Modern Actuarial Risk Theory
Chapter 1: Utility Theory and Insurance
Part 1

Yas Suttakulpiboon[1]

[1] Department of Statistics (Insurance)
Chulalongkorn Business School
Bangkok, Thailand
isariya@cbs.chula.ac.th

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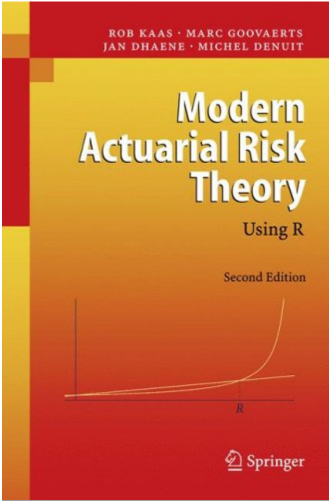
Agenda

Read Chapter 1



What is Utility Function

- $u(x)$
- It is a tool used by economist to explain human choosing behavior
- ‘Law of diminishing marginal utility’
- Graphical presentation
- A few calculus involved

Properties

- Increasing in consumption... the more we consume, the happier we become i.e. $u'(X) > 0$
- Concavity in consumption i.e. $u''(X) < 0$

Classes of Utility Functions

- Linear
- Quadratic
- Logarithmic
- Exponential
- Power

Example

- Check whether the utility functions above have the two properties
- (Do it yourself) you can try plotting the function in Excel to see the different shape of each utility function

Risk Aversion Coefficients

- Given a utility function, one can measure a risk aversion coefficient
- Absolute Risk Aversion Coefficient : $R(w) = \frac{-u''(w)}{u'(w)}$
- Relative Risk Aversion Coefficient : $r(w) = w * R(w)$
- Example : try computing R(w) and r(w) of exponential utility function and quadratic utility function

DARA, CARA, IARA, CRRA

- DARA : *Decreasing* Absolute Risk Aversion
- CARA : *Constant* Absolute Risk Aversion
- IARA : *Increasing* Absolute Risk Aversion
- CRRA : *Constant Relative* Risk Aversion

Example

- Question : Is an expected utility function DARA/CARA/IARA or CRRA ?
- Question : Is quadratic utility function DARA/CARA/IARA or CRRA ?
- Question : Is a linear utility function DARA/CARA/IARA or CRRA ?

Summary

- Utility function and its properties
- Classes of utility function
- Risk aversion coefficients

Agenda

1. Utility Theory : The Basics
   - Utility Theory : The Basics
   - Classes of Utility Functions
   - Example
2. Expected Utility Theory
   - Expected Utility Theory
   - Expected Utility Theory with Insurance
3. Conclusion

Expected Utility Theory

- Describe an individual preference over risky asset
- $\mathbb{E}[u(x)]$
- Math presentation
  - Discrete : $\mathbb{E}[u(x)] = \Sigma p_i u(x_i)$
  - Continuous : $\mathbb{E}[u(x)] = \int u(x)f(x)dx$
- Graphical representation

St.Petersberg Paradox

- This is the paradox
- For a price P, one may enter the following game. A fair coin is tossed until a head appears. If this takes n trials, the gain is an amount 2^n. Therefore, the expected gain from the game equals $\Sigma 2^n \left(\frac{1}{2^n}\right)$
- How much would you pay to enter this game ?
- Still, unless P is small, it turns out that very few are willing to enter the game, which means no one merely looks at expected profits.

Jensen's Inequality

- This is a crucial property for our study
- If u( ) is concave, then $\mathbb{E}[u(x)] \leq u(\mathbb{E}[x])$
- If u( ) is convex, then $\mathbb{E}[u(x)] \geq u(\mathbb{E}[x])$

Example

- Consider the graphical representation of the concave / convex utility function again.

Summary

- Expected Utility Function
- St.Petersberg Paradox
- Jensen's Inequality

Insured Problem : Demand for Insurance

- Notation : W = Wealth, X = Random loss, u( ) = utility function
- Utility of a person's net wealth after loss = u(w-X)
- Expected utility of this person = $\mathbb{E}[u(w - X)]$
- Consider the case when loss is discrete i.e. bernoulli loss
- Consider the case when loss is continuous e.g. exponential loss

Insured Problem : Demand for Insurance

- P = Premium paid to insurer to cover the loss
- Q = Indemnity received from insurer in case of loss. Assume Q = w (full insurance)
- We can derive a condition how the insured will purchase an insurance policy or not :
- $\mathbb{E}[u(w - X)] = u(w - P^{+})$

Insurer Problem : Supply of Insurance

- Insurer sells insurance policy : receive premiums from the insured and pay indemnity to the insured in the case of a loss
- Assume that insurer has a utility function U(x)... an upper case U
- Insurer will decide whether to sell insurance policy or not using the following decision equation :
- $U(W) = \mathbb{E}[U(W + P^{-} - X)]$
- P^{-} is the lowest premium price insurer will offer in the market.

When Demand Meets Supply

- Insured : P^{+}
- Insurer : P^{-}
- If $P^{-} \leq P^{+}$, the market exists.

Example

- Given u( ) and U( ), you should be able to compute P^- and P^+
- Consider the case when both u( ) and U( ) are exponential utility functions
- Consider the case when both u( ) and U( ) are exponential utility functions and the loss distribution (distribution of the loss X) is exponential with parameter β
- Consider the case when u( ) and U( ) are quadratic utility functions
- (Do it yourself) Consider the case when u( ) and U( ) are exponential utility functions with Gamma-distributed loss.

Conclusion

- Utility Function
- Expected Utility
- Expected Utility with Insurance Application

Next Class

- Approximate Premiums using Arrow-Pratt Approximation
- Stop-loss Reinsurance
- Proportional Reinsurance

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