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# SPM SEMINAR 2019

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PART 2

## ADD MATHS

VIDEO PEMBELAJARAN LENGKAP DI

Tingkatan 4

Tingkatan 5



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# Additional Mathematics

Presented by: Mathew Pang

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## Our Agenda for Today

Integration  
Integration skill | Area and Volume  
  
Trigonometry Function  
Basic of Trigonometry | Graph | Formulae  
  
Q&A Session  
Interaction session with students



Linear Law  
 $y=mx+c$  | Relationship of two diagram



Vectors  
Concept of Vectors | Vectors in Cartesian Planes



Probability Distribution  
Binomial Distributions | Normal Distribution





## Linear Law

$Y=mx+c$  | Relationship of two diagram

4

$$Y=mx+c$$

$m$  = Gradient |

$c$  = Vertical Intercept |

| Convert non linear using linear law

$$\begin{aligned} xy &= \textcolor{red}{p} x^3 + \textcolor{green}{k} \\ Y &= \textcolor{red}{m}X + \textcolor{green}{c} \end{aligned}$$

Upward arrows point from the variables  $x$ ,  $y$ ,  $p$ ,  $k$  in the first equation to the variables  $X$ ,  $m$ ,  $c$  in the second equation.

$$Y=mx+c$$

m = Gradient |

c = Vertical Intercept

| Convert non linear using linear law

$$\log y = p \log x + k^{10}$$

↑      ↑      ↑      ↑  
 $Y = m X + c$

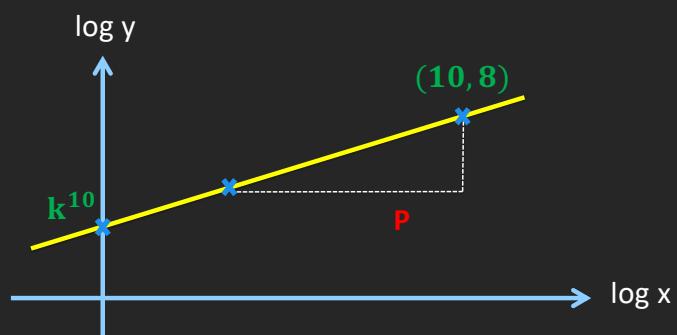
$$Y=mx+c$$

m = Gradient |

c = Vertical Intercept

| Convert non linear using linear law

$$\log y = p \log x + k^{10}$$



$$Y=mx+c$$

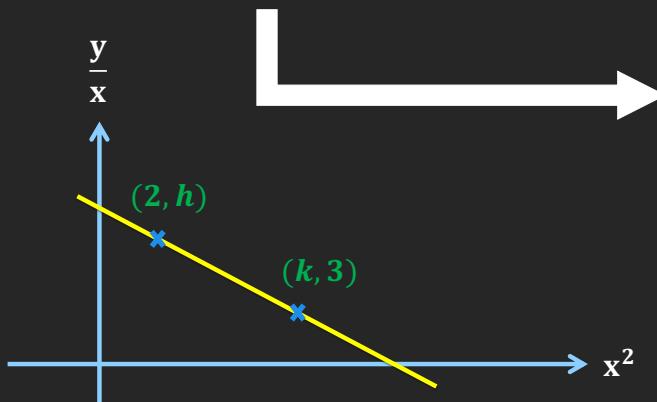
$m$  = Gradient |  $c$  = Vertical Intercept

| Convert non linear using linear law

Given  $y = 9x - x^3$ , find  $h$  and  $k$ .

(From the diagram)

$$Y = m X + c$$



- 10 Table 10 shows the values of two variables,  $x$  and  $y$ , obtained from an experiment. The variables  $x$  and  $y$  are related by the equation  $y = \frac{p}{x} + qx$ , where  $p$  and  $q$  are constants.  
*Jadual 10 menunjukkan nilai-nilai dua pemboleh ubah,  $x$  dan  $y$ , yang diperoleh daripada satu eksperimen. Pemboleh ubah  $x$  dan  $y$  dihubungkan oleh persamaan  $y = \frac{p}{x} + qx$  dengan keadaan  $p$  dan  $q$  ialah pemalar.*

$x$	1	2	3	4	5	6
$y$	5.60	6.55	8.53	10.38	13.12	15.52

Table 10 / Jadual 10

- (a) Plot  $xy$  against  $x^2$ , using a scale of 2 cm to 5 units on the  $x^2$ -axis and 2 cm to 10 units on the  $xy$ -axis. Hence, draw the line of best fit. [5 marks]  
*Plot  $xy$  melawan  $x^2$ , dengan menggunakan skala 2 cm kepada 5 unit pada paksi- $x^2$  dan 2 cm kepada 10 unit pada paksi- $xy$ . Seterusnya, lukis garis lurus penyeuaian terbaik. [5 markah]*
- (b) Use the graph in 10(a) to find the value of  
 Gunakan graf anda di 10(a) untuk mencari nilai  
 (i)  $p$   
 (ii)  $q$   
 (iii)  $y$  when  $x = 4.5$   
 $y$  apabila  $x = 4.5$   
 [5 marks]  
 [5 markah]

### 1 Draw the new table

\*Get hint from part (a)

### 2 Rearrange the equation

\*make it into  $y = mx + c$

### 3 Get the m and c values from your graph

### 4 Find the x or y values from the graph

- 11 Diagram 11 shows the graph of  $\log_2 y$  against  $\log_2 x$ . The variables  $x$  and  $y$  are related by the equation  $y = px^3$ , where  $p$  is a constant.

Rajah 11 menunjukkan graf  $\log_2 y$  melawan  $\log_2 x$ . Pembelahan hubungan  $x$  dan  $y$  dihubungkan oleh persamaan  $y = px^3$  dengan keadaan  $p$  adalah pemalar.

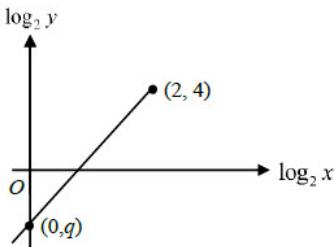


Diagram 11/ Rajah 11

Find the values of  $p$  and  $q$ .

Cari nilai  $p$  dan nilai  $q$ .

[3 marks]

Terengganu 2018 P1 Q11

- 19 The variables,  $x$  and  $y$  are related by the equation  $y = px - qx^3$ .

Diagram 19 shows a straight-line  $MN$  is obtained by plotting  $\frac{y}{x}$  against  $x^2$ .

Pembelahan hubungan  $x$  dan  $y$  dihubungkan oleh persamaan  $y = px - qx^3$ .

Rajah 19 menunjukkan garis lurus  $MN$  yang diperoleh dengan memplot  $\frac{y}{x}$  melawan  $x^2$ .

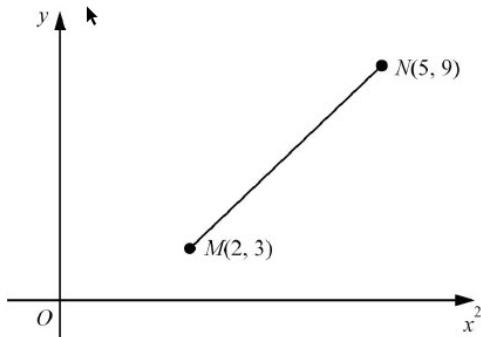


Diagram 19 / Rajah 19

Find the values of  $p$  and  $q$ .

Cari nilai  $p$  dan nilai  $q$ .

[3 marks]

[3 markah]

Perlis 2018 P1 Q19

Variables  $x$  and  $y$  are related by the equation  $4y = 2(x - 1)^2 + 3k$ , where  $k$  is a constant. Find

Pemboleh ubah  $x$  dan  $y$  dihubungkan oleh persamaan  $4y = 2(x - 1)^2 + 3k$ , dengan keadaan  $k$  ialah pemalar. Cari

- (a) the gradient of the straight line obtained when graph  $(y + x)$  against  $x^2$  is plotted.  
*kecerunan garis lurus yang diperoleh apabila graf  $(y + x)$  melawan  $x^2$  diplot.*

- (b)  $y$ -intercept in terms of  $k$ .  
*pintasan- $y$  dalam sebutan  $k$ .*

Answer/ Jawapan:

(a)

[3 marks]

[3 markah]

Negeri Sembilan 2018 P1 Q19

Table 9 shows the values of two variables,  $x$  and  $y$ , obtained from an experiment. Variables  $x$  and  $y$  are related by the equation  $y = p\sqrt{q}$ , where  $p$  and  $q$  are constants.

Jadual 9 menunjukkan nilai-nilai bagi dua pembolehubah  $x$  dan  $y$ , yang diperoleh daripada suatu eksperimen. Pembolehubah  $x$  dan  $y$  dihubungkan oleh persamaan  $y = p\sqrt{q}$ , dengan keadaan  $p$  dan  $q$  ialah pemalar.

$x$	1	4	9	16	25	36
$y$	1.78	2.63	3.72	5.75	8.91	12.59

Table 9  
*Jadual 9*

- (a) Based on Table 9, construct a table for the values of  $\log_{10} y$  and  $\sqrt{x}$ . [2 marks]

Berdasarkan Jadual 9, bina satu jadual bagi nilai-nilai  $\log_{10} y$  dan  $\sqrt{x}$ .

[2 markah]

- (b) Plot  $\log_{10} y$  against  $\sqrt{x}$ , using a scale of 2 cm to 1 unit on the  $\sqrt{x}$ -axis and 2 cm to 0.1 unit on the  $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[3 marks]

Plot  $\log_{10} y$  melawan  $\sqrt{x}$ , menggunakan skala 2 cm kepada 1 unit pada paksi- $\sqrt{x}$  dan 2 cm kepada 0.1 unit pada paksi- $\log_{10} y$ .

Seterusnya, lukis garis lurus penyuai terbaik.

[3 markah]

- (c) Using the graph in 9(b), find the value of

Menggunakan graf di 9(b), cari nilai

- (i)  $p$ ,  
(ii)  $q$ .

MRS M 2018 P2 Q9



# Integration

Integration skill | Area and Volume

14

## Integration skill

Two type of Integration | Basic law of integrals | Area and Volume

$$\int ax^n = \frac{ax^{n+1}}{n+1} + c$$

$$\int 3x^2 = \frac{3x^{2+1}}{2+1} + c$$

## Integration skill

Two type of Integration

Basic law of integrals

Area and Volume

$$\int (ax + b)^n = \frac{\overline{(ax + b)^{n+1}}}{a(n+1)} + c$$

$$\int (3x + 5)^4 = \frac{(3x + 5)^{4+1}}{3(4+1)} + c$$

## Integration skill

Two type of Integration

Basic law of integrals

Area and Volume

1  $\int_a^b kf(x) dx = k \int_a^b f(x) dx$

2  $\int_a^b f(x) dx = - \int_b^a f(x) dx$

3  $\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx$

4  $\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$

## Integration skill

Two type of Integration

| Basic law of integrals

| Area and Volume

$$\int \frac{3x^3}{4} dx$$

$$\int \frac{x - 5x^2}{x^4} dx$$

$$\int -\frac{2}{3(8 - 2x)^5} dx$$

## Integration skill

Two type of Integration

| Basic law of integrals

| Area and Volume

$\int \frac{d^2y}{dx^2} dx = \frac{dy}{dx}$ , where  $\frac{dy}{dx}$  is the gradient function

$\int \frac{dy}{dx} dx = y$ , where  $y$  is the equation of curve.

## Integration skill

Two type of Integration

Basic law of integrals

Area and Volume

$$\text{Area} = \int_a^b y \, dx$$

$$\text{Volume} = \pi \int_a^b y^2 \, dx$$

$$\text{Area} = \int_a^b x \, dy$$

$$\text{Volume} = \pi \int_a^b x^2 \, dy$$

- 4 Given  $\frac{d}{dx} \left( \frac{3-x}{x+1} \right) = g(x)$  and  $\int_{-2}^2 [hg(x) - 3] \, dx = \frac{32}{3}$ , find the value of  $h$ .

[3 marks]

NS 2018 P1 Q4

- 17 Given the gradient function of a curve is  $kx^2 + 5x$ , where  $k$  is a constant. A straight line,  $y - 3x - 7 = 0$  is the tangent to the curve at point  $(-3, 2)$ . Find the equation of the curve.

Answer/ Jawapan:

[3 markah]

NS 2018 P1 Q17

- 18 Given that  $y = \frac{2x+1}{3x^2}$  and  $\frac{dy}{dx} = 3g(x)$ . Find the value of  $\int_{-1}^1 g(x) \, dx$ . [3 marks]

Diberi bahawa  $y = \frac{2x+1}{3x^2}$  dan  $\frac{dy}{dx} = 3g(x)$ . Cari nilai  $\int_{-1}^1 g(x) \, dx$ . [3 markah]

Perlis 2018 P1 Q18

**24** Given that  $y = \frac{2x}{(x+3)^3}$  and  $\frac{dy}{dx} = \frac{6-4x}{(x+3)^4}$ , find  $\int_{-1}^0 \frac{3-2x}{(x+3)^4} dx$ .

MRSMP 2018 P1 Q24

**25** The gradient function of a curve is  $\frac{10}{(2-x)^3}$ .

Find the equation of the curve if the curve passes through point  $(1, 2)$ .

MRSMP 2018 P1 Q25

**3** Diagram 3 shows part of a curve.

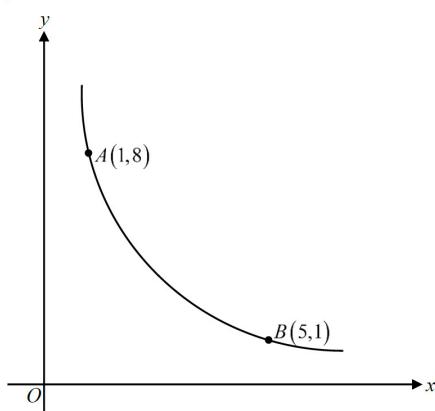


Diagram 3 / Rajah 3

Given that the points  $A(1, 8)$  and  $B(5, 1)$  lie on the curve and  $\int_1^5 y dx = 18$ ,

find the value of  $\int_1^8 x dy$ .

Terengganu 2018 P1 Q3

- 7 Diagram 7 shows part of the graph  $y = \frac{1}{2}x^2 + 4$  which passes through  $R(2, 6)$ . The straight line  $RS$  is the tangent to the curve at  $R$ .

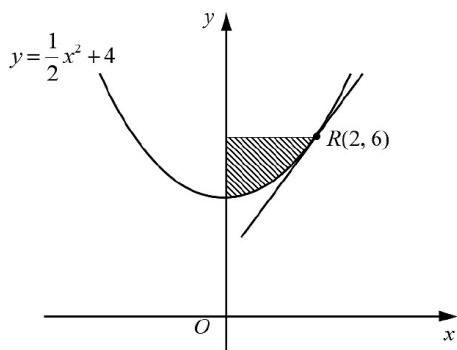


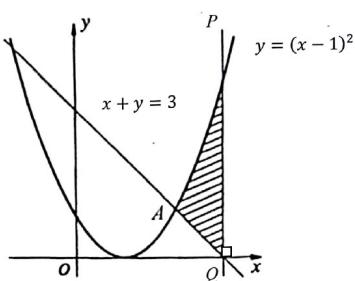
Diagram 7 / Rajah 7

Find / Cari

- (a) the equation of the straight line  $RS$ , [3 marks]
- (b) the area, in  $\text{cm}^2$ , of the shaded region, [4 marks]
- (c) the volume generated, in terms of  $\pi$ , when the shaded region is rotated through  $360^\circ$  about the  $y$ -axis. [3 marks]

Perlis 2018 P2 Q7

- 10 Diagram 7 shows a curve  $y = (x - 1)^2$ , straight line  $x + y = 3$  and straight line  $PQ$ . The curve intersects the straight line  $x + y = 3$  at point  $A$ .

Diagram 7  
Rajah 7

- (a) Find the coordinates of point  $A$ . [3 markah]
- (b) Calculate
  - (i) the area bounded by the curve,  $x$ -axis and the straight line  $x + y = 3$ , [3 markah]
  - (ii) the volume of revolution, in terms of  $\pi$ , when the shaded region is rotated through  $360^\circ$  about the  $x$ -axis. [4 markah]

Negeri Sembilan 2018 P2 Q10

- 3 Diagram 3 shows front view of an arch. Company A is assigned to paint the front wall of the arch. The entrance of the arch is represented by the equation of  $y = ax^2 + c$ , where  $a$  and  $c$  are constants.

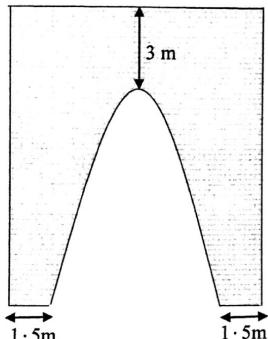


Diagram 3  
Rajah 3

The width and the height of the wall is 7 m and 11 m respectively. Paint is sold in 5 litre bucket and 1 litre of paint can cover  $10 \text{ m}^2$  of area of a single coating.

Find the minimum number of bucket of paint that is needed if the company has to complete the painting with 3 coatings.

[7 markah]

MRS M 2018 P2 Q3

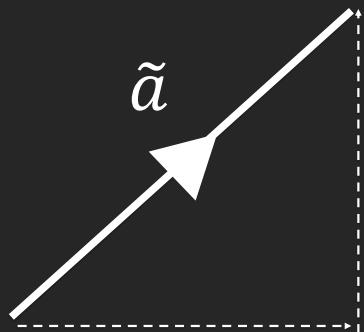


## Vectors

Concept of vectors | Cartesian Planes

## Vector

Basic knowledge of vector | Vector in Cartesian Plane



$$\tilde{a} = 3i + 4j$$

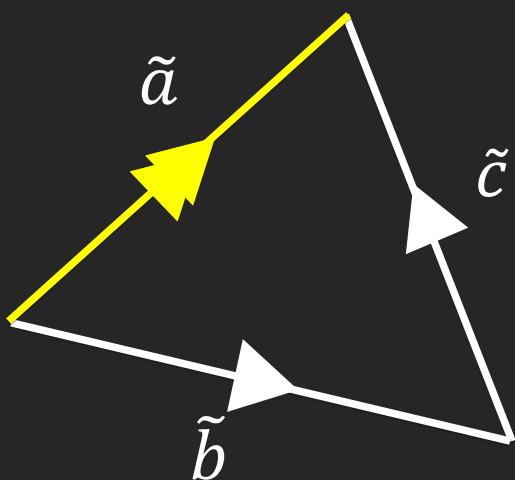
$$|\tilde{a}| = \sqrt{3^2 + 4^2}$$

$$\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \frac{3i + 4j}{5}$$

## Vector

Basic knowledge of vector | Vector in Cartesian Plane

*Triangle Law*



If  $\tilde{a}$  is resultant vector,

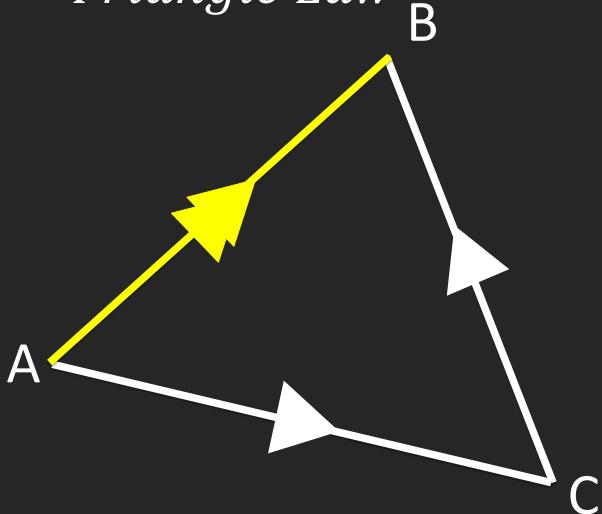
$$\text{Then, } \tilde{a} = \tilde{b} + \tilde{c},$$

$$\text{Or, } \tilde{a} = \tilde{c} + \tilde{b},$$

## Vector

Basic knowledge of vector | Vector in Cartesian Plane

*Triangle Law*



$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$$

$$\overrightarrow{BA} =$$

$$\overrightarrow{CA} =$$

## Vector

Basic knowledge of vector | Vector in Cartesian Plane

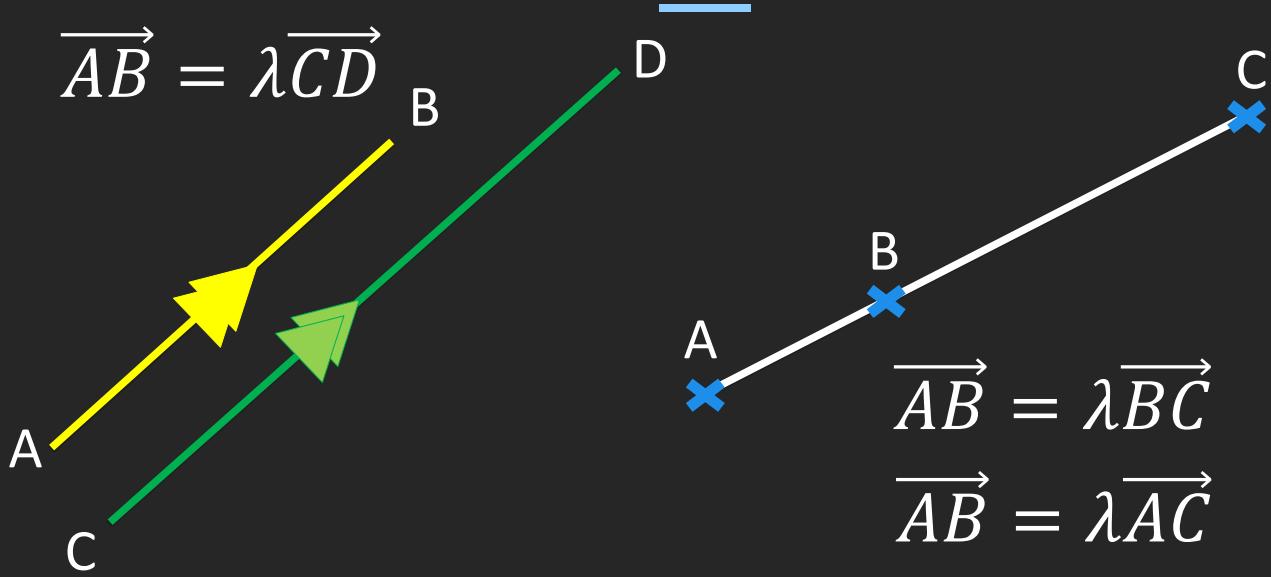
*Position Vector of A* =  $\overrightarrow{OA}$

*Position Vector of M* =  $\overrightarrow{OM}$

*Position Vector of P* =

$$\overrightarrow{AB} = 3i - 6j \rightarrow \overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

## Parallel and Collinear



- 2 Point  $P$  lies on line  $QR$ . Given that  $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ ,  $\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$  and  $\overrightarrow{OR} = \begin{pmatrix} h \\ 9 \end{pmatrix}$ ,

find the value of  $h$ .

Answer/ Jawapan:

[3 markah]

Negeri Sembilan 2018 P1 Q2

- 4 It is given that  $\overrightarrow{OA} = m\hat{i} + 2\hat{j}$ ,  $\overrightarrow{OB} = -\hat{i} + 5\hat{j}$  and  $\overrightarrow{OC} = p\hat{i} + 14\hat{j}$  where  $m$  and  $p$  are constants. If  $ABC$  are collinear, express  $p$  in terms of  $m$ .

Diberi bahawa  $\overrightarrow{OA} = m\hat{i} + 2\hat{j}$ ,  $\overrightarrow{OB} = -\hat{i} + 5\hat{j}$  dan  $\overrightarrow{OC} = p\hat{i} + 14\hat{j}$  dengan keadaan  $m$  dan  $p$  adalah pemalar. Jika  $ABC$  adalah segaris, ungkapkan  $p$  dalam sebutan  $m$ .

[3 marks]

Terengganu 2018 P1 Q4

- 6 It is given that  $\underline{a} = 5\hat{i} - 7\hat{j}$  and  $\underline{b} = 4\hat{i} + 3\hat{j}$ .

Find the unit vector in the direction of  $\underline{a} - \underline{b}$ .

MRSMS 2018 P1 Q6

- 14 The vectors  $\underline{a}$  and  $\underline{b}$  are non-zero vectors and non-parallel vectors. Given that  $(h-2)\underline{a} = (2k+3-h)\underline{b}$ , where  $h$  and  $k$  are constants.

Find the value of  $h$  and of  $k$ . [3 marks]

*Vektor  $\underline{a}$  dan  $\underline{b}$  adalah vektor bukan sifar dan vektor tidak selari. Diberi bahawa  $(h-2)\underline{a} = (2k+3-h)\underline{b}$ , dengan keadaan  $h$  dan  $k$  ialah pemalar.*

*Cari nilai  $h$  dan nilai  $k$ .* [3 markah]

Perlis 2018 P1 Q14

- 15 Given points  $A(-1, 6)$  and  $O$  is the origin.

Find the unit vector in the direction of  $\overrightarrow{OA}$ . [3 marks]

*Diberi titik  $A(-1, 6)$  dan  $O$  adalah asalan.*

*Cari vektor unit dalam arah vektor  $\overrightarrow{OA}$ .* [3 markah]

Perlis 2018 P1 Q15

- 5 Diagram 4 shows two vectors  $\underline{a}$  and  $\underline{b}$  on a Cartesian plane.

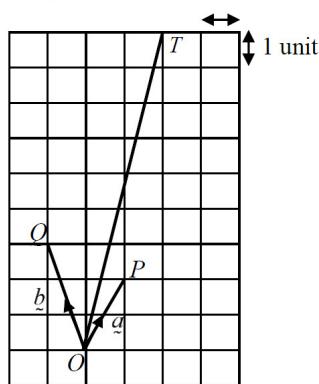


Diagram 4 / Rajah 4

Given that  $\overrightarrow{OP} = \underline{a}$  and  $\overrightarrow{OQ} = \underline{b}$ . Diberi bahawa  $\overrightarrow{OP} = \underline{a}$  dan  $\overrightarrow{OQ} = \underline{b}$ .

(a) Find

$$|\overrightarrow{OT}|$$

(b) Express the vector  $\overrightarrow{OT}$  in terms of  $\underline{a}$  and  $\underline{b}$ .

Terengganu 2018 P1 Q5

7 Diagram 7 shows triangle  $ADF$ . Points  $B$  and  $E$  are the midpoint of  $AC$  and  $DF$  respectively.

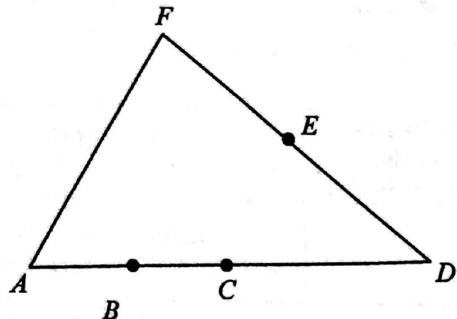


Diagram 7  
Rajah 7

It is given that  $\overrightarrow{AF} = \underline{p}$ ,  $\overrightarrow{DF} = 2\underline{q}$  and  $AB = \frac{1}{4}AD$ .

Express  $\overrightarrow{BE}$  in terms of  $\underline{p}$  and  $\underline{q}$ .

[3 marks]

MRSIM 2018 P1 Q7

1 Diagram 1 shows a trapezium  $OPQR$  and point  $T$  lies on  $PR$ .

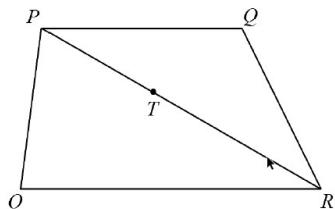


Diagram 1 / Rajah 1

Given  $\overrightarrow{OR} = 9\underline{b}$ ,  $\overrightarrow{OP} = 3\underline{a}$  and  $\overrightarrow{OR} = \frac{3}{2}\overrightarrow{PQ}$ .

(a) Express in terms of  $\underline{a}$  and  $\underline{b}$ ,

(i)  $\overrightarrow{PR}$

(ii)  $\overrightarrow{OQ}$

[3 marks] / [3 markah]

(b) Given  $\overrightarrow{PT} = k\overrightarrow{PR}$ , where  $k$  is a constant.

Find the value of  $k$  if the points  $O$ ,  $T$  and  $Q$  are collinear.

[3 marks]

Perlis 2018 P2 Q1

- 4 Diagram 4 shows a rectangle. It is given  $\vec{BC} = k\underline{a}$ ,  $\vec{AD} = h\underline{a}$  and  $\vec{AB} = h\underline{b}$ , where  $h$  and  $k$  are constant.

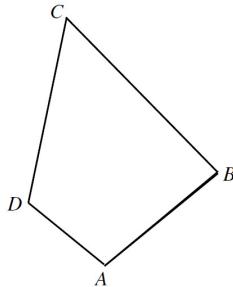


Diagram 4 / Rajah 4

If  $\vec{DC} = 4\underline{a} + \left(\frac{h+3}{2}\right)\underline{b}$ , find

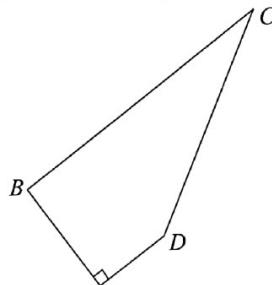
- (a) the value of  $h$  and of  $k$ , [4 marks]

- (b) Given vector  $\underline{a} = -\underline{i} + \frac{3}{4}\underline{j}$  and  $\underline{b} = 4\underline{i} + 4\underline{j}$ ,

find the unit vector in the direction of  $\vec{DC}$  [4 marks]

Terengganu 2018 P2 Q4

5. Diagram 3 shows a quadrilateral  $ABCD$ .

Diagram 3  
Rajah 3

Given  $\vec{AD} = n\underline{x}$ ,  $\vec{BC} = m\underline{x}$  and  $\vec{AB} = n\underline{y}$  where  $m$  and  $n$  are constants.

If  $\vec{DC} = 5\underline{x} + \left(\frac{6-n}{2}\right)\underline{y}$ , find

- a) the value of  $m$  and  $n$  in terms of  $\underline{x}$  and  $\underline{y}$ ,  
 b) the area of triangle  $ABC$  if the area of triangle  $ABD$  is 18 unit<sup>2</sup>.

[7 markah]

Negeri Sembilan 2018 P2 Q5



## Trigonometric Function

Basic of Trigonometry | Graph | Formulae

40

### Formulae

#### TRIGONOMETRY/ TRIGONOMETRI

1 Arc length,  $s = r\theta$

Panjang lengkok,  $s = j\theta$

2 Area of sector,  $A = \frac{1}{2}r^2\theta$

Luas sektor,  $L = \frac{1}{2}j^2\theta$

3  $\sin^2 A + \cos^2 A = 1$

4  $\sec^2 A = 1 + \tan^2 A$

5  $\operatorname{cosec}^2 A = 1 + \cot^2 A$

6  $\sin 2A = 2 \sin A \cos A$

7  $\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$

8  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

9  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

10  $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

11  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

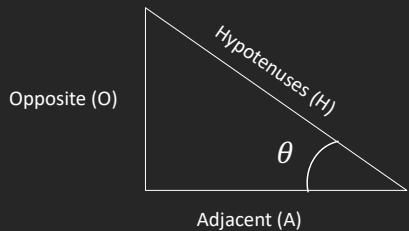
12  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

13  $a^2 = b^2 + c^2 - 2bc \cos A$

14 Area of triangle / Luas segitiga  
 $= \frac{1}{2}ab \sin C$

## Basic of trigonometry

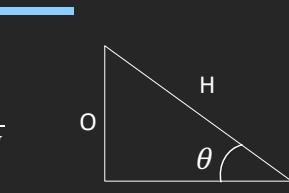
**SOH CAH TOA**



**$\tan\theta$**

**SOH**

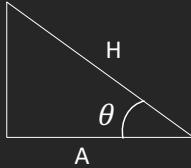
$$\sin \theta = \frac{O}{H}$$



**$cosec\theta$**

**CAH**

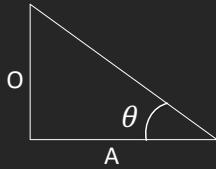
$$\cos \theta = \frac{A}{H}$$



**$sec\theta$**

**TOA**

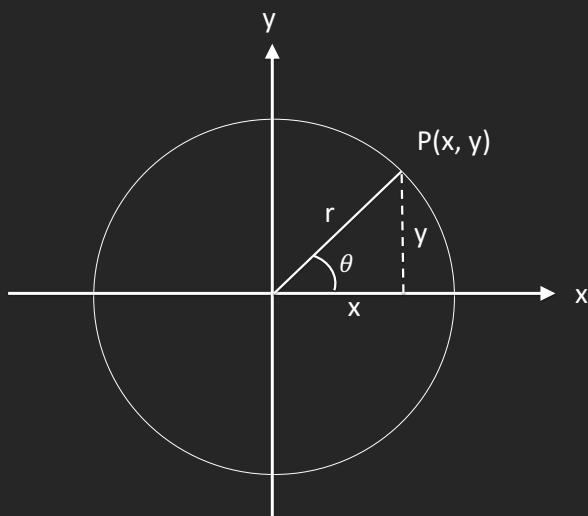
$$\tan \theta = \frac{O}{A}$$



**$cot\theta$**

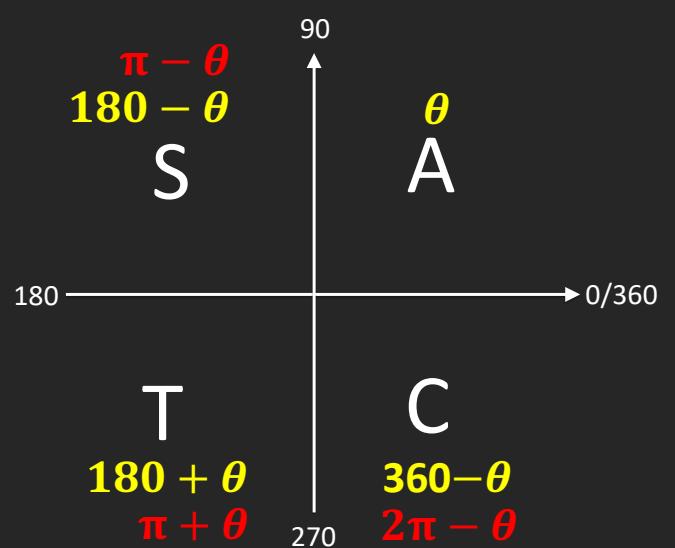
## Triangle in 4 different quadrant

**Circle with radius r**

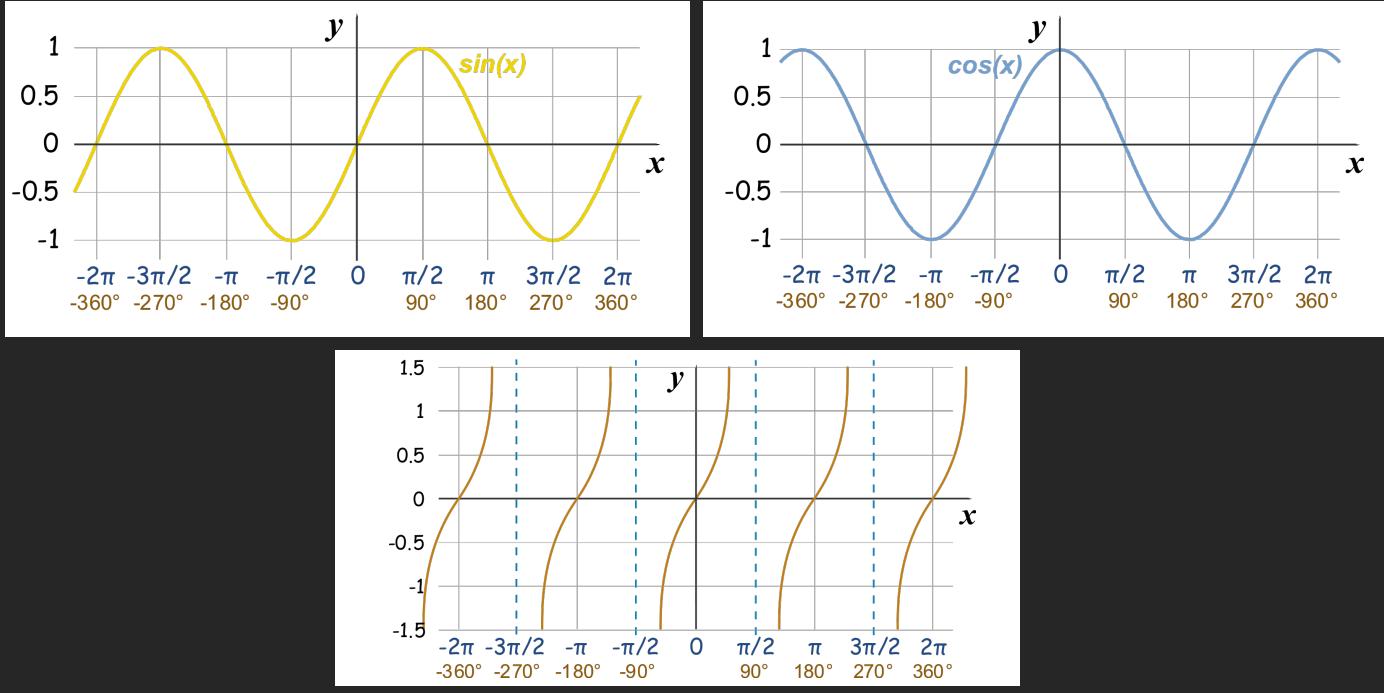


$\pi - \theta$   
180 -  $\theta$   
**S**

$180 + \theta$   
 $\pi + \theta$   
**T**



## Sine, Cosine and Tangent Graph



## General equation of sin cos tan

$$y = a \sin bx + c$$

$$y = a \cos bx + c$$

$$y = a \tan bx + c$$

$a$  = amplitude

$b$  = number of cycles in  $2\pi$

$c$  = shifting your graph up or down

## Half angle formulae for KBAT

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

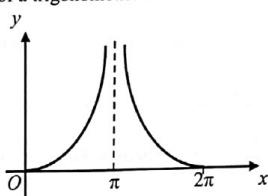
$$\cos \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$$

**8** Solve the trigonometric equation  $6 \sin x = \sec x$  for  $0^\circ \leq \theta \leq 360^\circ$ .

Negeri Sembilan 2018 P1 Q8

**7** Diagram 3 shows the graph of a trigonometric function.



State the trigonometric function.

Answer/Jawapan:

[2 markah]

Negeri Sembilan 2018 P1 Q7

**16** Solve the equation  $4\sin x \cos x + 1 = 0$  for  $0^\circ \leq x \leq 360^\circ$ .

Selesaikan persamaan  $4\sin x \cos x + 1 = 0$  untuk  $0^\circ \leq x \leq 360^\circ$ .

[3 marks]

Terengganu 2018 P1 Q16

17 Solve the equation  $\sin(2\beta + 60^\circ) = 3\sqrt{3} \cos 2\beta$  for  $0^\circ \leq \beta \leq 360^\circ$ . [4 marks]

*Selesaikan persamaan*  $\sin(2\beta + 60^\circ) = 3\sqrt{3} \cos 2\beta$  *for*  $0^\circ \leq \beta \leq 360^\circ$ . [4 markah]

Perlis 2018 P1 Q17

17 It is given that  $\sin x = k$ , where  $k$  is a constant and  $0^\circ < x < 90^\circ$ .

Express in terms of  $k$

(a)  $\cos(180^\circ - x)$ ,

(b)  $\operatorname{cosec} 2x$ .

[4 marks]

Terengganu 2018 P1 Q17

2 (a) Sketch the graph of  $y = 3\sin x - 1$  for  $0 \leq x \leq 2\pi$ . [3 marks]

(b) By using the same axes, sketch a suitable straight line to find the number of solutions for the equation  $3\pi\sin x + 2x = 3\pi$  for  $0 \leq x \leq 2\pi$ . [3 marks]

Terengganu 2018 P2 Q2

4. a) Sketch the graph of  $y = 1 - 3 \cos 2x$  for  $0 \leq x \leq \pi$ .

*Lakar graf bagi*  $y = 1 - 3 \cos 2x$  *untuk*  $0 \leq x \leq \pi$ .

b) Hence, by using the same graph, determine the value of  $k$  such that

$2 = k + 3 \cos 2x$  has only one solution for  $0 \leq x \leq \pi$ .

*Seterusnya, dengan menggunakan graf yang sama, tentukan nilai k supaya*

$2 = k + 3 \cos 2x$  *mempunyai satu penyelesaian sahaja untuk*  $0 \leq x \leq \pi$ .

[6 marks]

Negeri Sembilan 2018 P2 Q4

5 (a) Show  $\cos^2 \frac{3}{4}x - \sin^2 \frac{3}{4}x = \cos \frac{3}{2}x$ . [2 marks]

(b) (i) Sketch the graph of  $y = 2 \left| \cos \frac{3}{2}x \right|$  for  $0 \leq x \leq 2\pi$ . [3 marks]

(ii) Hence using the same axes, sketch a suitable straight line to find the number of solution for the equation  $10 \left| \cos^2 \frac{3}{4}x - \sin^2 \frac{3}{4}x \right| + \frac{3}{\pi}x = 5$  for  $0 \leq x \leq 2\pi$ .

State the number of solutions. [3 marks]

Perlis 2018 P2 Q5



## Probability Distribution

Binomial Distributions | Normal Distributions

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## Binomial Distribution

The formulae

$$P(X = r) = {}^n C_r p^r q^{n-r}$$

***n = number of trials***

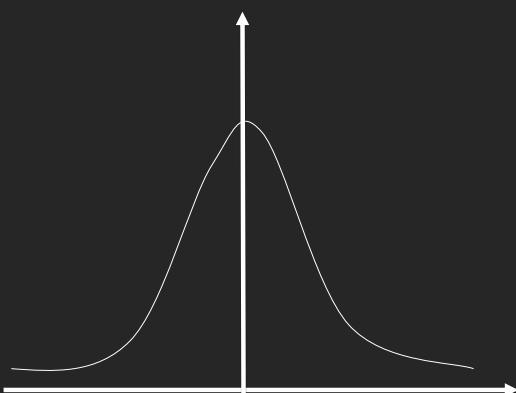
***p = probability of success***

***q = probability of failure***

***r =?***

## Relationship between Z-Table and bell-shape diagram

z	0	1	2	3	4	5	6	7	8	9	Minus / Tolak									
											10	11	12	13	14	15	16	17	18	19
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641	4	8	12	16	20	24	28	32	36	
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4403	0.4364	0.4325	0.4286	0.4247	4	8	12	16	20	24	28	32	36	
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859	4	8	12	15	19	23	27	31	35	
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483	4	7	11	15	19	22	26	30	34	
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121	4	7	11	15	18	22	25	29	32	
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776	3	7	10	14	17	20	24	27	31	
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451	3	7	10	13	16	19	23	26	29	
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148	3	6	9	12	15	18	21	24	27	
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867	3	5	8	11	14	16	19	22	25	
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611	3	5	8	10	13	15	18	20	23	
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379	2	5	7	9	12	14	16	19	21	
1.1	0.1357	0.1334	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170	2	4	6	8	10	12	14	16	18	
1.2	0.1151	0.1133	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0986	2	4	6	7	9	11	13	15	17	
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823	2	3	5	6	8	10	11	13	14	
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681	1	3	4	6	7	8	10	11	13	
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559	1	2	4	5	6	7	8	10	11	
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455	1	2	3	4	5	6	7	8	9	
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0384	0.0375	0.0367	1	2	3	4	4	5	6	7	8	
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	1	1	2	3	4	5	6	6	6	
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	1	1	2	2	3	4	4	5	5	
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	0	1	1	2	2	3	3	4	4	
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	0	1	1	2	2	3	3	4		
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	0	1	1	1	2	2	2	3		
2.3	0.0107	0.0104	0.0102		0.00990	0.00964	0.00939	0.00914		0.00889	0.00866	0.00842	2	5	7	9	12	14	16	19
													3	5	8	10	13	15	18	20
2.4	0.00820	0.00798	0.00776	0.00755	0.00734						2	4	6	8	11	13	15	17	19	
2.5	0.00621	0.00600	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480	2	3	5	6	8	9	11	12	14	
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357	1	2	3	5	6	7	9	10		
2.7	0.00347	0.00333	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264	1	2	3	4	5	6	7	8		
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193	1	1	2	3	4	4	5	6		
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00149	0.00144	0.00139	0	1	1	2	2	3	3	4	4		
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100	0	1	1	2	2	3	3	4		



## Examples

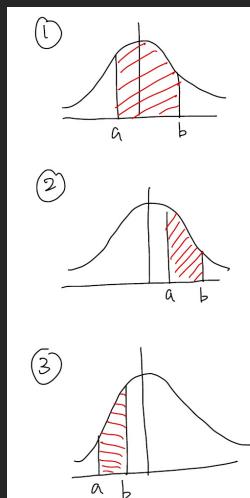
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- 1  $P(Z \geq 0.3)$
- 2  $P(Z > -0.3)$
- 3  $P(Z < 0.3)$
- 4  $P(Z \leq -0.3)$
- 5  $P(-0.3 \leq Z \leq 0.3)$
- 6  $P(2 \leq Z \leq 1)$
- 7  $P(-2 \leq Z \leq -1)$

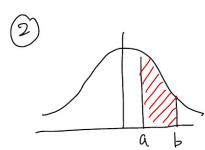
- 8  $P(Z \geq m) = 0.3464$
- 9  $P(Z \geq m) = 0.6704$
- i  $P(Z \leq m) = 0.0532$
- j  $P(Z < m) = 0.5383$

## Basic Rules for bell-shape diagram

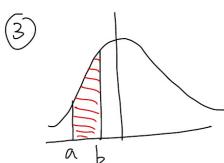
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$$1 - P(z < a) - P(z > b)$$

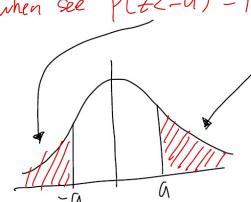


$$P(z > a) - P(z > b)$$



$$P(z < a) - P(z < b)$$

\*Remember, when see  $P(z < a) = P(z > a)$



$$Z = \frac{X - \mu}{\sigma} ?$$

**10** The random variable  $X$  has a normal distribution with a mean of 4.8 and a standard deviation of  $\sigma$ . Given  $X = 6.6$  and  $Z = 1.2$ , find the value of

- (a)  $\sigma$
- (b)  $k$  if  $P(-k < Z < k) = \frac{1}{2}\sigma$ .

[4 markah]

MRSM 2018 P1 Q10

**24** 8 % of the screws produced by factory A is defective. If 10 screws are randomly chosen, find the probability that

- (a) 2 screws are defective.
- (b) none of the screws is defective.

[3 marks]

Terengganu 2018 P1 Q24

**25** The ages of 1 000 residents in a village are normally distributed with a mean of 70 years and a variance,  $\sigma^2$ . Given that the probability of selecting a resident who is more than 60 years old is 0.7499, find the value of standard deviation.

Answer/ Jawapan:

[3 markah]

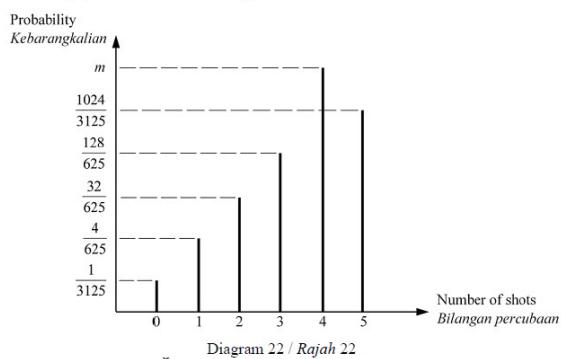
Negeri Sembilan 2018 P1 Q25

**25** Given  $Z \sim N(0,1)$  and  $P(z_1 < Z < 1.4) = 0.1612$ . Find  $P(0 < Z < z_1)$ .

[3 marks]

Terengganu 2018 P1 Q25

**22** Adam is one of the archers who represents his school in Secondary School's Archery Tournament. During the practice, Adam has done 5 trial shots. The probability that Adam strikes the target is  $p$ , where  $p$  is a constant. The coach then produces a sketch of the graph of probability distribution for the number of shots that strikes the target obtained by Adam as shown in Diagram 22.



- (a) Find the values of  $p$  and of  $m$ .  
*Cari nilai p dan nilai m.*
- (b) Calculate the standard deviation of the number of shots that strikes the target obtained by Adam.

Perlis 2018 P1 Q22

- 7 (a) The probability of a student walks to school is  $p$ . A sample of 6 students is selected at random.
- If the probability of all the students walk to school is 0.046656, find the value of  $p$ .
  - Find the probability that more than 4 students walk to school.

[5 marks]

- (b) Diagram 7 shows a standard normal distribution graph representing the volume of tomato sauce in bottles produced by a factory.

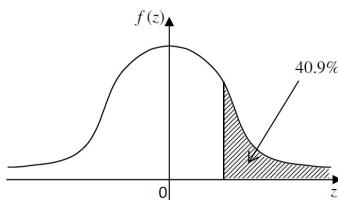


Diagram 7 / Rajah 7

It is given the mean is  $900 \text{ m}^3$  and variance is  $289 \text{ m}^6$ . If the percentage of the volume more than  $v$  is 40.9%, find

- the value of  $v$ ,
- the probability that the volume between  $866 \text{ m}^3$  and  $951 \text{ m}^3$

[5 marks]

Terengganu 2018 P2 Q7

- 7 (a) In a school, 250 students sat for an examination. The marks obtained is normally distributed with a mean,  $\mu$  and a standard deviation of 5.

*Cari*

- the value of  $\mu$  if the probability of a student chosen at random get marks less than 58 is 0.3085,
- the number of students who achieved marks more than 65.

[6 markah]

- (b) The probability of a consumer shops online is  $p$ . A sample of 6 consumers are chosen at random from a particular area.

*Calculate*

- the value of  $p$  if the probability that none of the consumers shop online is  $\frac{1}{729}$ ,
- the probability that less than 2 consumers shop online.

[4 markah]

MRSM 2018 P2 Q7

8 (a) The result of a study shows that in certain school, 65% of the students pass Additional Mathematics test.

- (i) If 10 students from the school are chosen at random, calculate the probability that at least 8 of the students pass Additional Mathematics test.

[3 marks] / [3 markah]

- (ii) If the total number of students in the school is 960 students, find the variance number of students who fail Additional Mathematics test.

[2 marks] / [2 markah]

(b) The height of students in Form 4 Arif is normally distributed with mean 150 cm and variance  $225 \text{ cm}^2$ .

- (i) If a student is chosen at random from the class, find the probability that the height of the students is less than 140 cm,

[2 marks] / [2 markah]

- (ii) Find the number of students which is chosen randomly falls in the range of 160 cm and 180 cm, if the total number of students is 27 students.

[3 marks] / [3 markah]

**Sila lengkapkan borang penilaian bagi Seminar SPM yang telah anda hadiri. Penilaian anda dapat membantu kami memahami tahap keberkesanan program ini dan seterusnya membolehkan kami meningkatkan kualiti perkhidmatan kami di masa hadapan.**

**Terima kasih!**

***Please fill up this form for the session that you are attending. Your evaluation will help us improve our service and help us understand the effectiveness of this program.***

***Thank you!***

1. Nombor Telefon

*Phone Number*

2. Apakah subjek bagi seminar yang sedang anda sertai sekarang?

*What is the seminar's subject that you're attending now?*

- |                                       |  |
|---------------------------------------|--|
| <input type="radio"/> Bahasa Malaysia | <input type="radio"/> Kimia              |
| <input type="radio"/> English         | <input type="radio"/> Chemistry          |
| <input type="radio"/> Sejarah         | <input type="radio"/> Fizik              |
| <input type="radio"/> Sains           | <input type="radio"/> Physics            |
| <input type="radio"/> Science         | <input type="radio"/> Matematik Tambahan |
| <input type="radio"/> Matematik       | <input type="radio"/> Additional Maths   |
| <input type="radio"/> Mathematics     | <input type="radio"/> Perniagaan         |
| <input type="radio"/> Biologi         | <input type="radio"/> Prinsip Perakaunan |
| <input type="radio"/> Biology         | <input type="radio"/> Ekonomi            |

3. Pernahkah anda menonton mana-mana video BACfreeschool (sebelum ini dikenali sebagai EduNation)?

*Have you ever watched any BACFreeschool's (previously known as EduNation) videos?*

- |                                   |
|-----------------------------------|
| <input type="radio"/> Ya<br>Yes   |
| <input type="radio"/> Tidak<br>No |

4. Nilai kefahaman guru terhadap isi kandungan yang diajar bagi subjek ini.

*Rate the teacher's understanding of this particular subject.*

Sangat Rendah

*Very Low*

Rendah

*Low*

Sederhana

*Intermediate*

Tinggi

*High*

Sangat Tinggi

*Very High*

5. Nilai cara penyampaian guru bagi subjek ini.

*Rate the teacher's delivery of the subject.*

Sangat Tidak Menarik

*Very Uninteresting*

Tidak Menarik

*Not Interesting*

Sederhana

*Intermediate*

Menarik

*Interesting*

Sangat Menarik

*Very Interesting*

6. Nilai tahap kepuasan terhadap nota tambahan yang telah diberikan.

*Rate your satisfaction level with the notes given.*

Sangat

Tidak Berpuashati

*Very Unsatisfied*

Tidak Berpuashati

*Not Satisfied*

Sederhana

*Intermediate*

Berpuashati

*Satisfied*

Sangat Berpuashati

*Very Satisfied*

7. Nilai tahap kebergunaan isi kandungan seminar.

*Rate the usefulness of the seminar's content to your SPM preparation.*

Sangat Tidak Berguna

*Not Very Useful*

Tidak Berguna

*Not Useful*

Sederhana

*Intermediate*

Useful

*Berguna*

Sangat Useful

*Very Useful*

8. Bagi pendapat anda, 3 jam untuk satu sesi seminar adalah...

*In your opinion, 3 hours per session is...*

terlalu pendek.  
*too short.*

bersesuaian.  
*just right.*

terlalu panjang.  
*too long.*

9. Adakah anda mempunyai sebarang maklum balas/komen bagi meningkatkan prestasi kami?

*Do you have any additional comments, questions, or concerns you would like to share?*