

## NEW JERSEY CENTER FOR TEACHING \& LEARNING

## PM1 ${ }^{\circ}$

## Progressive Mathematics Initiative ${ }^{\circ}$

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# NEW JERSEY CENTER FOR TEACHING \& LEARNING 

## Algebra I

# Graphing Linear Equations 

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## Table of Contents

- Linear Equations
- Graphing Linear Equations Using Intercepts
- Horizontal and Vertical Lines
- Slope of a Line
- Point-Slope Form
- Slope-Intercept Form
- Working with Linear Equations
- Scatter Plots and the Line of Best Fit
- PARCC Sample Questions
- Glossary and Standards


## Linear Equations



Return to Table of Contents

## Linear Equations

Any equation must have at least one variable.
Linear equations have either one or two variables and may also have a constant.

The variables in a linear equation are not raised to any power (beyond one); they are not squared, cubed, etc.

The standard form of a linear equation is

$$
A x+B y=C
$$

Where:

- $x$ and $y$ are variables
- $A$ and $B$ are coefficients and $C$ is a constant
- $A, B$ and $C$ are integers
- $A \geq 0$


## Linear Equations

There are an infinite number of solutions to a linear equation.

In general, each solution is an ordered pair of numbers representing the values for the variables that make the equation true.

For each value of one variable, the value of the other variable is determined.

## Linear Equations

The fact that the solutions of linear equations are part of an infinite set of ordered pairs led to the idea that those pairs could be treated as points on a graph, and that those points would then form a line.

This reasoning is why these types of equations are called "Linear Equations."

The idea of merging algebra and geometry led to analytic geometry in the mid 1600's.

## Graphing Equations

Analytic Geometry

- is a powerful combination of algebra and geometry.
- was independently developed and published in France during 1637 by René Descartes and Pierre de Fermat.
- The Cartesian Plane is named for Descartes.

```
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        Ia veritéd dansksformees.
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        LAGEOMETRIE,
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        *****)
    Defluptimeriedelan Maine.
        <13 1) < XXXVAL
        Smac Primigge
```


## Graphing Equations

How would you describe to someone the location of these five points so they could draw them on another piece of paper without seeing your drawing?

Discuss.

## Graphing Equations

Adding this Cartesian coordinate plane makes the previous task simple since the location of each point can be given by just two numbers:
an $x$ - and $y$-coordinate, written as an ordered pair $(x, y)$.


## Graphing Equations

With the Cartesian Plane providing a graphical description of locations on the plane, solutions of equations (noted in ordered pairs) can be analyzed using algebra.


## Graphing Equations

The Cartesian Plane is formed by the intersection of the $x$-axis and $y$-axis, which are perpendicular. It's also called a Coordinate Plane or an $X Y$ Plane.

The $x$-axis is horizontal (side-to-side) and the $y$-axis is a vertical (up and down).

The axes intersect at the origin, $(0,0)$.


## Graphing Equations

An ordered pair represents a solution to a linear equation, and a point on the plane.

The numbers represent the $x$-and $y$-coordinates, $(x, y)$.

The point $(4,8)$ is shown.


## Graphing Equations

A linear equation has an infinite set of solutions.

Graphing the pairs of $x$ and $y$ values which satisfy a linear equation forms a line (hence the name "linear" equation).

|  |  |  | v |  |  |  |  |  | , |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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|  |  |  | $\bigcirc$ |  |  |  |  |  |  | - | $x$ |
| $-10$ |  | -5 | 0 |  |  | 5 |  |  | 10 | 0 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 5 |  |  |  |  |  |  |  |  |
|  |  |  | -5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | - |  |  |  |  |  |  |  |  |
|  |  |  | $-10$ |  |  |  |  |  |  |  |  |
|  |  |  | $-10$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

## Graphing Equations

One way to graph the line that represents the solutions to a linear equation is to use a table to find a few sets of solutions.

Since a line is uniquely defined by any two points, finding three or more points provides the line and a check to make sure the points are correct.

| $\cdots$ |  |  | $y$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | - |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| -10 | -5 |  | 0 |  | 5 |  | 10 |
|  |  |  |  |  |  |  |  |
|  |  |  | 5 |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  | $0 \cdot$ |  |  |  |  |
|  |  |  |  |  |  |  |  |

## Graphing Equations

Let's graph the line

$$
y=2 x+3
$$

We'll make a table, pick some $x$-values and then calculate the matching $y$-values to create ordered pairs to graph.

We can pick any values for $x$, but we will choose them so that the resulting points: - are easy to plot.

- are far enough apart to allow us to draw an accurate line.


## Graphing Equations

$$
y=2 x+3
$$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 0 |  |
| -3 |  |

While we only need two points to determine the line, it's good to check with some extra points.

Use the equation to fill in the $y$-values in the table.


## Graphing Equations

$$
y=2 x+3
$$

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |
| 0 | 3 |
| -3 | -3 |

These are just a few points on the line.

There are an infinite number of ordered pairs that satisfy the equation.


Let's draw the line that represents the infinite set of solutions to this equation.

## Graphing Equations

$$
y=2 x+3
$$

The arrows on both ends of the line indicate that it continues forever in both directions.

Because it is a line, it includes an infinite number of points representing all the real numbers.


## Graphing Equations

| $y=-\frac{1}{3} x+9$ |
| :---: |
| $\boldsymbol{x}$ |$] \boldsymbol{y} \quad |$| -3 | 10 |
| :---: | :---: |
| -1 | $9.3333 \ldots$ |
| 0 | 9 |
| +2 | $8.3333 \ldots$ |
| +6 | 7 |



Note: click to reveal
Click on the points that are integers and the line to graph

1 Given the equation, $y=2 x-5$, what is $y$ when $x=0$ ?
○ $A$
○ -5
○ -3
OD 0
OE I need help

2 Given an equation of $y=2 x-5$, what is $y$ if $x=\frac{1}{2}$ ?
OA -5
○ $\quad-4$
○C -1
OD 7
OE I need help


3 Is $(3,-5)$ on the line $y=2 x-12$ ?
OA Yes
OB No
OC Not enough information
OD I need help

https://njctl.org/video/?v=AFdcmY86gVU

## 4 Which point is on the line $4 y-2 x=0$ ?

OA $(-2,1)$
$O_{B}(0,1)$
OC $(-2,-1)$
$O_{D}(1,2)$
OE I need help

https://njctl.org/video/?v=_prlhQTNF94

5 Which point lies on the line whose equation is $2 x-3 y=9$ ?
○A

$$
(0,3)
$$

B $(-3,1)$
$O_{C}(-3,0)$
$O_{D}(6,1)$
OE I need help

https://njctl.org/video/?v=cHDzJjEErqM

## 6 Point $(k,-3)$ lies on the line $x-2 y=-2$.

What is the value of $k$ ?
○ A -8
○ -6
OC 6
OD 8
OE I need help

https://njctl.org/video/?v=asRYiD2rVcl

# Graphing Linear Equations Using Intercepts 

Return to Table of Contents

## $x$ - and $y$ - intercepts

To graph a line, two points are required. One technique uses the $x$ - and $y$-intercepts.


## $x$ - and $y$ - intercepts

To graph a line, two points are required. One technique uses the $x$ - and $y$-intercepts.

The $y$-intercept is where a graph of an equation passes through the $y$-axis. The coordinates of the $y$-intercept are $(0, b)$, where $b$ is any real number.

The $y$-intercept of the linear equation shown is $(0,-2)$.


## 7 What is the $y$-intercept of this line?




8 What is the $x$-intercept of this line?


9 What is the $y$-intercept of this line?


ว A (0,3)
ว B $(3,0)$
Ј $\subset(-5,0)$
ว D ( $0,-5$ )
ว E I need help

10 What is the $y$-intercept of this line?


$$
\begin{aligned}
& \partial A(-1,0) \\
& \partial B(1,0) \\
& \partial C(0,1) \\
& \partial D(0,-1) \\
& \partial E \text { I need help }
\end{aligned}
$$

11 What is the $x$-intercept of this line?


JA (0,2)
ว B $(0,4)$
ว C ( 2,0 )
ว D (4, 0)
ว E I need help

12 What is the $x$-intercept of this line?


https://njctl.org/video/?v=svooHUHvotQ

## Graphing Linear Equations Using Intercepts

The technique of using intercepts works well when an equation is written in Standard Form.

Recall that a linear equation written in standard form is $A x+B y=C$, where $A, B$, and $C$ are integers and $A \geq 0$.
https://njctl.org/video/?v=FYOIWItpNqw

## Graphing Linear Equations Using Intercepts

Example: Find the $x$ - and $y$-intercepts in the equation $3 x+5 y=15$. Then graph the equation.
$x$-intercept: Let $y=0: 3 x+5(0)=15$

$$
\begin{array}{r}
3 x+0=15 \\
3 x=15
\end{array}
$$

$$
x=5 \quad \text { so the } x \text {-intercept is }(5,0) .
$$

$y$-intercept: Let $x=0$ :

$$
\begin{aligned}
3(0)+5 y & =15 \\
0+5 y & =15 \\
5 y & =15 \\
y & =3
\end{aligned}
$$

$$
\text { so the } y \text {-intercept is }(0,3)
$$

## Graphing Linear Equations Using Intercepts

Example: Find the $x$ - and $y$ intercepts in the equation $3 x+5 y=15$.
Then graph the equation.
$x$-intercept is Click
$y$-intercept is Click

Click on the points \& the line in the coordinate plane to reveal.


## Graphing Linear Equations Using Intercepts

Example: Find the $x$ - and $y$-intercepts in the equation $4 x-3 y=12$. Then graph the equation.
$x$-intercept: Let $y=\mathbf{o}$ :
$y$-intercept: Let $x=0$ :


## Graphing Linear Equations Using Intercepts

Example: Find the $x$-and $y$-intercepts in the equation $4 x-3 y=12$.
Then graph the equation.
$x$-intercept is

$y$-intercept is


Click on the points \& the line in the coordinate plane to reveal.


## Graphing Linear Equations Using Intercepts

Does anyone see a shortcut to finding the $x$ - and $y$ intercepts?
How could your shortcut make the problem easier?

## Graphing Linear Equations Using Intercepts

Given the equation $4 x-3 y=12$, another way to look at the intercept method is called the "cover-up method."

If $y=0$, we can cover $-3 y$ up (because zero times anything is 0 ) and solve the remaining equation.

$$
4 x-3 y=12
$$

press $-3 y$
that leaves us with

$$
\overline{\text { click }}
$$

Then solve for $x$.
the $x$-intercept is

## Graphing Linear Equations Using Intercepts

If $x=0$, we can cover that up and solve the remaining equation.
press $4 x \longrightarrow 4 x-3 y=12$
that leaves us with

$$
\overline{\text { Click }}
$$

Then solve for $y$.
the $y$-intercept is


## Graphing Linear Equations Using Intercepts

Try This:
Find the $x$ - and $y$ intercepts of $y=3 x-9$. Then graph the equation.

Click on the points \& the line in the coordinate plane to reveal.


Answer

13 Given the equation $y=\frac{1}{2} x-7$, what is the $x$-intercept?
OA (3.5, 0)
OB $(14,0)$
OC $(0,3.5)$
OD $(0,14)$
OE I need help

14 Given the equation $y=\frac{1}{2} x-7$, what is the $y$-intercept?
OA $(0,-7)$
OB $(14,0)$
OC (0,7)
$O D(0,14)$
OE I need help


15 Given the equation $y-3=4(x+2)$, what is the $x$ intercept?

OA ( 11,0 )
OB $(2,0)$
OC $(-3,0)$
OD (-2.75, 0 )
OE I need help

https://njctl.org/video/?v=7TdjzyvAL3w

16 Given the equation $y-3=4(x+2)$, what is $y$-intercept?
OA $(0,11)$
OB $(0,2)$
OC $(0,-3)$
OD ( $0,-2.75$ )
OE I need help

17 Given the equation $x+3 y=3$, what is the $y$-intercept?
OA $(3,0)$
$O_{B}(0,1)$
○ $C(0,4)$
OD $(0,3)$
OE I need help

18 Given the equation $x+3 y=3$, what is the $x$-intercept?
OA $(3,0)$
B $(0,1)$
OC $(0,4)$
OD ( 0,3 )
OE I need help

## Horizontal and Vertical Lines

https://njctl.org/video/?v=zrZ9SBmiZSc

## Horizontal \& Vertical Lines

Horizontal and vertical lines are different from slanted lines in the coordinate plane.

A vertical line goes "up and down."

Select random points on each line shown to the left.

What are the similarities and differences between the points on the vertical lines?


## Horizontal \& Vertical Lines

Notice that each point on the line furthest to the left all have $x$-coordinates of -7 . Examples of points on this line are $(-7,2),(-7,0)$, $(-7,-3)$, etc.

The same holds true for the points on all of the vertical lines that follow. What is the common $x$-coordinate shared on the remaining lines?


## Horizontal \& Vertical Lines

A vertical line has the equation $x=a$, where $a$ is the $x$-intercept and the common $x$-coordinate shared by all of the points on the line.

Notice that no " $y$ " is contained in the equation.


## Horizontal \& Vertical Lines

A horizontal line goes "sideways."

Select random points on each line shown to the left.

What are the similarities and differences between the points on the horizontal lines?

Discuss!


Math Practice

## Horizontal \& Vertical Lines

Notice that each point on the top line have $y$ coordinates of 10 . Examples of points on this line are $(-5,10),(-2,10)$, $(0,10)$, etc.

The same holds true for the points on all of the horizontal lines that follow. What is the common $y$ coordinate shared on the remaining lines?


## Horizontal \& Vertical Lines

A horizontal line has the equation $y=b$, where $b$ is the $y$-intercept and the common $y$-coordinate shared by all of the points on the line.

Notice that no " $x$ " is contained in the equation.


## Math Practice

19 Is the following equation that of a vertical line, a horizontal line, neither, or cannot be determined?

$$
y=4
$$

(1) Vertical

OB Horizontal
© C Neither
OD Cannot be determined
OE I need help

20 Is the following equation that of a vertical line, a horizontal line, neither, or cannot be determined?

$$
x+2 y=9
$$

A Vertical
OB Horizontal
OC Neither
OD Cannot be determined
OE I need help

21 Is the following line that of a vertical, a horizontal, neither, or cannot be determined?

$$
x=-23
$$

OA Vertical
OB Horizontal
OC Neither
OD Cannot be determined
OE I need help

22 Is the following equation that of a vertical line, a horizontal line, neither, or cannot be determined:

$$
2 x-3=0
$$

OA Vertical
OB Horizontal
OC
Neither
OD Cannot be Determined
OE I need help

## 23 Which statement describes the graph of $x=3$ ?

OA It passes through the point $(0,3)$
OB It is parallel to the $y$-axis
OC It is parallel to the $x$-axis
OD I need help

24 The intercepts method (cover-up method) of graphing could NOT have been used to graph which of the following lines?


25 Which of the following equations can't be graphed using the intercepts method?

$$
\begin{array}{lll}
\text { I } & y=-3 & \text { V } \\
\text { II } & y-2=\frac{1}{2}(x+9) & \text { VI } \\
\text { III } & y x-4 y=-3 x \\
\text { IV } & x=-4 & \text { VII } \\
& & x=2 y-8 \\
\text { VIII } & y=x \\
& & \\
\square \text { A } & \text { II, V, VI, VII } & \square \text { C } \\
\square \text { I, III, IV, VIII } \\
\square & \text { I, IV } & \square \text { D } \\
\text { II, III, V, VII }
\end{array}
$$

$\square E \quad$ I need help

## Slope of a Line

https：／／njctl．org／video／？v＝LBRhjUAfoiM

## Return to Table of Contents



## Slopes and Points

It's possible, and often easier, to graph lines using a slope and a point as opposed to a table.

Also, it's not difficult to write an equation for a line from finding the slope and a point from a graph.

Let's first define slope, and then we can use that idea.

## Slope

The slope of a line is a number that describes both the direction and steepness of a line. The letter $m$ is typically used as the variable for slope.

The slope of a line can have 4 types of direction:

- positive: rising from left to right
- negative: falling from left to right
- zero: horizontal
- undefined: vertical

To measure the steepness of a line we use the ratio
 of "rise" over "run."

## Slope

The "rise" is the change in the value of the $y$-coordinate while the "run" is the change in the value of the $x$ coordinate.

The symbol for "change" is the Greek letter delta, " $\Delta$," which just means "change in." So the slope is equal to the change in $y$ divided by the change in $x$, or $\Delta y$ divided by $\Delta x \ldots$ delta $y$ over delta $x$.

$$
m=\frac{\Delta y}{\Delta x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$



## Slope

$$
m=\frac{\Delta y}{\Delta x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

In this case:
The rise is from 4 to 11
$\Delta y=11-4=7$
And the run is from 2 to 8 , $\Delta x=8-2=6$

So the slope is

$$
m=\frac{\Delta y}{\Delta x}=\frac{7}{6}
$$



## Slope

Any points on the line can be used to calculate its slope, since the slope of a line is the same everywhere.

The values of $\Delta y$ and $\Delta x$ may be different for other points, but their ratio will be the same.

You can check that with the red and green triangles shown here.

$$
m=\frac{\Delta y}{\Delta x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$



## Slope

Slope is also referred to as a constant rate of change. Here is an application of slope:

A road might rise 1 foot for every 10 feet of horizontal distance.


The ratio, $\frac{1}{10}$, which is also called slope, is a measure of the
steepness of the hill. Engineers call this use of slope grade and measure the grade with percentages. The grade of the road above is $10 \%$.

## Slope

Horizontal and vertical lines have special slopes. A horizontal line has a slope of 0 , and a vertical line has an undefined slope. Let's see what makes these slopes special.
Two points on the horizontal line are $(0,4)$ and $(3,4)$. If we look at the graph, the $\Delta y$ is 0 and the $\Delta x$ is 3 ,
so $m=\frac{0}{3}=0$.

Slope of horizontal \& vertical lines using Rise \& Run


## Slope

Horizontal and vertical lines have special slopes. A horizontal line has a slope of 0 , and a vertical line has an undefined slope. Let's see what makes these slopes special.
Two points on the vertical line are $(7,0)$ and $(7,2)$. If we look at the graph, the $\Delta y$ is 2 and the $\Delta x$ is 0 , so...
$m=\frac{2}{0}=$ undefined because you cant divide by 0 .

Slope of horizontal \& vertical lines using Rise \& Run


26 The slope of the indicated line is:

OA negative
$O_{B}$ positive
Oc zero
OD undefined
OE I need help
https://njctl.org/video/?v=ZxaVmzQ9gcE


Answer

27 The slope of the indicated line is:

A negative
OB positive
OC zero
OD undefined
OE I need help


28 The slope of the indicated line is:

A negative
OB positive
OC zero
OD undefined
OE I need help



Answer

29 The slope of the indicated line is:

OA negative
OB positive
Oc zero
OD undefined
OE I need help



Answer

## 30 The slope of the indicated line is:

OA negative
OB positive
OC zero
OD undefined
OE I need help


## Answer

31 The slope of the indicated line is:

OA negative
OB positive
OC zero
OD undefined
OE I need help

https://njctl.org/video/?v=QRJ_MuQ3Jkg


## Answer

## 32 What's the slope of this line?

A $m=-2$
$O_{B} m=2$
$\bigcirc \mathrm{C} m=0$
OD $m=\frac{1}{2}$
OE I need help

https://njctl.org/video/?v=iV9i8hE2xGg

## 33 What's the slope of this line?

A $m=-\frac{1}{2}$
$O_{B} m=\frac{1}{2}$
OC $m=2$
OD $m=-2$
OE I need help
https://njctl.org/video/?v=fjsrKWIYmNc

34 What's the slope of this line?

○A $m=6$
○B $m=1$
OC $m=0$
OD undefined
OE I need help

https://njctl.org/video/?v=4vNTNhfcCxQ

## 35 What's the slope of this line?

A $m=-\frac{3}{5}$
$O_{B} m=\frac{3}{5}$
$\bigcirc \mathrm{C} m=-\frac{5}{3}$
OD $m=\frac{5}{3}$
OE I need help

https://njctl.org/video/?v=x3iA9DYceKY

## 36 What's the slope of this line?

A $m=7$
В B $m=3$
○C $m=-\frac{7}{3}$
OD $m=\frac{7}{3}$
OE I need help

https://njctl.org/video/?v=m3BcuVoX4So

## 37 What is the slope of this line?

○ A $m=3$
$O_{B} m=1$
OC $m=0$
OD undefined
OE I need help


Answer

38 What is the slope of the line passing through the indicated points?

OA
OB -2


Answer

## Slope

Let's try an example that does not have a graph.
Calculate the slope of the line that passes through $(-5,4)$ and $(5,0)$.

First identify $(-5,4)$ as your $\left(x_{1}, y_{1}\right)$ and ( 5,0 ) as your ( $x_{2}, y_{2}$ ).

Second, substitute your numbers into the slope formula for their assigned variables.

$$
\frac{0-4}{5-(-5)}=\frac{0-4}{5+5}=\frac{-4}{10}=\frac{-2}{5}
$$



39 What is the slope of the line through $A(-2,1)$ and $B(3,-1)$ ?

A $m=-\frac{2}{5}$
B $m=-2$
OC $m=0$
OD undefined
OE I need help

# 40 What is the slope of $\overrightarrow{M N}$ given $M(1,7)$ and $N(3,-4)$ ? 

OA $m=-\frac{6}{7}$
$O_{B} m=-\frac{2}{11}$
OC $m=-\frac{11}{2}$
OD $m=-\frac{7}{6}$
OE I need help

41 What is the slope of the line containing $(-1,7)$ and $(3,-7) ?$

○A $m=-7$
$O_{B} m=-\frac{7}{2}$
$\bigcirc$ C $m=\frac{7}{2}$
OD $m=2$
OE I need help

https://njctl.org/video/?v=pJFHjRuL2Mc

42 What is the slope of the line that passes through the points $(3,5)$ and $(-2,2)$ ?
$O A \frac{1}{5}$
$O_{B} \frac{3}{5}$

OC $\frac{5}{3}$
OD 5
OE I need help

https://njctl.org/video/?v=Up1DD3EpsmY

43 A straight line with a slope of 5 contains the points $(1,2)$ and $(3, k)$. Find the value of $k$.

A $k=12$
○B $k=5$
$\bigcirc \mathrm{C} k=2$
○ $\mathrm{D} k=0$
OE I need help


## Constant Rate of Change

Slope formula can be used to find the constant rate of change in a "real world" problem.

When traveling on the highway, drivers will set the cruise control and travel at a constant speed this means that the distance traveled is a constant increase.

The graph at the right represents such a trip. The car passed mile-marker 60 at 1 hour and mile-marker 180 at 3 hours. Find the slope of the line and what it represents.


Time
(hours)

$$
m=\frac{180 \text { miles }-60 \text { miles }}{3 \text { hours }-1 \text { hour }}=\frac{120 \text { miles }}{2 \text { hours }}=\frac{60 \text { miles }}{h o u r}
$$

So the slope of the line is 60 and the rate of change of the car is 60 miles per hour.


## Constant Rate of Change

If a car passes mile-marker 100 in 2 hours and mile-marker 200 in 4 hours, then how many miles per hour is the car traveling?

Use the information to write ordered pairs $(2,100)$ and $(4,200)$.

44 If a car passes mile-marker 90 in 1.5 hours and mile-marker 150 in 3.5 hours, how many miles per hour is the car traveling?

A 60 miles per hour
O 30 miles per hour
OC 2 miles per hour
OD 40 miles per hour
OE I need help


45 How many meters per second is a person running if they are at 10 meters in 6 seconds and 100 meters in 15 seconds?

OA 90 meters per second
O B 10 meters per second
OC 9 meters per second
${ }^{\circ} \mathrm{D} 0.9$ meters per second
OE I need help

## Point-Slope Form

Return to Table of Contents

## Point-Slope Form

A useful form of the equation of a line is the pointslope form. It's equation is $y-y_{1}=m\left(x-x_{1}\right)$.

It's based on the use of the slope and any point that is on the line.

This equation is the most effective when you are given the slope and a point on the line because you can use it to write the equation in multiple forms.

Let's get started.

## Point-Slope Form

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

Point-slope form starts using the definition of slope, in which two points on a line are given by the ordered pairs

$$
\left(x_{1}, y_{1}\right) \text { and }\left(x_{2}, y_{2}\right)
$$

As a first step, let's just name the coordinates for the second point $(x, y)$ rather than $\left(x_{2}, y_{2}\right)$.

That will be true for any point on the line, not just one point, and will allow us to write an equation for all points on the line.

Then our slope formula becomes:

$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

## Point-Slope Form

$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

Now let's solve that equation for $y$ using what we've learned about solving equations.

Try it yourself, before we show you our answer.
(Hint: Remember to treat the denominator $\left(x-x_{1}\right)$ like it's in parentheses.)

## Point-Slope Form

$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

$m\left(x-x_{1}\right)=y-y_{1}$ Multiply both sides by $\left(x-x_{1}\right)$ to get rid of the fraction.
$y-y_{1}=m\left(x-x_{1}\right) \quad$ The last step is to switch the expressions to the opposite sides of the equals sign.

## Point-Slope Form

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Point-Slope Form where:

- $m$ is the slope,
- $\left(x_{1}, y_{1}\right)$ is any of the infinite points that satisfy the equation.


## Point-Slope Form

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

If you are provided a graph of a line, you can calculate m and locate a point directly from the graph.

That allows you to write the equation of the line directly, which you can then use to find any other needed points.

## Using Point-Slope to Draw a Line

For instance, if I know that the equation of a line is $y-2=2(x-1)$, then one point on the line is $(1,2)$ and the slope of the line is 2 . Using this information, I can find a second point, and then draw the line.


## Using Point-Slope to Draw a Line

I do this by recognizing that the slope of 2 means that if I go up 2 units on the $y$-axis I have to go 1 unit to the right on the $x$-axis.

Or if I go up 10 , I have to go over 5 units, etc.


## Using Point-Slope to Draw a Line

Then I draw the line through any two of those points.

This method is the easiest to use if you just have to draw a line given a point and slope.


## Using Point-Slope to Draw a Line

Graph the equation

$$
y-3=3(x-7)
$$

Based on the equation, we know that the line passes through ${ }_{\text {click }}$
and has a slope of $\frac{}{\text { click }}$
Now the graph can be drawn.

Click on the point to reveal it. Click slightly above it to show the rise \& slightly to the right to show the run. Click on the new point.
 Click in between to reveal the line.

## Using Point-Slope to Draw a Line

Example: Given the equation

$$
y+4=\frac{1}{3}(x+2)
$$

Determine the point on the line and the slope.
The point on the line is click
The slope is

## click

Graph the line representing the equation.
Click on the point to reveal it. Click slightly above it to show the slope. Click on the new point. Click in between to reveal the line.


46 What is the slope of $y-3=4(x+2) ?$
○ A $m=2$
○B $m=4$
○C $m=-3$
○D $m=8$
OE I need help

47 Which point is on the line $y-3=4(x+2)$ ?
OA

$$
(-3,2)
$$

OB $(3,-2)$
OC $(2,-3)$
$O D(-2,3)$
OE I need help

48 What is the slope and a point on the line

$$
y+5=-3(x-4) ?
$$

$\bigcirc$ A $m=-3 ;(4,-5)$
○B $m=-3 ;(-4,5)$
○С $m=3 ;(4,-5)$
$\bigcirc \mathrm{D} m=3 ;(-4,5)$
OE I need help

https://njctl.org/video/?v=Pfm_vl_Zc24

49 Which is the slope and a point on the line

$$
y-1=\frac{1}{3}(x) ?
$$

○A $m=\frac{1}{3} ;(-1,0)$
OB $\quad m=-\frac{1}{3} ;(0,-1)$
OC $m=\frac{1}{3} ;(0,1)$
OD $\quad m$ is undefined; $(0,1)$
OE I need help

https://njctl.org/video/?v=2hW4RxiaVzA

50 Which line represents $y+5=-3(x-4)$ ?

OA Line $A$
OB Line $B$
OC Line $C$
OD Line $D$
OE I need help


https://njctl.org/video/?v=AjQOuuSBpuE

51 Which line represents $y+1=-\frac{1}{2}(x+5)$ ?

OA Line $A$
OB Line $B$
OC Line $C$
OD Line $D$
OE I need help


Answer


52 Which line represents $y-6=3(x+4)$ ?
OA Line $A$
OB Line $B$
OC Line $C$
OD Line $D$
OE I need help


## Answer



## Point-Slope Form

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

You can determine and graph equations in point-slope form even when you are given limited information.

For example, if you are given the slope ( $m$ ) and any point $\left(x_{1}, y_{1}\right)$, then by substituting the point into the equation for $x_{1}$ and $y_{1}$ and the slope for $m$, you have the equation of the line with no additional work required.

## Point-Slope Form

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Or, if you are given any two points, it's always possible to determine the slope, $m$.

By then substituting one of those points in for $x_{1}$ and $y_{1}$, write the equation of a line from two points.

Let's clarify the steps required by doing some examples for both cases.

## Point-Slope Form

Example:

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Write the equation of a line in point-slope form that has a slope of 7 and passes through the point $(-9,3)$.

We already know that the slope is 7 , or $m=7$.
We also know that a point that the line passes through is $(-9,3)$, which represent $\left(x_{1}, y_{1}\right)$ respectfully.

By substituting these numbers into our equation, we have:

$$
y-3=7(x-(-9)) \text { or }
$$

$y-3=7(x+9)$, which is our equation in point-slope form.

## Point-Slope Form

Example:

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Write the equation of a line in point-slope form that passes through the points $(5,-9)$ and $(3,0)$.

This time, we do not know the slope, so we need to calculate it.

$$
m=\frac{0-(-9)}{3-5}=\frac{9}{-2}=-\frac{9}{2}
$$

We also know that a point that the line passes through is (5, -9), which represent $\left(x_{1}, y_{1}\right)$ respectfully.

By substituting these numbers into our equation, we have:

$$
y-(-9)=-\frac{9}{2}(x-5) \text { which simplifies to } y+9=-\frac{9}{2}(x-5) .
$$

## Point-Slope Form

## Example:

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Write the equation of a line in point-slope form that passes through the points $(5,-9)$ and $(3,0)$.

You might be asking if the equation on the previous slide is the only answer. Actually, you can also write the equation in point-slope form using the other point $(3,0)$ as $\left(x_{1}, y_{1}\right)$. Therefore, the equation could also be:
$y-0=-\frac{9}{2}(x-3)$, which simplifies to $y=-\frac{9}{2}(x-3)$.

## Point-Slope Form

## Example:

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Write the equation of a line in point-slope form that passes through the points $(5,-9)$ and $(3,0)$.

The reason that we have two possible answers is because they would both simplify into the same equation in Standard Form. The work for each equation is below.

$$
\begin{array}{rlrl}
y+9 & =-\frac{9}{2}(x-5) & y & =-\frac{9}{2}(x-3) \\
y+18 & =-9(x-5) & 2 y & =-9(x-3) \\
2 y+18 & =-9 x+45 & 2 y & =-9 x+27 \\
9 x+2 y+18 & =45 & 9 x+2 y & =27 \\
9 x+2 y & =27 &
\end{array}
$$

53 What is the equation of the line in point-slope form if its slope is -2 and it passes through the point $(9,-7)$.

OA $y-7=-2(x-9)$
○В $y+7=-2(x+9)$
○с $y+7=-2(x-9)$
OD $y-7=-2(x+9)$
OE I need help


54 What is the equation of the line in point-slope form if its slope is 5 and it passes through the point $(-1,-4)$.

○A $y-4=5(x-1)$
○В $y+4=5(x+1)$
○ $y+4=5(x-1)$
○D $y-4=5(x+1)$
OE I need help


55 What is the equation of the line in point-slope form if its
slope is $-\frac{4}{9}$ and it passes through the point $(3,8)$.

$$
\begin{aligned}
y-8 & =-\frac{4}{9}(x-3) \\
\text { A } y+8 & =-\frac{4}{9}(x+3) \\
\text { C } y+8 & =-\frac{4}{9}(x-3) \\
\text { D } y-8 & =-\frac{4}{9}(x+3)
\end{aligned}
$$

OE I need help

56 What is the equation of the line in point-slope form that passes through the points $(3,8)$ and $(-2,-2)$.

A $y+8=2(x+3)$
○В $y-8=2(x-3)$
○c $y+2=\frac{1}{2}(x+2)$
OD $y-2=\frac{1}{2}(x-2)$
OE I need help


57 What is the equation of the line in point-slope form that passes through the points $(7,9)$ and $(5,-1)$.

A $y-9=5(x-7)$
○B $y-1=5(x-5)$
С $y+1=\frac{1}{5}(x-5)$
$O D^{y-9}=\frac{1}{5}(x-7)$
OE I need help


58 What is the equation of the line in point-slope form that passes through the points $(4,-7)$ and $(-2,-3)$.

A $y-7=-\frac{3}{2}(x+4)$
B $y-3=-\frac{3}{2}(x-2)$
C $y+3=-\frac{2}{3}(x+2)$
$\mathrm{D}^{y-7}=-\frac{2}{3}(x-4)$
OE I need help


## Slope-Intercept Form



## Slope-Intercept Form

Another very useful form of the equation of a line is the slope-intercept form.

It's based on the use of the slope and the $y$-intercept of a line.

Similar to point-slope, we are going to start by showing a derivation proof of slope-intercept form.

## Slope-Intercept Form

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

Let's start with the definition of slope, in which two points on a line are given by the ordered pairs

$$
\left(x_{1}, y_{1}\right) \text { and }\left(x_{2}, y_{2}\right)
$$

As a first step, let's just name the coordinates for the second point $(x, y)$ rather than $\left(x_{2}, y_{2}\right)$.

That will be true for any point on the line, not just one point, and will allow us to write an equation for all points on the line.

Then our slope formula becomes:

$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

## Slope-Intercept Form

$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

Now let's solve that equation for $y$ using what we've learned about solving equations.

Try it yourself, before we show you our answer.
(Hint: Remember to treat the denominator $\left(x-x_{1}\right)$ like it's in parentheses.)

## Slope-Intercept Form

$m=\frac{y-y_{1}}{x-x_{1}} \quad \begin{aligned} & \text { Multiply both sides by }\left(x-x_{1}\right) \text { to get rid of the } \\ & \text { fraction. }\end{aligned}$
$m\left(x-x_{1}\right)=y-y_{1}$ Now, add $y_{1}$ to both sides.
$m\left(x-x_{1}\right)+y_{1}=y$ Now switch the sides.
$y=m\left(x-x_{1}\right)+y_{1}$ This is fine, and allows us to graph a line given any point $\left(x_{1}, y_{1}\right)$.

But, one additional step is taken to get the most useful equation for a line.

## Slope-Intercept Form

$y=m\left(x-x_{1}\right)+y_{1}$ Use the $y$-intercept for $\left(x_{1}, y_{1}\right)$.
The $y$-intercept is the point where the line crosses the $y$-axis.

The coordinates of that point are $(0, b)$.

$$
y=m x+b
$$

- $x$ is zero anywhere on the $y$-axis
- We name the $y$-intercept " $b$."

Then, this becomes: $y=m x+b$

## Slope-Intercept Form

$$
y=m x+b
$$

Slope-Intercept Form is where:

- $m$ is the slope,
- $b$ is the $y$-intercept $(0, b)$
- $(x, y)$ is any of the infinite points that satisfy the equation.


## Slope-Intercept Form

$$
y=m x+b
$$

This is the form of the equation of a line that is most often used.
If you are provided a graph of a line, you can calculate $m$ and read $b$ directly from the graph.

That allows you to write the equation of the line directly, which you can then use to find any other needed points.

## Slope-Intercept Form

The line shown in the coordinate plane to the left has a $y$-intercept at the point $(0,-2)$ and has a slope
of $\frac{2}{3}$. Therefore, the equation of this line is

$$
y=\frac{2}{3} x-2 .
$$



## Slope-Intercept Form

Example: Write the equation for the line shown in the graph to the right.

The slope of the line $(m)$ is

$$
\overline{\text { click }}
$$

The $y$-intercept of the line $(b)$ is

$$
\overline{\text { click }}
$$

Therefore the equation of the line in slope intercept form is


## Slope-Intercept Form

Example: Write the equation for the line shown in the graph to the right.

The slope of the line $(m)$ is

```
click
```

The $y$-intercept of the line (b) is

```
click
```

Therefore the equation of the line in slope intercept form is


59 What is the equation of the graphed line?

$$
\begin{array}{ll}
\text { OA } & y=3 x-1 \\
\text { OB } & y=3 x \\
\text { OC } & y=3 x+1 \\
\text { OD } & y=\frac{1}{3} x+1 \\
\text { OE } & \text { I need help }
\end{array}
$$



60 Which equation is that of the graphed line?

$$
\begin{array}{ll}
\text { OA } & y=3 x-1 \\
\text { OB } & y=3 x+1 \\
\text { OC } & y=\frac{1}{3} x+1 \\
\text { OD } & y=\frac{1}{3} x-1 \\
\text { OE } & \text { I need help }
\end{array}
$$

https://njctl.org/video/?v=n62Hdx1tIPs


Answer

61 Which equation is that of the graphed line?
OA $y=3 x-1$
OB $y=3 x$
OC $y=3 x+1$
OD $y=\frac{1}{3} x$
OE I need help


62 Which equation is that of the graphed line?
A $y=4 x-2$
OB $y=-\frac{1}{2} x-2$
○ $y=-\frac{1}{2} x+4$
OD $y=-2 x+4$
OE I need help


## Graphing a Line Using Slope-Intercept Form

Having the equation of a line in slope-intercept form also allows us to quickly graph a line, using the $y$-intercept and the slope.
For instance, if we know that the equation of the line is

$$
y=-\frac{1}{2} x+5
$$

then we can graph the $y$ intercept $(0,5)$ and use the slope of $-\frac{1}{2}$ to count the number of spaces, down 1 unit and right 2 units (or up
 one and left 2 units), to get an additional point on the line.

## Graphing a Line Using Slope-Intercept Form

Then we draw the line through any two of those points.

This method is the easiest to use if you just have to draw a line given the $y$-intercept and slope.


## 63 What is the slope of the linear equation below?

$$
y=-\frac{3}{2} x-5
$$

○A 5
○ $\quad-5$
$O C \frac{3}{2}$

OD $-\frac{3}{2}$
OE I need help


64 What is the $y$-intercept of the linear equation below?

○ $\quad(0,5)$

$$
y=-\frac{3}{2} x-5
$$

○ $\quad(0,-5)$
$O C\left(0, \frac{3}{2}\right)$

OD $\left(0,-\frac{3}{2}\right)$
OE I need help


65 Which line in the graph below represents the linear
equation $y=-\frac{3}{2} x-5$ ?

OA Red
OB Blue
OC Purple
OD Black
OE I need help



66 What is the slope of the linear equation below?

○A 5

$$
y=\frac{2}{3} x+5
$$

○B -5
$O C \frac{2}{3}$
OD $-\frac{2}{3}$
OE I need help


67 What is the $y$-intercept of the linear equation below?
$\mathrm{A} \quad(0,5) \quad y=\frac{2}{3} x+5$
○ $\quad(0,-5)$
$O C\left(0, \frac{2}{3}\right)$
$O D\left(0,-\frac{2}{3}\right)$
OE I need help


68 Which line in the graph below represents the linear equation
$y=\frac{2}{3} x+5 ?$
OA Red
OB Blue
Oc Purple
OD Black
OE I need help
https://njctl.org/video/?v=4ptUullgwM8

## Slope-Intercept Form

$$
y=m x+b
$$

You can also determine and graph equations in slope-intercept form even when you are given limited information.

For example, if you are given the slope ( $m$ ) and any point $(x, y$ ), then by substituting the point into the equation for $x$ and $y$, you can solve for $b$.

Then, you can write the equation of a line when given the slope and a point that the line passes through.

## Slope-Intercept Form

$$
y=m x+b
$$

Or, if you are given any two points, it's always possible to determine the slope, $m$.

By then substituting one of those points in for $x$ and $y$, you can solve for $b$.

Then, you can write the equation of a line from two points.

Let's clarify the steps required by doing some examples for both cases.

## Slope-Intercept Form

$$
y=m x+b
$$

## Example:

Write the equation of a line in slope-intercept form that has a slope of 8 and passes through the point $(3,10)$.

We already know that the slope is 8 , or $m=8$.
We also know that a point that the line passes through is $(3,10)$, which represent $(x, y)$ respectfully.

By substituting these numbers into our equation, we have:

$$
10=8(3)+b
$$

## Slope-Intercept Form

$$
y=m x+b
$$

Now, we can solve for $b$.

$$
\begin{aligned}
10 & =8(3)+b & & \text { Multiply } 8 \text { and } 3 . \\
10 & =24+b & & \text { Subtract } 24 \text { from both sides. } \\
-14 & =b & & \\
b & =-14 & & \text { Switch the order. }
\end{aligned}
$$

Our equation is then $y=8 x-14$.

## Slope-Intercept Form

## Example:

$$
y=m x+b
$$

Write the equation of a line in slope-intercept form that has a slope of 8 and passes through the point $(3,10)$.

This problem can also be solved using point-slope and using the rules of algebra to get it into slope-intercept form. We have our point $(3,10)$ as $\left(x_{1}, y_{1}\right)$ and our slope of 8 . If I substitute the numbers into pointslope, we will get this equation:

$$
\begin{aligned}
& y-10=8(x-3) \\
& y-10=8 x-24 \\
& y \text { Distribute the } 8 \text { to } x \text { and }-3 . \\
& y x-14
\end{aligned}
$$

## Slope-Intercept Form

## Example:

$$
y=m x+b
$$

Write the equation of a line in slope-intercept form that passes through the points $(4,0)$ and $(6,5)$.

This time, we do not know the slope, so we need to calculate it.

$$
m=\frac{5-0}{6-4}=\frac{5}{2}
$$

We also know that a point that the line passes through is $(4,0)$, which represent $(x, y)$ respectfully.

By substituting these numbers into our equation, we have:

$$
0=\frac{5}{2}(4)+b
$$

## Slope-Intercept Form <br> $$
y=m x+b
$$

Now, we can solve for $b$.

$$
\begin{array}{rlrlrl}
0 & =\frac{5}{2}(4)+b & & \text { Multiply }{ }^{\frac{5}{2}} \text { and } 4 . \\
0 & =10+b & & \text { Subtract } 10 \text { from both sides. } \\
-10 & =b & & \\
& b & =-10 & & \text { Switch the order. }
\end{array}
$$

Our equation is then $y=\frac{5}{2} x-10$.

## Slope-Intercept Form

## Example:

$$
y=m x+b
$$

Write the equation of a line in slope-intercept form that passes through the points $(4,0)$ and $(6,5)$.

This problem can also be solved using point-slope and using the rules of algebra to get it into slope-intercept form. We can select one of our
points as $\left(x_{1}, y_{1}\right)$ and our slope that we calculated was $\frac{5}{2}$. Let's use
$(4,0)$. If we substitute the numbers into point-slope, we will get this equation:

$$
y-0=\frac{5}{2}(x-4)
$$

Distribute the slope \& drop the 0.

$$
y=\frac{5}{2} x-10
$$

69 What is the equation of the line that passes through the point $(-3,-7)$ and has a slope of 4 ?

OA $y=4 x+25$
OB $y=4 x+5$
OC $y=-3 x+5$
OD $y=7 x-17$
OE I need help

70 What is the equation of the line that passes through the point $(6,-2)$ and has a slope of $\frac{5}{6}$ ?
A $y=\frac{5}{6} x-7$
○В $y=\frac{5}{6} x+7.67$
OC $y=6 x-7$
OD $y=-2 x+7.67$
OE I need help

https://njctl.org/video/?v=GgstSLpBaFM

71 What is the equation of the line that passes through the point $(-1,-8)$ and has a slope of $-\frac{1}{2}$ ?
A $y=-\frac{1}{2} x-8.5$
B $y=-\frac{1}{2} x+4$
CC $y=-\frac{1}{2} x-5$
OD $y=-\frac{1}{2} x-7.5$
OE I need help

72 What is the equation of the line that passes through the points $(-1,-8)$ and $(2,1)$ ?

A $y=\frac{1}{3} x-\frac{1}{3}$
(B $y=\frac{1}{3} x+\frac{5}{3}$
OC $y=3 x-5$
OD $y=3 x-1$
OE I need help

https://njctl.org/video/?v=t7wbz5uZSJ0

73 What is the equation of the line that passes through the points $(7,0)$ and $(3,2)$ ?

A $y=-\frac{1}{2} x+\frac{7}{2}$
OB $y=-\frac{1}{2} x+7$
OC $y=-2 x+7$
OD $y=-2 x+8$
OE I need help

https://njctl.org/video/?v=lukLOJnB81w

74 What is the equation of the line that passes through the points $(-3,-5)$ and (3,2)?

OA

$$
y=\frac{6}{7} x+\frac{17}{7}
$$

OB $y=\frac{6}{7} x-\frac{4}{7}$
OC $\begin{gathered}y=\frac{7}{6} x-8.5\end{gathered}$
$y=\frac{7}{6} x-1.5$

OE I need help

## Lab: Marble Masters

Students use a linear equation, slope, and $x$ and $y$ intercepts to aim a marble launch tube so the marble will cross a specified set of Cartesian coordinates and hit the "target"!

## Working with Linear Equations

Return to Table

## Working with Linear Equations

Throughout this unit, you have learned how to calculate the slope of a line and write the equation of a line in three different forms. What if you are given the equation of a line, and they ask you to find the slope? or an intercept?

Depending on what form the equation is in, the information might be easy to find. Sometimes, it might require some algebraic steps to manipulate the equation into the desired form.

## Working with Linear Equations

## Example:

What is the slope of the equation $8 x-10 y=40 ?$
This equation is in standard form, which can be used to find the intercepts. It doesn't tell you the slope, though. So we have to manipulate it using what we know from algebra.

Which form(s) of a linear equation can help us find the slope of this line?

```
click to reveal
```

Which form(s) of a linear equation would be the most appropriate for this situation? Why?

## Working with Linear Equations

## Example: (cont.)

What is the slope of the equation $8 x-10 y=40 ?$

$$
\begin{aligned}
8 x-10 y & =40 & & \\
-10 y & =-8 x+40 & & \text { Subtract } 8 x \text { from both sides. } \\
y & =\frac{4}{5} x-4 & & \text { Divide both sides (or all terms) by }-10
\end{aligned}
$$

Therefore, the slope of the equation is $\overline{\text { click }}$

## Working with Linear Equations

## Example:

Write the equation of the line with an $x$-intercept of 5 and a $y$-intercept of 10 .

Since two points are given, we can use the slope formula. The two points are $(5,0)$ and $(0,10)$.


Since this problem does not specify a form for the equation, you can select any form that you desire. The equation of the line in point-slope form, using the $x$-intercept, is

Or you could also use the $y$-intercept to write the equation in pointslope form:

Or since you have the $y$-intercept and you calculated the slope, you could have written the equation in slope-intercept form:

## Working with Linear Equations

Example: Write the equation $y-7=\frac{3}{4}(x+4)$ in standard form. Then determine the $x$ - and $y$-intercepts.

We need to get this equation into $A x+B y=C$, where $A \geq 0$. Therefore, we need to use our inverse operations.

$$
y-7=\frac{3}{4}(x+4) \text { Multiply both sides of the equation by the LCD. }
$$

$4(y-7)=3(x+4)^{\text {Distributive Property }}$
$4 y-28=3 x+12$ Subtract $3 x$ from both sides.
$-3 x+4 y-28=12$
$-3 x+4 y=40$
Add 28 to both sides.
Multiply all terms by -1 , because $A \geq 0$.
$3 x-4 y=-40$

## Working with Linear Equations

Example: Write the equation $y-7=\frac{3}{4}(x+4)$ in standard form.
Then determine the $x$ - and $y$-intercepts.

## Standard Form:

$x$-intercept: $y=0$
$y$-intercept: $x=0$


75 What is the slope of the line formed by the equation below?

$$
3 x-7 y=9
$$

OA
OB ${ }_{\frac{3}{7}}^{-\frac{3}{7}}$
OC ${ }_{\frac{7}{3}}^{-\frac{7}{3}}$
OD
OE I need help

76 What is the slope of the line formed by the equation below?

$$
5 x+9 y=45
$$

OA
OB $\frac{-\frac{9}{5}}{5}$
OC $-\frac{5}{9}$
$\frac{5}{9}$
OD
OE I need help

77 What is the equation of a line that has an $x$-intercept of 2 and a $y$-intercept of 7 ?

OA $y=-\frac{7}{2}(x-2)$
OB $y=-\frac{7}{2} x+7$
OC $7 x+2 y=14$
O All of the above
OE I need help

78 Which equation represents $y+6=2(x-1)$ in Standard Form?

ค $2 x-y=8$
○В $-2 x+y=-8$
OC $y=2 x-8$
$y=-2 x+8$

OE I need help

https://njctl.org/video/?v=sO5JJIEtDKY

79 Which equation represents $y=-\frac{3}{5} x+9$ in
Standard Form？
ค $3 x-5 y=-45$
○В $3 x+5 y=45$
○С $5 y=-3 x+9$
OD $y-2=-\frac{3}{5}(x-5)$
OE I need help

80 What is the equation of a line that has an $x$-intercept of 5 and a $y$-intercept of -7 ?

OA

$$
y=\frac{7}{5} x+5
$$

OB $\quad y=\frac{7}{5} x-7$
OC $7 x-5 y=70$
O All of the above
$O_{E}$ I need help

https://njctl.org/video/?v=WR-CLimGOos

## Comparing Rates of Change

To compare the rates of change of two different types of functions, we find the rate of change of each and compare their absolute values.

For example, if a graph has a slope of -4 and an equation has a slope of 3 , the slope of the graph is steeper because the absolute value of -4 is greater than the absolute value of 3 , i.e. $|-4|>|3|$.

Since the graph has a steeper slope it also has a larger rate of change.

## Comparing Rates of Change

Let's try one!
Which has a greater rate of change?

A $y=-5 x+6$

Slope =




Slope =

## Greater Rate of Change

Let's compare a verbal model and a table.
A Chris and Shari are going to have a bowling party. It costs $\$ 10$ to rent a lane and $\$ 2$ per pair of shoes.

B | $\boldsymbol{x}$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 7 | 13 | 19 | 25 | 31 | 37 | 41 |

Which has the greater rate of change?

## Greater Rate of Change

A Chris and Shari are going to have a bowling party. It costs $\$ 10$ to rent a lane and $\$ 2$ per pair of shoes.

Let's model this with a function.
The initial cost is $\$ 10$ to rent a lane, a constant.
The cost for shoes changes depending on the amount of people.
Let $x$ be the number of people bowling and $y$ represent the cost. This makes the cost of the shoes $2 x$.

So the cost equation is $y=10+2 x$.
Its rate of change $=$

## Greater Rate of Change

В | $x$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 7 | 13 | 19 | 25 | 31 | 37 | 41 |

We can use the slope formula to find the rate of change.

## Greater Rate of Change

Comparing rates from a table and a verbal model.
A Chris and Shari are going to have a bowling party. It costs $\$ 10$ to rent a lane and $\$ 2$ per pair of shoes.

В | $x$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 7 | 13 | 19 | 25 | 31 | 37 | 41 |

Which has the greater rate of change?
The rate of change of $\mathrm{A}=2$ and the rate of change of $\mathrm{B}=3$.

$$
2<3
$$

Therefore, B has the greater rate of change.

81 Which has the greater rate of change?

A $y=\frac{1}{3} x+5$
OB The school store is selling book covers 2 for $\$ 1$.
OC I need help

## 82 Which has the greater rate of change？

OA $y=x-4$

○B | $x$ | -9 | -6 | -3 | 0 | 3 | 6 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |

OC I need help

83 Which has the greatest rate of change?
$\bigcirc A\{(1,3),(2,4),(3,5),(4,6),(5,7)\}$
OB Ryan and Andrew jump down the stairs 3 steps at a time.
OC $y=\frac{1}{8} x-2$


OE I need help

84 Which has the greatest rate of change?
OA A cable company charges $\$ 12$ for every 2 premium channels.
B $y=5 x+6$
$\bigcirc \subset\{(9,3),(6,2),(3,1),(0,0),(-3,-1)\}$
OD


OE I need help

## Function Notation

Sometimes, you will see an equation in the form $f(x)=m x+b$ instead of $y=m x+b$. This can occur because every linear equation is a function.
A function is a relationship that exists when every $x$ value has exactly one $y$ value. When this happens, nothing changes mathematically. Yet, instead of writing $y$ mathematicians use $f(x)$, read " $f$ of $x$."
For example:

* $y \Leftrightarrow f(x)$ given that $y$ is a function.

$$
y=5 x+7 \text { becomes } f(x)=5 x+7
$$

$f(x)=5 x+7$ is still a line with a slope of 5 and a $y$-intercept of ( 0,7 ). Since we haven't yet covered the material for functions, it would be easiest for you, at this point, to switch the $f(x)$ to $y$ and solve the equation from there.

## Function Notation

The letter $f$ is often used to name a function, but other letters, such as $g$ and $h$, could also be used.

For example:

$$
\begin{aligned}
& f(x)=5 x+7 \\
& g(x)=8 x-3 \\
& h(x)=9 x+1
\end{aligned}
$$

$f, g$, and $h$ are all examples of linear functions.

Since we haven't yet covered the material for functions, it would be easiest for you, at this point, to substitute $y$ for $f(x), g(x), h(x)$, etc. and solve the equation from there.

## Solving Linear Equations

Example: The graph shown to the right represents the function

$$
f(x)=-\frac{7}{3} x+14
$$

For what value of $x$ does $f(x)=0$ ?
If we start with the advice that we started with, substituting $y$ in for $f(x)$, we get the equation

$$
y=-\frac{7}{3} x+14
$$



## Solving Linear Equations

Example: (cont.)The graph shown to the right represents the function

$$
f(x)=-\frac{7}{3} x+14
$$

For what value of $x$ does $f(x)=0$ ?

$$
f(x)=-\frac{7}{3} x+14
$$

If $f(x)=0$, that means that $y=0$. This occurs when the graph crosses the $x$-axis, giving us our $x$-intercept. Looking at the graph, we see that our $x$-intercept is $(6,0)$,
 so the value of $x$ is 6 .

85 The graph shown below represents the function $g(x)=2 x-8$. For what value of $x$ does $g(x)=0$ ?

$$
\begin{aligned}
& \text { OA }-8 \\
& \text { OB } 8 \\
& \text { OC } 4 \\
& \text { OD }-4 \\
& \text { OE I need help }
\end{aligned}
$$



86 The graph shown below represents the function

$$
h(x)=-\frac{1}{3} x+2
$$

For what value of $x$ does $h(x)=0$ ?

○A 6
OB -6
OC 2
OD -2
OE I need help


87 The graph shown below represents the function

$$
j(x)=-\frac{3}{5} x-3
$$

For what value of $x$ does

$$
j(x)=0 ?
$$

○A 3
○B-3
○C 5
○D -5
OE I need help



# Scatter Plots and the Line of Best Fit 

Return to Table of Contents

## Scatter Plot

A scatter plot is a graph that shows a set of data that has two variables.

| Time <br> Studying | Test <br> Score |
| :---: | :---: |
| 45 | 89 |
| 30 | 78 |
| 50 | 90 |
| 60 | 92 |
| 40 | 85 |
| 48 | 87 |
| 55 | 95 |
| 35 | 82 |

Test Score vs. Time Spent Studying


## Scatter Plot

There are three types of linear association that are possible for scatter plots. What are they?

What type of linear association does the graph have to the right?

Test Score vs. Time Spent Studying


## Scatter Plot

What connection do you see between linear association and the slope of a line?

Test Score vs. Time Spent Studying


88 What type of scatter plot is shown in the graph below?

A non-linear
OB linear with positive association
OC linear with negative association
OD linear without association

OE I need help

Ice Cream Sales vs. Temperature


Temperature ( ${ }^{\mathbf{F}}$ )

89 What kind of association is shown in the graph?

OA non-linear
OB linear with positive association
OC linear with negative association
OD linear without association
OE I need help

## Number of Clothing Layers Worn vs. Temperature



Answer

Temperature $\left({ }^{\circ} \mathrm{F}\right.$ )

90 What kind of association is shown in the graph?
Shoe Size vs. Height
OA non-linear
OB linear with positive association
OC linear with negative association
OD linear without association

OE I need help




Answer

91 What association is shown in this graph?
Boy's Height vs. Weight
OA non-linear
OB linear with positive association
OC linear with negative association
OD linear without association
OE I need help


## Answer

92 Which of the following scenarios would produce a linear scatter plot with a negative association?

OA Miles driven and money spent on gas
OB Number of pets and how many shoes you own
OC Work experience and income
$O_{D}$ Time spent studying and number of bad grades
OE I need help

93 Which of the following would have no association if plotted on a scatter plot?

A Number of toys and calories consumed in a day
OB Number of books read and reading scores
OC Length of hair and amount of shampoo used
OD Person's weight and calories consumed in a day
OE I need help

## Draw a Line

Notice that the points form a linear like pattern. To draw a line of best fit, use two points so that the line is as close as possible to the data points.
Our line is drawn so that it fits as close as possible to the data points. The number of points above and below the line should be about the same. There are 3 points above our line and 3 points below our line. This line was drawn through $(35,82)$ and $(50,90)$.
https://njctl.org/video/?v=pJxw5KFuD10

## Test Score vs. Time Spent Studying



## Scatter Plot

Using the line of best fit shown in the graph to:

- Predict the test score of someone who spends 32 minutes studying.
- Predict the test score of someone who spends 58 minutes studying.



## Prediction Equation

Use the two points that formed the line to write an equation for the line.


Math Practice

## Prediction Equation

Use the two points that formed the line to write an equation for the line.

Find the slope ( $m$ ).

$$
\begin{aligned}
m & =\frac{90-82}{50-35} \\
m & =\frac{8}{15}
\end{aligned}
$$

Test Score vs. Time Spent Studying


## Prediction Equation

Use the two points that formed the line to write an equation for the line.

Find the equation of the line.

$$
\begin{aligned}
& y-90=\frac{8}{15}(x-50) \\
& y-90=\frac{8}{15} x-\frac{80}{3} \\
& y=\frac{8}{15} x+\frac{190}{3}
\end{aligned}
$$

where $y$ is the score for $x$ minutes of studying

The equation formed is called a Prediction Equation.

Test Score vs. Time Spent Studying


## Extrapolation

Prediction Equations can be used to predict other related values.

$$
y=\frac{8}{15} x+\frac{190}{3}
$$

If a person studies 15 minutes, what would be the predicted score?

$$
S=\frac{8}{15}(15)+\frac{190}{3} \approx 71.3
$$

This is an extrapolation, because the time was outside the range of the original times.

## Interpolation

If a person studies 42 minutes, what would be the predicted score?

$$
S=\frac{8}{15}(42)+\frac{190}{3} \approx 85.7
$$

This is an interpolation, because the time was inside the range of the original times.

## What is Wrong?

Interpolations are more accurate because they are within the set.
The farther points are away from the data set the less reliable the prediction.

Using the same prediction equation, consider:
If a person studies 120 minutes, what will be there score?

$$
S=\frac{8}{15}(120)+\frac{190}{3} \approx 127.3
$$

What is wrong with this prediction?

## What is the Prediction?

If a student got an 80 on the test, what would be the predicted length of their study time?

$$
\begin{aligned}
80 & =\frac{8}{15} x+\frac{190}{3} \\
16.7 & =\frac{9}{15} x \\
31.25 & =x
\end{aligned}
$$

The student studied about 31 minutes.

## Shoe Size vs. Height

Shoe Size vs. Height

- Draw the line of best fit for our data.
- Determine the equation for the line of best fit.
- Predict the height of a person who wears a size 8 shoe.
- Predict the shoe size of a person who is 50 inches tall.


Shoe Size

## Shoe Size vs. Height

Draw the line of best fit for our data.
Click on the graph to reveal the line of best fit.

Determine the equation for the line of best fit.

Find the slope.


Find the equation of the line.


Shoe Size

## Shoe Size vs. Height

Predict the height of a person who wears a size 8 shoe.

Shoe Size vs. Height

Predict the shoe size of a person who is 50 inches tall.


94 Consider the scatter graph to answer the following: Which two points would give the best line of fit?

○A $A$ and $D$
OB $B$ and $C$
○C $C$ and $D$
$O D$ There is no pattern

OE I need help
https://njctl.org/video/?v=F_y5YT3KFNs


| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 3 | 9 |
| 4.5 | 8 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

## 95 Consider the scatter graph to answer the following: Which two points would give the best line of fit?

○A $A$ and $D$
OB $B$ and $C$
○C $C$ and $D$
OD There is no pattern
OE I need help


| $x$ | $y$ |
| :---: | :---: |
| 5 | 2 |
| 6 | 4 |
| 7 | 3 |
| 8 | 4 |
| 9 | 4.5 |
| 9 | 5 |
| 10 | 3 |

Answer

96 Consider the scatter graph to answer the following. What is the slope of the line of best fit going through $A$ and $D$ ?

- A $\frac{3}{4}$
- $B \frac{-3}{4}$

○ C 1

○ D -1

https://njctl.org/video/?v=u4PbRDhlkBQ

97 Consider the scatter graph to answer the following. What is the $y$-intercept of the line of best fit going through $A$ and $D$ ?

○A 9

- 10

OC 11
OD 12
OE I need help


| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 3 | 9 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

98 Consider the scatter plot to answer the following. Using the line of best fit shown in the graph below, what would the prediction be if $x=7$ ? Is this an interpolation or extrapolation?

OA 5, interpolation
OB 5, extrapolation
OC 6, interpolation
OD 6, extrapolation
OE I need help


99 Consider the scatter graph to answer the following.Using the line of best fit shown in the graph below, what would the prediction be if $x=14$ ? Is this an interpolation or extrapolation?

A -4 , interpolation
OB -4, extrapolation
OC -2, interpolation
OD -2, extrapolation
OE I need help



100 Consider the scatter graph to answer the following: Using the line of best fit shown in the graph below, what would the prediction be if $y=11$ ? Is this an interpolation or extrapolation?

OA 1, interpolation
OB 1, extrapolation
OC 2, interpolation
OD 2, extrapolation
OE I need help

https://njctl.org/video/?v=0Tggsam1p6s


101 In the previous questions, we began by using the table at the right. Which of the predicted values
$(7,5)$ or $(14,-2)$ will be more accurate and why?

OA (7,5); it is an interpolation
OB (7,5); there already is a 5 and a 7 in the table

OC $(14,-2)$ it is an extrapolation
OD (14, -2); the line is going down and will become negative

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 3 | 9 |
| 5 | 7 |
| 6 | 5 |
| 8 | 4 |
| 9 | 3 |
| 10 | 1 |

## Candle Lab

Students measure the height of a burning candle, graph the data and find the line of best fit.

## PARCC Sample Questions

The remaining slides in this presentation contain questions from the PARCC Sample Test.

After finishing this unit, you should be able to answer these questions.

Good Luck!

Return to Table of Contents

102 Which points are on the graph of the equation $-3 x+6 y+5=-7$ ？Select all that apply．

$$
\begin{array}{ll}
\text { A }(-3,6) & \text { D }(6,-3) \\
B(-2,0) & E(8,2) \\
C(0,-2) &
\end{array}
$$

$\square \mathrm{A} A, \mathrm{E}$
$\square$ C C，E
$\square$ B B ，C

$$
\square \mathrm{D} \text { B, D }
$$

$\square E$ I need help

103 The graph of the function $f(x)=-1+.5 x$ is shown on the coordinate plane. For what value of $x$ does $f(x)=0$ ?


A 2
B -1
C 0
D 0.5
E I need help

104 Graph the equation $6 x-4 y=12$ on the $x y$-coordinate plane. Identify the $x$-intercept of the graph.

O $(2,0)$
$O_{B}(0,2)$
○C $(0,-3)$
OD $(-3,0)$
OE I need help

|  |  |  | $y$ | $y$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 10 |  |  |  |  |  |  |  |
|  |  |  | 10 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 5 |  |  |  |  |  |  |  |
|  |  |  | 5 |  |  |  |  |  |  |  |
|  |  |  | , |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | $x$ |
| -10 |  |  | 0 |  |  | 5 |  |  | 10 |  |
|  |  |  | 5 |  |  |  |  |  |  |  |
|  |  |  | -5 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | -10† |  |  |  |  |  |  |  |
|  |  |  | - |  |  |  |  |  |  |  |

105 The ordered pairs $(20,-29.5),(21,-31)$ and $(22,-32.5)$ are points on the graph of a linear equation. Graph the line that shows all of the ordered pairs in the solution set of this linear equation.

OA Red
OB Blue
OC Purple
OD Black
OE I need help


From PARCC Sample Test - Calculator

ләмsu*

## Glossary and Standards

Return to
Table of Contents

## Constant Rate of Change

A rate that describes how one quantity changes in relation to another. This rate never changes.


## Direct Variation

A relationship between two variables in which one is a constant multiple of the other. When one variable changes the other changes in proportion to the first.

$$
y=m x
$$



Goes through the origin.

## Extrapolation

## A data point that is outside the range of data.



## Grade

## A unit engineers use to measure the steepness of a hill.



## Horizontal Line

A line whose direction is left and right. All of the $y$-coordinates on the line are equal.


## Interpolation

## A data point that is inside the range of data.



## Line of Best Fit

## A line on a graph showing the general direction that a group of points seem to be heading. Trend line.





## Negative Slope

When a line falls down from left to right.


Back to
Instruction

## Point-Slope Form

The point-slope equation for a line is

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

where $m$ is the slope and $\left(x_{1}, y_{1}\right)$ is a point on the line.


## Positive Slope

When a line rises from left to right.


## Prediction Equation

An equation that is created using the line of best fit. A line that can predict outcomes using the given data.


## Scatter Plot

A graph of plotted points that show the relationship between two sets of data.


Back to
Instruction

## Slope

## How much a line rises or falls. Steepness of a line. The ratio of a line's rise over its run.

$$
y=m x+b
$$

" $m$ " = slope or how the line "moves"
formula for slope:

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$



## Slope-Intercept Form

One type of straight line equation that utilizes the slope and $y$-intercept to graph.


## Standard Form

## Standard form looks like

$$
\mathrm{A} x+\mathrm{B} y=\mathrm{C}
$$

where $\mathrm{A}, \mathrm{B}$ and C are integers and $\mathrm{A}>0$.

$$
\begin{array}{c|c}
-3 x+2 y=-6 \\
-3 x=-6 & 2 y=-6 \\
x=2 & y=-3 \\
(2,0) & (0,-3)
\end{array}
$$



Back to
Instruction

## Undefined Slope

When a line does not run at all as one reads from bottom to top on the $y$-axis.


## Vertical Line

A line whose direction is only up and down.
All of the $x$-coordinates on the line are equal.


## $x$-Intercept

## Where a line crosses the $x$-axis.





## $y$-Intercept

Where a line crosses the $y$-axis.


## Zero Slope

When a line does not rise at all as one reads it from left to right on the $x$-axis.


Back to
Instruction

Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems \& persevere in solving them.
MP2: Reason abstractly \& quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for \& make use of structure.
MP8: Look for \& express regularity in repeated reasoning.
Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

