Résumé : intégrales

Définition

$(F'(x) = f(x))$

Si f est continue sur $[a,b]$ et F est sa primitive, alors :

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

exemples :

- $$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \ldots$$
- $$\int_1^e \frac{1}{x}\,dx = \Big[\ln x\Big]_1^e = \ln(e) - \ln(1) = 1 - 0 = 1$$

Tableau des primitives

$$\int a = \Big[ax\Big] \qquad a \in \mathbb{R}$$

$$\int x = \left[\frac{x^2}{2}\right]$$

$$\int x^n = \left[\frac{1}{n+1}x^{n+1}\right]$$

$$\int \frac{1}{x}\,dx = \Big[\ln x\Big]$$

$$\int \frac{U'(x)}{U(x)} = \Big[\ln(|U(x)|)\Big]$$

$$\int \sqrt[n]{x^m} = \int x^{m/n} = \left[\frac{1}{\frac{m}{n}+1}x^{\frac{m}{n}+1}\right]$$

$n \in \mathbb{R} \setminus \{0,1\}$ $\quad \int \frac{1}{x^n} = \int x^{-n} = \left[\frac{1}{-n+1} x^{-n+1}\right]$

$$\int U'(x) \cdot U^n(x) = \left[\frac{1}{n+1} U^{n+1}(x)\right]$$

$n \in \mathbb{Z} \setminus \{0,1\}$ $\quad \int \frac{U'(x)}{U^n(x)} = \int U'(x)\, U^{-n}(x) = \left[\frac{1}{-n+1} U^{-n+1}(x)\right]$

$$\int e^x = \left[e^x\right]$$

$$\int U'(x)\, e^{U(x)} = \left[e^{U(x)}\right]$$

$$\int \sin x = \left[-\cos x\right]$$

$$\int \cos x = \left[\sin x\right]$$

$$\int U'(x) \cos(U(x)) = \left[\sin(U(x))\right]$$

$$\int U'(x) \sin(U(x)) = \left[-\cos(U(x))\right]$$

$$\int 1 + \tan^2 x = \left[\tan x\right]$$

$$\int \frac{1}{\cos^2 x} = \left[\tan x\right]$$

$$\int U'(x) \cdot \frac{1}{\cos^2(U(x))} = \left[\tan(U(x))\right]$$

$$\int U'(x)\left(1 + \tan^2(U(x))\right) = \left[\tan(U(x))\right]$$

Intégration par parties :

$$\int_a^b f'g\,dx = [fg]_a^b - \int_a^b fg'\,dx$$

ALPES

exemples :

1) $\int_1^e x \ln x\,dx = \int_1^e \left(\frac{1}{2}x^2\right)' \ln x\,dx$

$= \left[\frac{1}{2}x^2 \ln x\right]_1^e - \int_1^e \frac{1}{2}x^2 \cdot (\ln x)'\,dx$

$= \frac{1}{2}e^2 \ln(e) - 0 - \int_1^e \frac{1}{2}x^2 \times \frac{1}{x}\,dx$

$= \frac{1}{2}e^2 - \frac{1}{2}\int_1^e x\,dx$

$= \frac{1}{2}e^2 - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e$

$= \frac{1}{2}e^2 - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1}{2}\right)$

$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4}$

$= \frac{1}{4}e^2 + \frac{1}{4}$

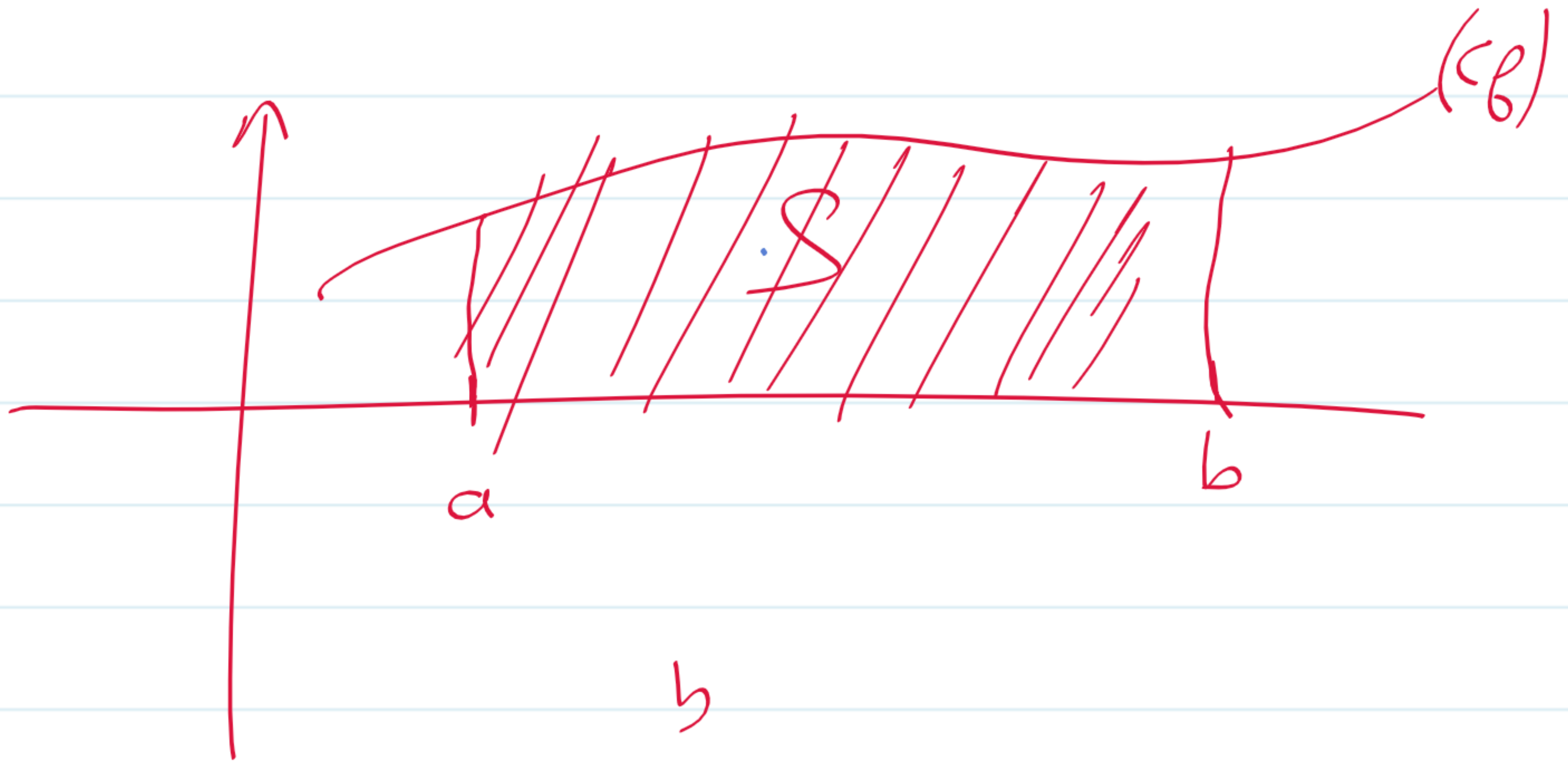
$\int_1^e x^2 \ln x\,dx = \int_1^e \left(\frac{1}{3}x^3\right)' \ln x\,dx$

$= \left[\frac{1}{3}x^3 \ln x\right]_1^e - \int_1^e \frac{1}{3}x^3 \times \frac{1}{x}\,dx$

$=$ calc .....

Calcul des surfaces :

1 Cas :

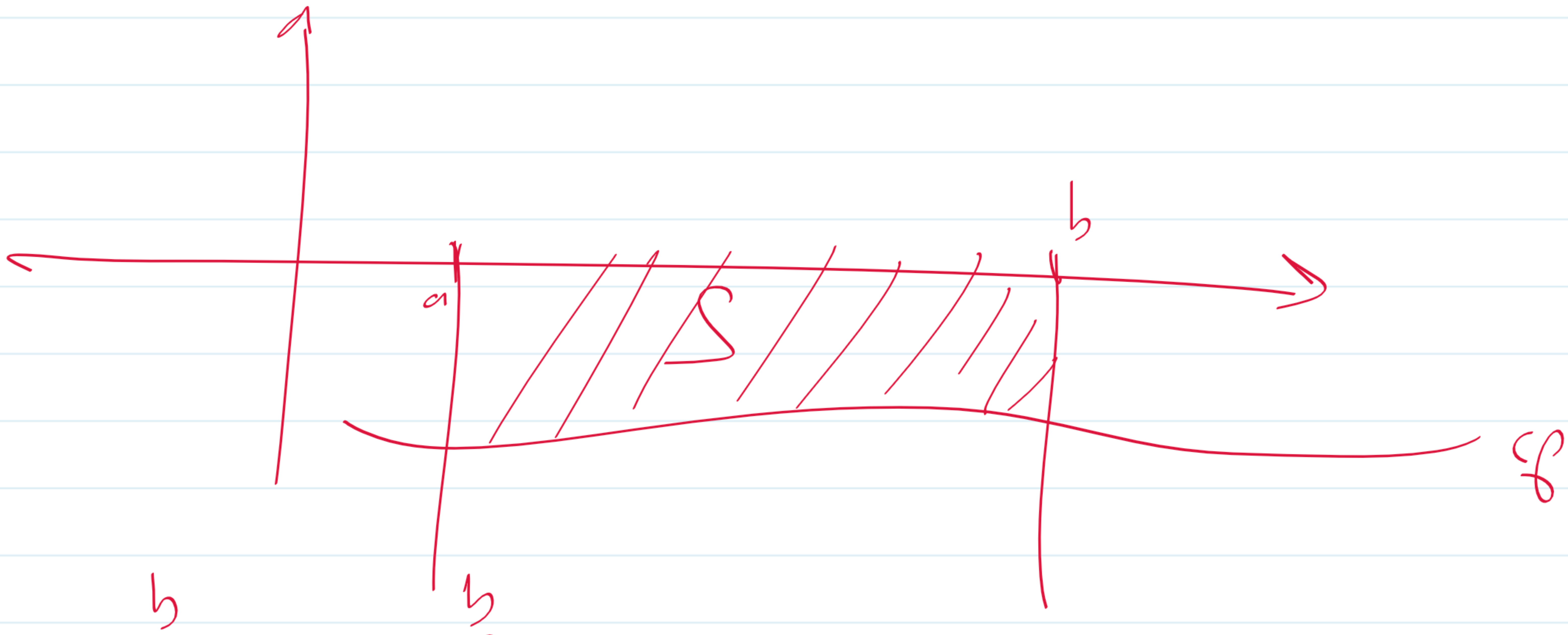


$\|\vec{i}\| = \|\vec{j}\| = \sim cm$

$$S = \int_a^b |f(x)|\,dx \cdot U \qquad U = \|\vec{i}\| \cdot \|\vec{j}\|$$

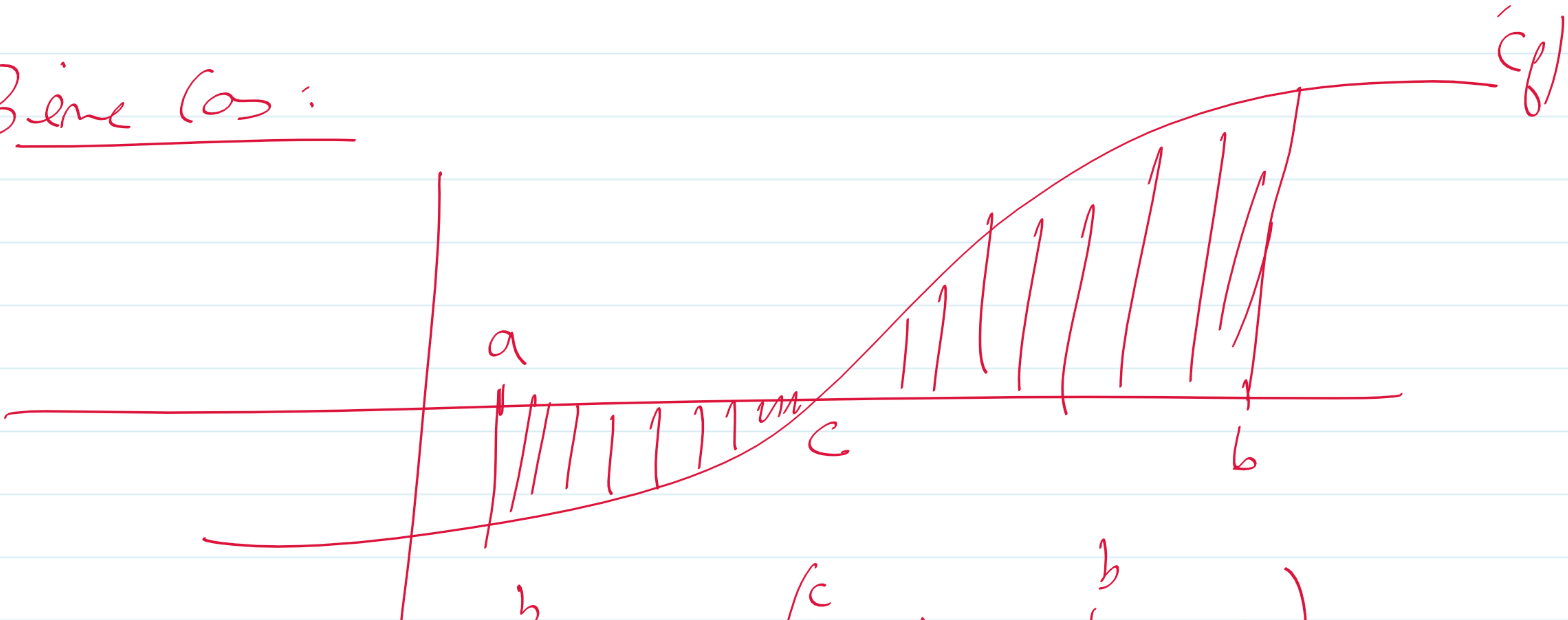
$$= \int_a^b f(x)\,dx \cdot U$$

2ème Cas :



$$S = \int_a^b |f(x)|\,dx = \int_a^b (-f(x))\,dx \cdot U$$

3ème Cas :

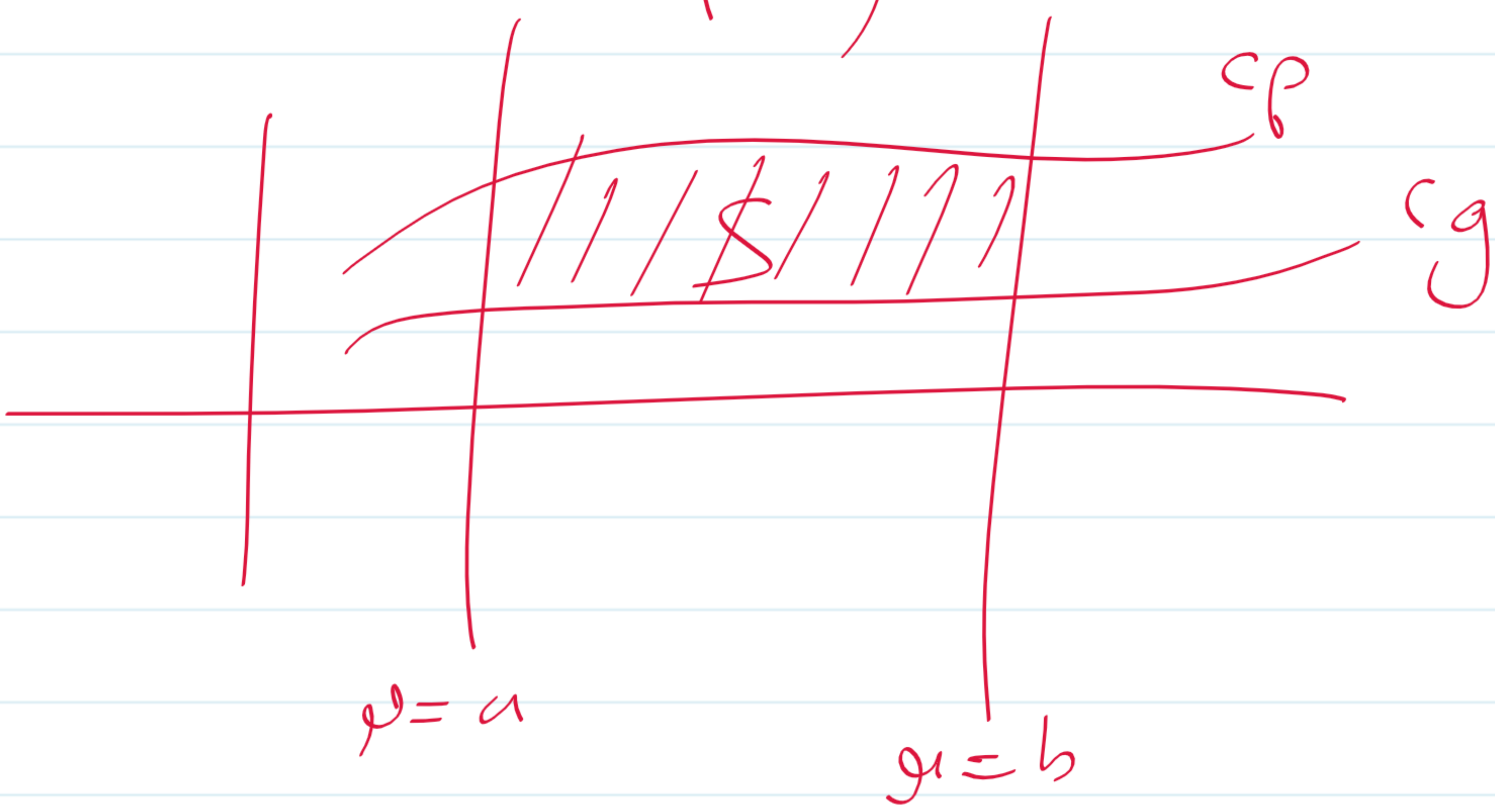


$$S = \int_a^b |f(x)|\,dx = \left(\int_a^c (-f(x))\,dx + \int_c^b f(x)\,dx \right) \cdot U$$

Dans tous les cas :

$$S = \int_a^b |f(x)|\,dx \cdot U \quad \text{où} \quad U = \|\vec{i}\| \cdot \|\vec{j}\|.$$

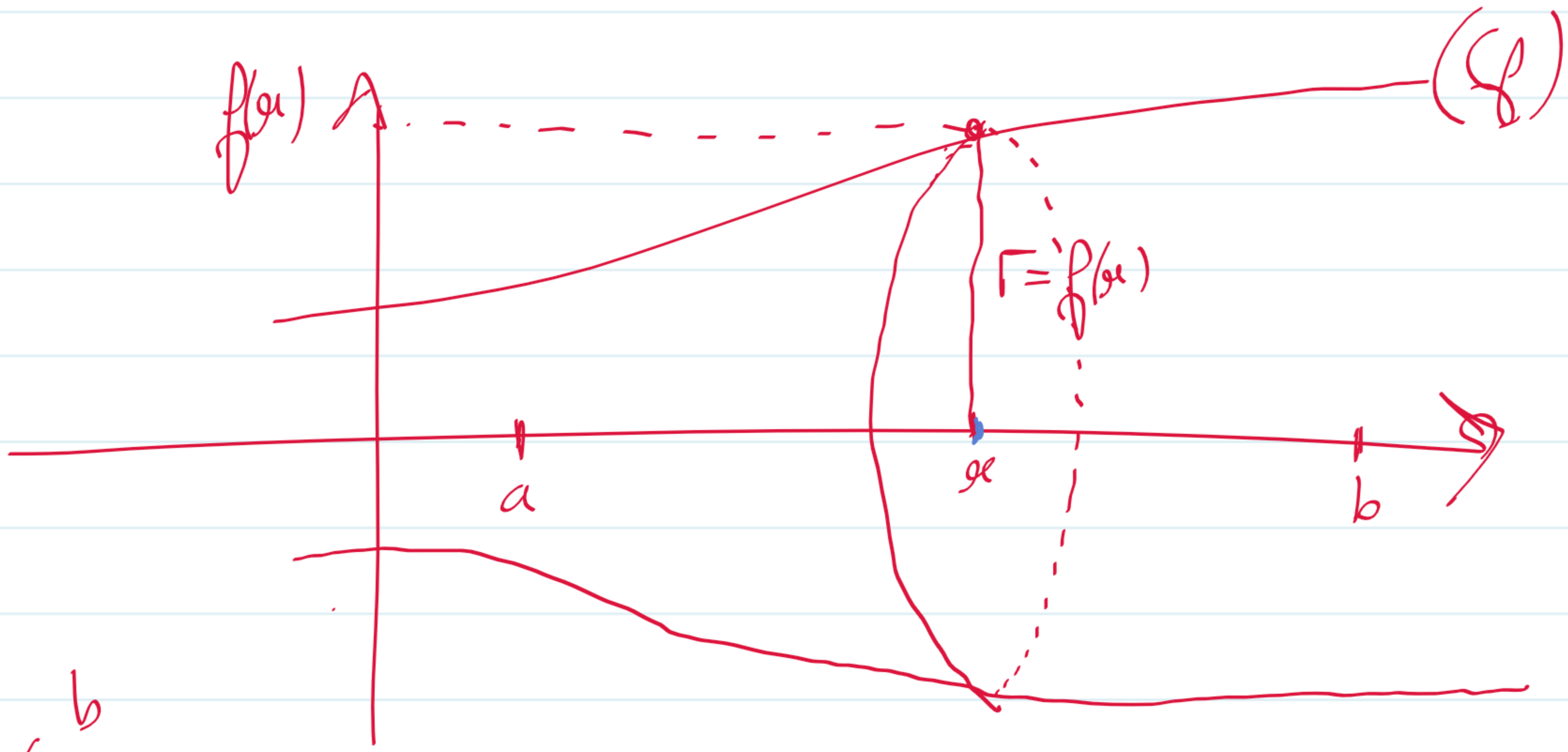
- Surface entre $(\mathcal{C}_f)$ et $(\mathcal{C}_g)$ et $x = a$ et $x = b$.



$$S = \int_a^b |f(x) - g(x)|\,dx.$$

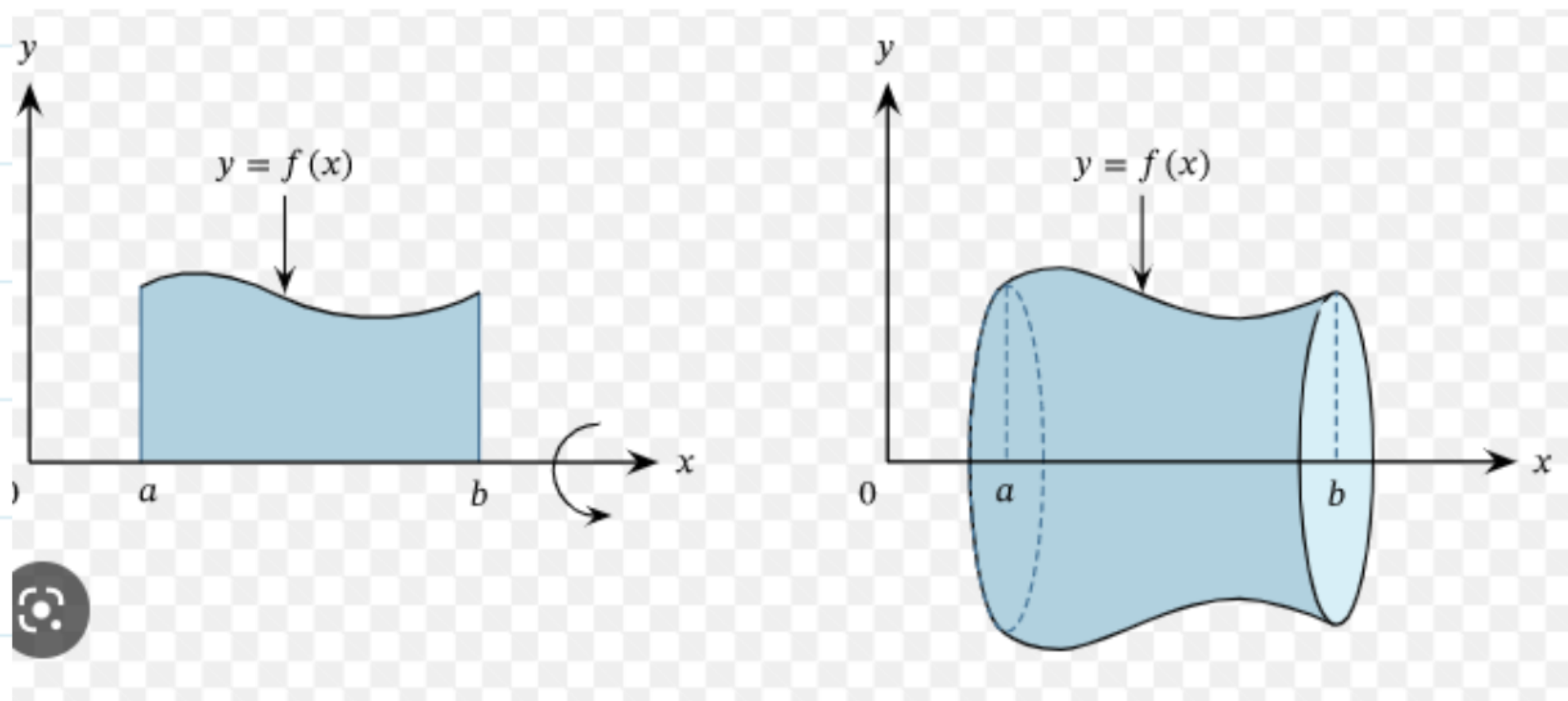
Calcul des volumes

• Volume généré par une rotation autour de l'axe des abscisses.

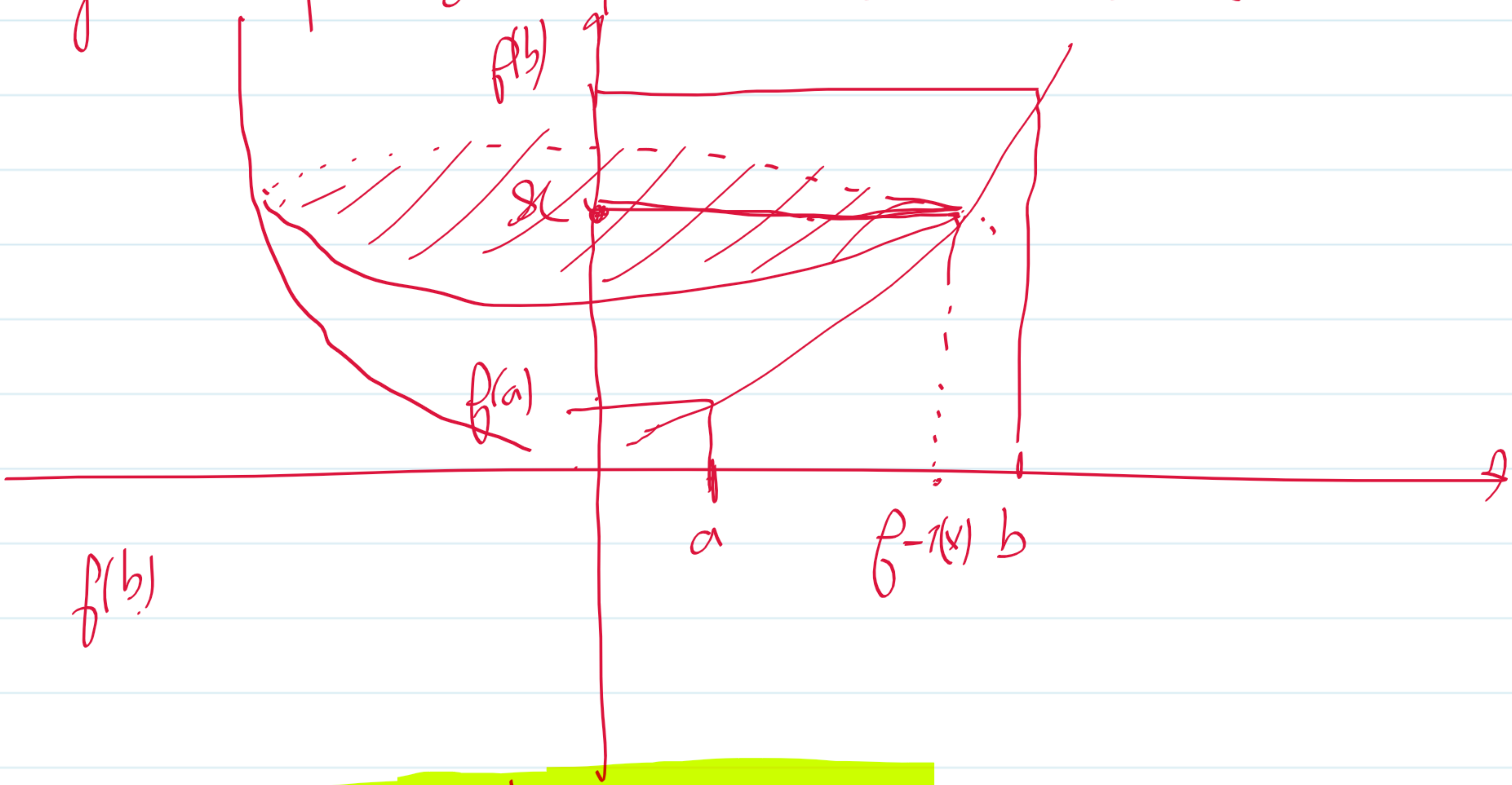


$$V = \left(\int_a^b \pi \left(f(x)\right)^2 dx \right) . U$$

On $U = \|\vec{i}\| . \|\vec{j}\| . \|\vec{k}\|$



• Volume généré par une rotation autour de l'axe des ordonnées



$$V = \int_{f(a)}^{f(b)} \pi \left(f^{-1}(x)\right)^2 dx \quad U$$