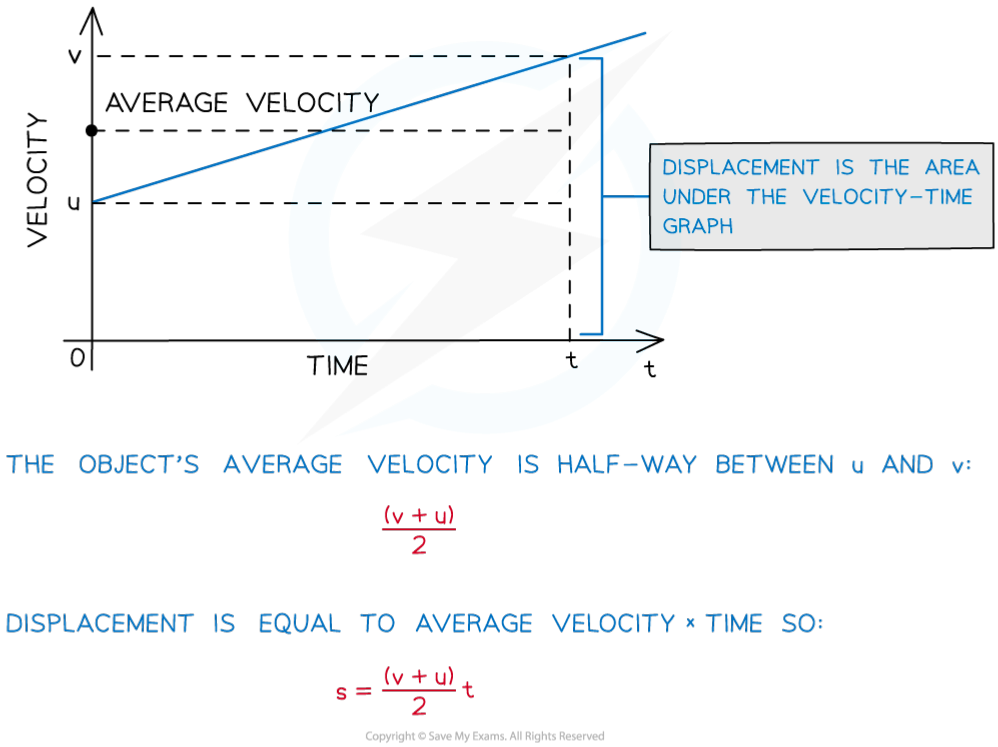
Deriving $s = \frac{(u + v)}{2} t$



The average velocity is halfway between u and v

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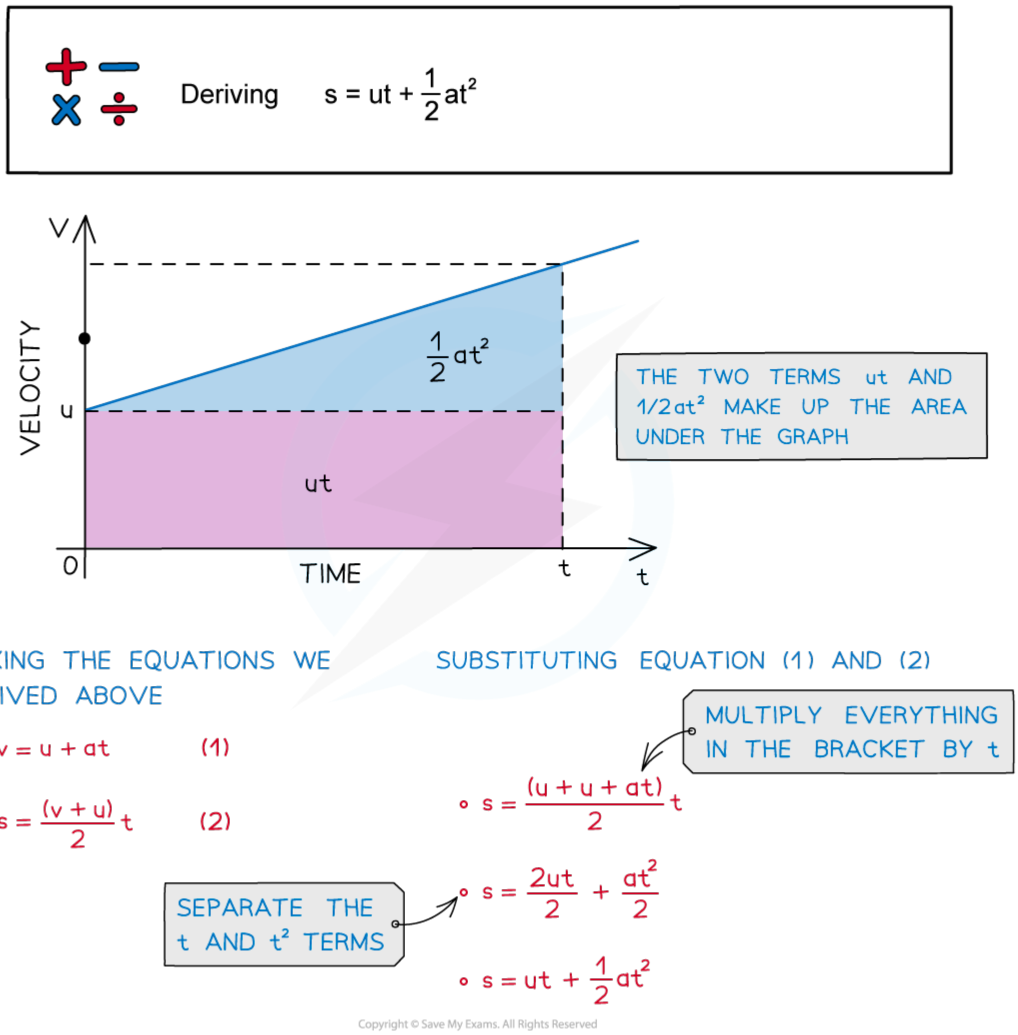

The two terms ut and ½at² make up the area under the graph

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Deriving $v^2 = u^2 + 2as$

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at \rightarrow t = \frac{v - u}{a}$ (1)

$s = \frac{(v + u)}{2} t$ (2)

SUBSTITUTING (1) INTO (2)

- $s = \frac{(v + u)}{2} \times \frac{(v - u)}{a}$
- $s = \frac{v^2 - u^2}{2a}$ ← MULTIPLY BOTH SIDES BY 2a
- $v^2 = u^2 + 2as$

This final equation can be derived from two of the others

SUMMARY

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{(v + u)}{2} t$

$v^2 = u^2 + 2as$

Summary of the four equations of uniformly accelerated motion

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2.1.7 SOLVING PROBLEMS WITH KINEMATIC EQUATIONS

Solving Problems with Kinematic Equations

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion a = ± 9.81 m s^{-2}, an object which starts or finishes at rest will have u = 0 or v = 0
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links *s, u, a* and *t* is $s = ut + ½at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

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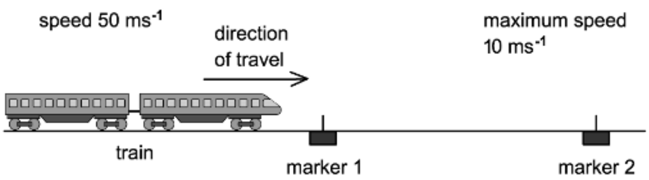
Worked Example

The diagram shows an arrangement to stop trains that are travelling too fast.



Trains coming from the left travel at a speed of 50 ms^{-1}. At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 ms^{-1}.

The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s.

How far from marker 2 should marker 1 be placed?

STEP 1

OUR KNOWN VARIABLES ARE

- $u = 50$ ms^{-1}
- $v = 10$ ms^{-1}
- $t = 20$ s

AND WE ARE ASKED TO FIND DISTANCE, s.

STEP 2

SO THE EQUATION THAT LINKS u, v, t AND s IS

$$s = \frac{(u+v)}{2}t$$

STEP 3

NO REARRANGING IS REQUIRED SO WE SIMPLY PLUG IN THE VARIABLES:

$$s = \frac{(50+10)}{2} \times 20 = 30 \times 20 = 600 \text{ m}$$

How to solve problems using the kinematic equations

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Exam Tip

- This is arguably the most important section of this topic, you can always be sure there will be one, or more, questions in the exam about solving problems with the kinematic equations
- The best way to master this section is to practice as many questions as possible

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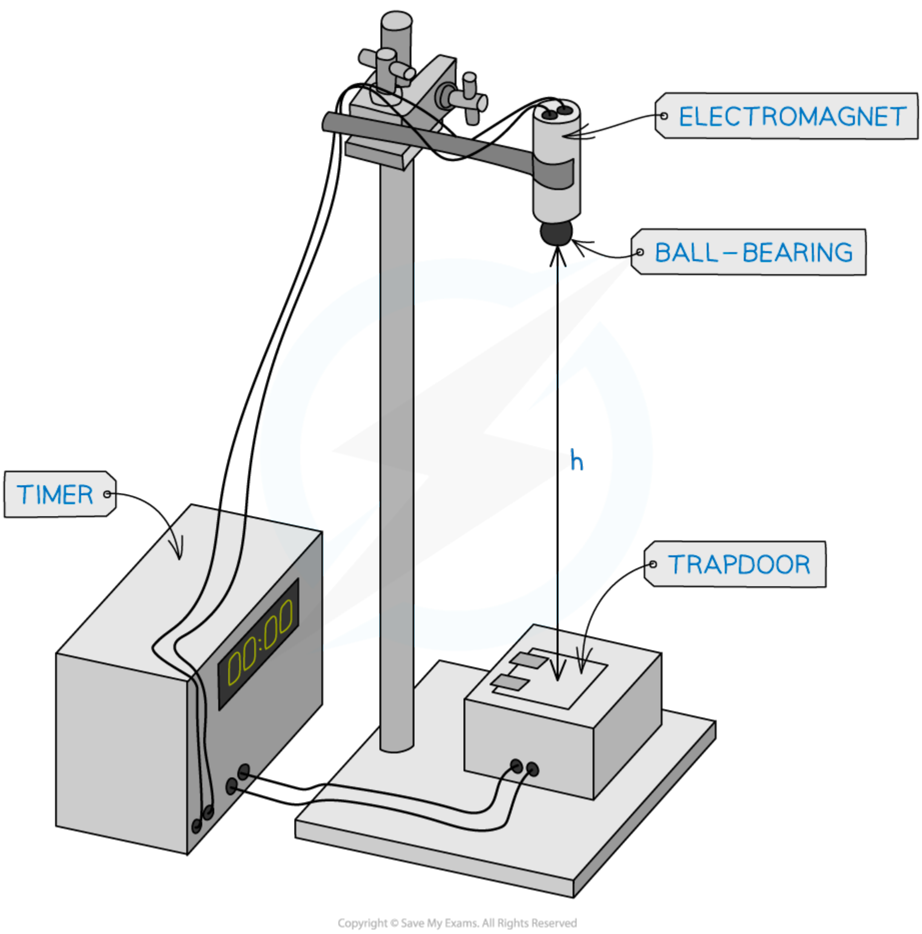
2.1.8 ACCELERATION OF FREE FALL EXPERIMENT

Acceleration of Free Fall Experiment

- A common experiment to determine acceleration of a falling object which can be carried out in the lab

Apparatus

- Metre rule, ball bearing, electromagnet, electronic timer, trapdoor



Apparatus used to measure g

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Method

- When the current to the magnet switches off, the ball drops and the timer starts
- When the ball hits the trapdoor, the timer stops
- The reading on the timer indicates the time it takes for the ball to fall a distance, *h*
- This procedure is repeated several times for different values of *h*, in order to reduce random error
- The distance, *h*, can be measured using a metre rule as it would be preferable to use for distances between 20 cm – 1 m

Analysing data

- To find *g* , use the same steps as in the problem solving section
- The known quantities are
  - Displacement $s = h$
  - Time taken = t
  - Initial velocity $u = 0$
  - Acceleration $a = g$
- The equation that links these quantities is
  - $s = ut + ½ at^2$
  - $h = ½ gt^2$
- Using this equation, deduce *g* from the gradient of the graph of *h* against t^2

Sources of error

- **Systematic error:** residue magnetism after the electromagnet is switched off may cause the time to be recorded as longer than it should be
- **Random error:** large uncertainty in distance from using a metre rule with a precision of 1mm, or from parallax error

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2.1.9 PROJECTILE MOTION

Projectile Motion

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance travelled by the projectile

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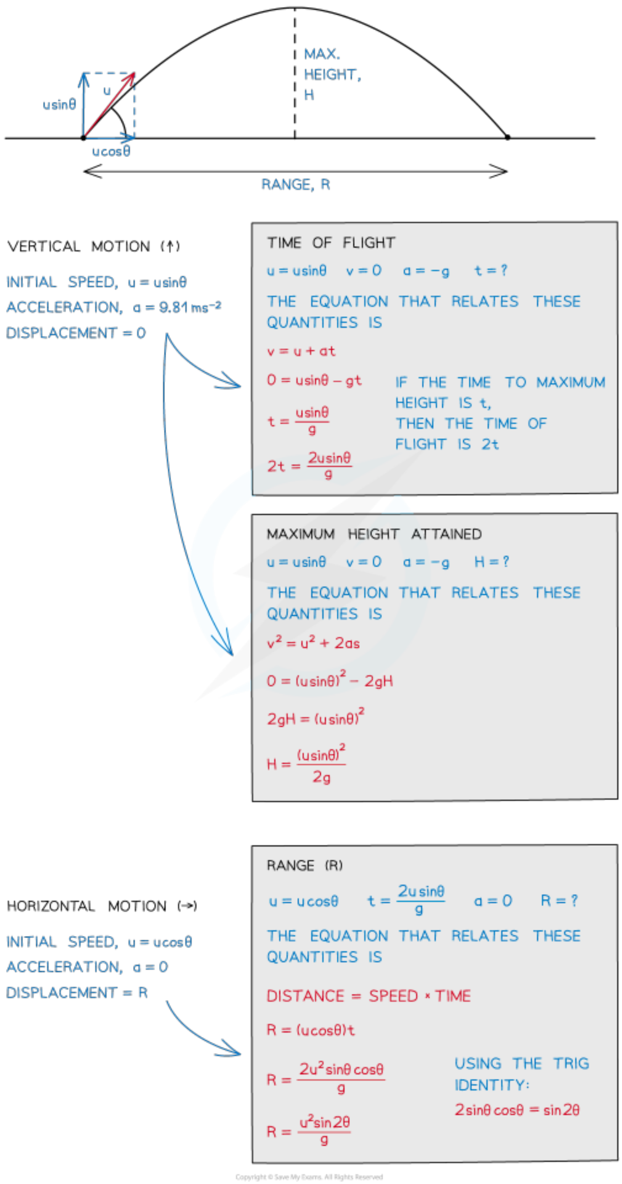

How to find the time of flight, maximum height and range

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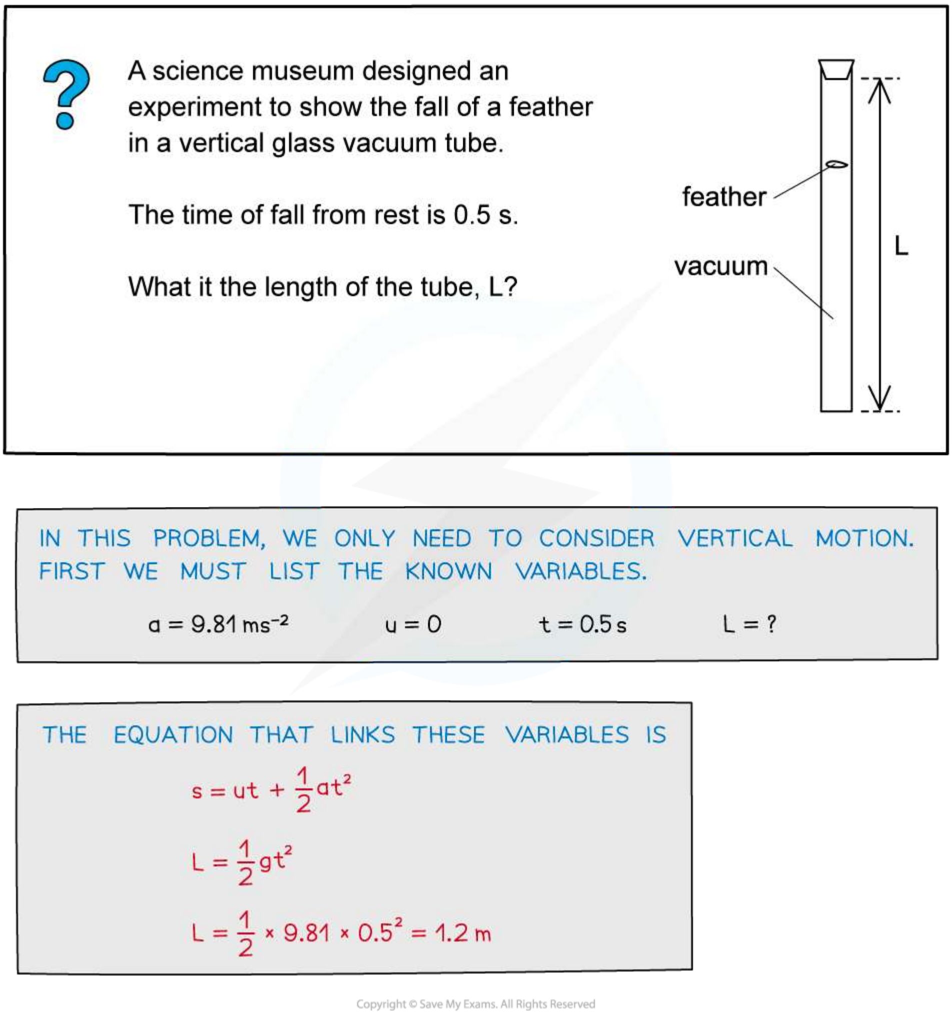
- **Remember:** the only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**
- Let's consider each in turn:

Worked Example 1



How to calculate vertical projection (free fall)

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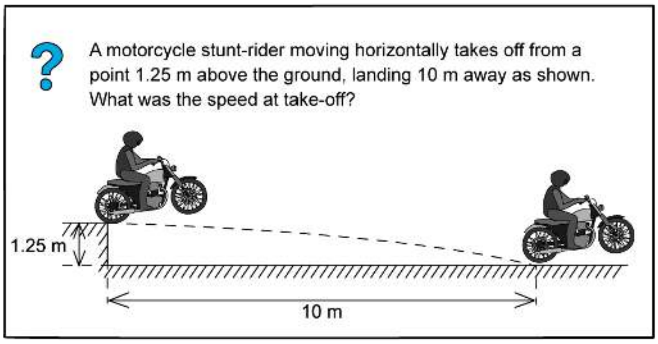
Worked Example 2



IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $\quad a = 9.81$ ms^{-2} $\quad u = 0 \quad t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $\quad a = 0 \quad t = 0.5$ s $\quad u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20\ \text{ms}^{-1}$$

How to calculate horizontal projection

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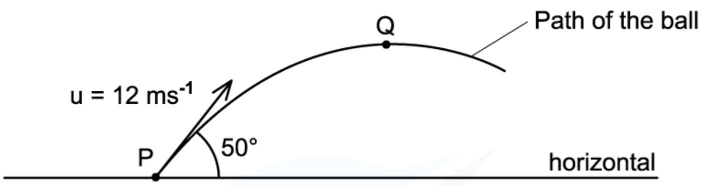
Worked Example 3

A ball is thrown from a point P with an initial velocity u of 12 ms^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q?



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $\quad a = -9.81\ ms^{-2}$ $\quad v = 0$ $\quad H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ m$$

How to calculate projection at an angle

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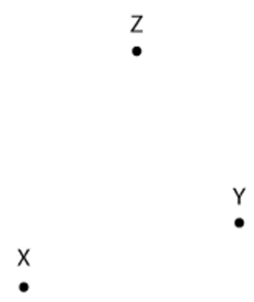
Exam Tip

Make sure you don't make these common mistakes:

- Forgetting that deceleration is negative as the object rises
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres

Exam Question: Easy

An object moves directly from X to Z



In a shorter time, a second object moves from X to Y to Z.

Which statement about the two objects is correct for the journey from X to Z?

A they have the same average speed

B they have the same average velocity

C they have the same displacement

D they travel the same distance

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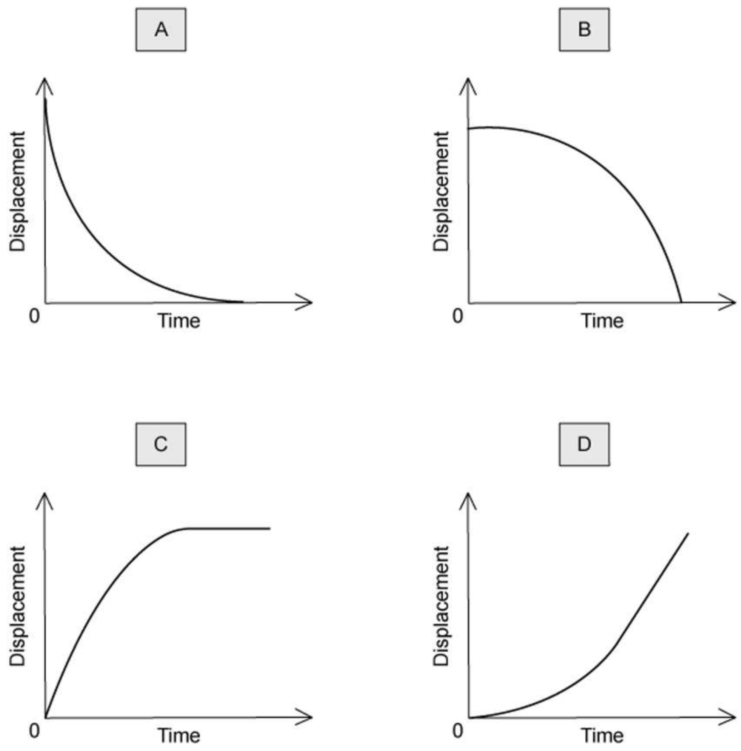
Exam Question: Medium

A sphere is released and falls. Its initial acceleration reduces until it eventually begins to travel at constant terminal velocity.

Which displacement-time graph best represents the motion of the sphere?



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Exam Question: Hard

A body having uniform acceleration *a* increases its velocity from *u* to *v* in time *t*.

Which expression would **not** give a correct value for the body's displacement during time *t*?

A $ut + \frac{1}{2}at^2$

B $vt - \frac{1}{2}at^2$

C $\frac{(v+u)(v-u)}{2a}$

D $\frac{(v-u)t}{2}$

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