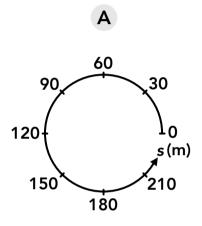
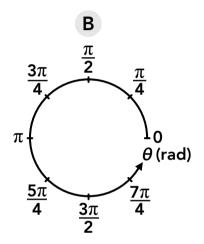
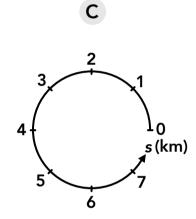
# **CIRCULAR & ROTATIONAL MOTION**

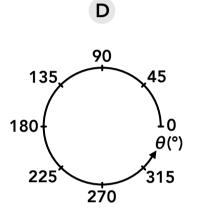
## Position and Displacement

- 1. 1 revolution is equivalent to which of the following? (Select all that apply)
  - A 1 circumference
  - B  $\pi$  radians
  - **c** 360°
  - D  $2\pi$  radians
- 2. If the circumference of a circle is 20 m, what is the radius?
  - **A** 10 m
  - B 6.37 m
  - **C** 3.18 m
  - D 2.52 m
- 3. Which of the following axes can be used with the tangential description of motion? (Select all that apply)

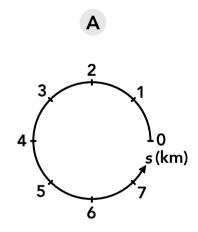


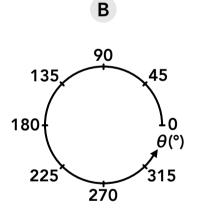


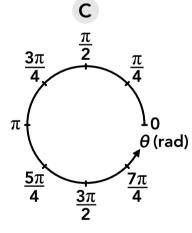


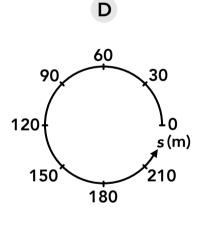


4. Which of the following axes can be used with the angular description of motion? (Select all that apply)









5. For the circle shown in Figure 1, what is s if  $\theta$  is  $\pi/4$  rad and r is 5 m?

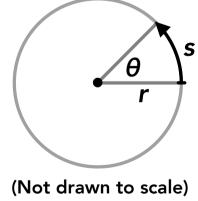


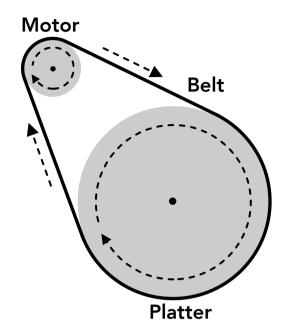
Figure 1

- 6. For the circle shown in Figure 1, what is  $\theta$  (in rad) if s is 2 m and r is 2.5 m?
- 7. For the circle shown in Figure 1, what is r if  $\theta$  is  $\pi/3$  rad and s is 12 m?

| 8.  | On a circular race track with a diameter of 20 m, what is the angular position (in rad) associated with a tangential position of 16 m?   |
|-----|--|
| 9.  | On a circular race track, a tangential position of 30 m corresponds to an angular position of 120°. What is the radius of the track?   |
| 10. | The handle on a door is 0.7 m from the door hinge. When the door is opened, if the handle moves an arc length (tangential displacement) of 0.9 m, what is the angular displacement of the door in deg?   |
| 11. | A person is on an amusement park ride that rotates so that the riders follow a circular path with a diameter of 6 m. The ride begins and turns 12 revolutions clockwise, then stops and turns 7 revolutions counterclockwise, then stops and turns 4 revolutions clockwise. What is the tangential displacement of a person from the beginning to the end of the ride, in m? |
| 12. | While a 30 cm diameter record is playing, a fly sitting on the outer edge of the record covers a tangential displacement of 35.7 m during a song. How many revolutions did the record turn during the song?  |
| 13. | While tightening a bolt, a wrench rotates from an angular position of $\pi/3$ rad to $2\pi/3$ rad. What is the tangential displacement, in cm, of a point on the wrench that is 15 cm away from the center of the bolt?  |
| 14. | Two people are riding on a large carousel. Person A is 5 m from the center and person B is 8 m from the center. If person A experiences a tangential displacement of 50 m during a period of time, what is the tangential displacement of person B during that time?   |

15. Car A is on circular track A with a radius of 0.6 km, and car B is on circular track B with a different radius. During a period of time, car A and car B both drive the same angular displacement. If car A travelled a tangential displacement of 1.2 km, and car B travelled a displacement of 1.8 km, what is the radius of track B in km?

16. A belt-driven record player like the one shown to the right (not drawn to scale) has a rubber belt wrapped around the outside of a motor wheel and the outside of the platter (a spinning plate that the record sits on while playing). As the motor wheel spins it moves the belt which turns the platter and the record. If the diameter of the motor wheel is 22 mm and the diameter of the platter is 330 mm, how many times does the motor wheel have to turn to get the platter to rotate once? (Consider how the belt's displacement relates to the motion of the motor wheel and platter, without slipping).



## Velocity

- 17. Two flies are sitting on a spinning record at different spots but the same distance from the edge of the record. Which of the following is true about the motion of the flies?
  - A They have the same tangential speed and the same angular speed
  - B They have the same tangential speed but different angular speeds
  - C They have different tangential speeds but the same angular speed
  - D They have different tangential speeds and different angular speeds
- 18. Two cars are driving around the same circular track that only has one lane. If Car A has a greater tangential speed than Car B, how are their angular speeds around the track related?
  - A  $\omega_{\text{Car A}} > \omega_{\text{Car B}}$
  - $\mathbf{B} \ \boldsymbol{\omega}_{\mathsf{Car} \; \mathsf{A}} = \boldsymbol{\omega}_{\mathsf{Car} \; \mathsf{B}}$
  - C  $\omega_{Car A} < \omega_{Car B}$
  - D Cannot be determined
- 19. A fly is sitting on a ceiling fan blade, 0.4 m from the center of rotation of the fan. If the tangential velocity of the fly is 8 m/s, what is the angular velocity of the fan in rad/s?

| 20. | A small rock gets stuck in the outer edge of a car's tire, which is 0.6 m in diameter. If the angular velocity of the wheel is 6 rad/s, what is the tangential velocity of the rock (from the tire's rotation, ignoring the car's motion)?  |
|-----|---|
| 21. | A car is driving on a circular track with a tangential velocity of 18 m/s. If the angular velocity swept out by the car is 1.5 rad/s, what is the radius of the track?  |
| 22. | A figure skater spins around with their arms stretched out and each hand is 0.8 m from their center of rotation. If the tangential velocity of their hands is 17 m/s, what is the angular velocity of the skater in rpm?  |
| 23. | If the minute hand on a (12-hour) clock is 120 mm long, what is the tangential speed of a point on the tip of the minute hand in mm/s?  |
| 24. | If a 22 m diameter Ferris wheel is spinning at 5 rpm, what is the tangential displacement of a rider (on the perimeter of the Ferris wheel) over a period of 30 s?  |
| 25. | The International Space Station (ISS) is in a circular orbit around the Earth. It takes 92.5 minutes for the ISS to circle the Earth and complete 1 orbit. If the radius of the Earth is 6371 km (assuming it's a sphere) and the tangential speed of the ISS is 7.66 km/s, how high is the ISS above the ground, in km?  |
| 26. | Two flies are sitting on the same spinning object. Fly A is 4 cm from the center and fly B is 14 cm from the center. If fly A travels a tangential displacement of 44 cm in 2 s, how many cm does fly B travel in 3 s?  |
| 27. | Two cars are on a circular track with 2 lanes. Car A follows the outside lane, a circular path with a radius of 100 m and car B follows the inside lane, a circular path with a radius of 95 m. The cars remain side by side as they drive around the track. If car A is travelling at a tangential speed of 65 km/h, what is car B's tangential speed in km/h? |

## Acceleration

28. A fly is sitting on a vinyl record when the "Start" button is pushed and the record accelerates at 2 rad/s². If the fly experiences a tangential acceleration of 0.2 m/s², how far is the fly from the center of the record? 29. An athlete performs a hammer throw by spinning around while holding a wire with a metal ball at the end, speeding up and eventually releasing the ball. If the ball is 1.5 m from the athlete's center of rotation and their angular acceleration is 4 rad/s<sup>2</sup>, what is the tangential acceleration of the ball? 30. A person riding 3.5 m from the center of a carousel experiences a tangential acceleration of 2 m/s², what is the carousel's angular acceleration in rad/s<sup>2</sup>? 31. A rider on a roller coaster travels through a circular loop with a radius of 9 m, slowing down as they move higher in the loop. If the tangential acceleration of the rider is -7 m/s², what is their angular acceleration in rad/s<sup>2</sup>? 32. A pole vaulter runs and plants the end of the 5 m long pole in the ground, propelling them upwards along a circular path with an initial tangential velocity of 8 m/s. When the pole vaulter reaches the peak 2 s later, their tangential velocity is 5 m/s. What was the angular acceleration of the pole in rad/s² (assuming it was constant)? 33. You're using a medieval slingshot, which involves swinging around a length of string with a rock in a pouch at the end. At first you swing the rock around in a circle with a tangential speed of 25 m/s, but then decide it's a little too fast and slow it down to a tangential speed of 16 m/s over a period of 3 s. If the angular acceleration of the rock and string was -1.5 rad/s<sup>2</sup> during that time, what is the total length of the string and your arm (how far is the rock from the center of rotation)?

| 34. | A car gets a small rock stuck in a tire while driving, which is 21 cm from the center of the wheel. The car is stopped at an intersection, and when the light turns green the car accelerates and the wheels speed up to 450 rpm in 5 s. What is the tangential acceleration of the rock during that period in cm/s² (from the persepective of the wheels rotating only, ignoring the motion of the car)? |
|-----|---|
| 35. | A race car on a 0.6 km diameter circular track is going 54 km/h. When the car is 80 m from the finish line, the driver speeds up such that the car's angular acceleration around the track is a constant 0.01 rad/s². From the moment the car began accelerating, how long did it take to cross the finish line?  |
| 36. | A vial spinning in a lab centrifuge is 0.1 m from the center of rotation. The technician sets the centrifuge to a higher speed, and the vial experiences a tangential acceleration of 2 m/s². During the 4 s it takes to go from one speed to the next, the centrifuge rotates 40 times. What was the original, slower angular speed of the centrifuge in rpm?  |
| 37. | On a hot summer day, you walk over and turn up the speed of your ceiling fan from 200 rpm to 300 rpm. During the period of time that the fan is increasing in speed, a fly sitting 75 cm from the fan's center of rotation travels a tangential displacement of 15 m. What was the angular acceleration of the fan during that time in rad/s²?  |

## **Answers**

| 1. | Α, | C, | D |
|----|----|----|---|
| 2  |    |    |   |

2. C 3. A, C

12.  $119/\pi$  rev or 37.9 rev 13.  $5\pi$  cm or 15.7 cm

11.  $-54\pi$  m or -170 m

4. B, C 5. 3.9 m

6. 0.8 rad

7. 11.5 m

8. 1.6 rad 9. 14.3 m

10. 74°

15. 0.9 km 16. 15 rev

14. 80 m

17. A 18. A

19. 20 rad/s 20. 1.8 m/s

21. 12 m

22. 202.9 rpm

23.  $\pi/15$  mm/s or 0.21 mm/s

24.  $55\pi$  m or 173 m

25. 395 km 26. 231 cm

27. 61.8 km/h

28. 0.1 m 29. 6 m/s<sup>2</sup>

30. 0.6 rad/s<sup>2</sup>

31. -0.8 rad/s<sup>2</sup>

32.  $-0.3 \text{ rad/s}^2$ 

33. 2 m

34.  $63\pi$  cm/s<sup>2</sup> or 198 cm/s<sup>2</sup>

35. 3.9 s

36. 218 rpm

37. 13.7 rad/s<sup>2</sup>

## **Answers - Position and Displacement**

## 1. Answer: A, C, D

When converting between tangential and angular quantities, we can use these equivalent relationships:

1 circumference  $2\pi$  radians 360° 1 revolution ' 1 revolution ' 1 revolution

#### 2. Answer: C

We can use the equation for the circumference of a circle:

 $(20 \text{ m}) = 2\pi r$  r = 3.18 m $C = 2\pi r$ 

## 3. Answer: A, C

The tangential description of motion refers to the motion along the circumference of a circle and uses a unit for length such as m or km.

#### 4. Answer: B, C

The angular description of motion refers to the angle swept out by a rotating object or an object in circular motion and uses a unit for angles such as degrees or radians.

#### 5. **Answer: 3.9 m**

We can use the equation that relates tangential position and angular position:

 $s = r \theta = (5 \text{ m})(\pi/4 \text{ rad}) = 3.9 \text{ m}$ 

#### 6. Answer: 0.8 rad

We can use the equation that relates tangential position and angular position:

 $(2 \text{ m}) = (2.5 \text{ m})\theta$ 

#### 7. Answer: 11.5 m

We can use the equation that relates tangential position and angular position:

 $(12 \text{ m}) = r(\pi/3 \text{ rad})$ r = 11.5 m

### 8. Answer: 1.6 rad

We can use the equation that relates tangential position and angular position. The radius is half of the diameter.

 $(16 \text{ m}) = (20 \text{ m} / 2)\theta$  $\theta = 1.6 \text{ rad}$ 

#### 9. Answer: 14.3 m

First we need to convert the angular position from degrees to radians:

$$\frac{120^{\circ}}{360^{\circ}} \times \frac{2\pi \text{ rad}}{360^{\circ}} = 2\pi/3 \text{ rad}$$

Then we can use the equation that relates the tangential position and angular position:

$$s = r \theta$$
 (30 m) =  $r(2\pi/3 \text{ rad})$   $r = 14.3 \text{ m}$ 

#### 10. Answer: 74°

First we can find the angular displacement of the door in radians:

$$\Delta s = r \Delta \theta$$
 (0.9 m) = (0.7 m) $\Delta \theta$   $\Delta \theta = 1.29$  rad

Then we can convert the angular displacement from radians to degrees:

$$\frac{1.29 \text{ rad}}{2\pi \text{ rad}} \times \frac{360^{\circ}}{2\pi \text{ rad}} = 74^{\circ}$$

#### 11. Answer: $-54\pi$ m or -170 m

The total angular displacement of the person is the sum of the angular displacements. Using counterclockwise as the positive direction:

$$\Delta\theta = (-12 \text{ rev}) + (7 \text{ rev}) + (-4 \text{ rev}) = -9 \text{ rev}$$

Then we can convert the angular displacement from revolutions to radians:

$$\frac{-9 \text{ rev}}{1 \text{ rev}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = -18\pi \text{ rad}$$

Then we can find the tangential displacement from the angular displacement. The radius is half of the diameter.

$$\Delta s = r \Delta \theta = (6 \text{ m} / 2)(-18\pi \text{ rad}) = -54\pi \text{ m}$$

### 12. Answer: $119/\pi$ rev or 37.9 rev

First we can find the angular displacement in radians. The radius is half of the diameter.

$$\Delta s = r \Delta \theta$$
 (35.7 m) = (0.3 m/2) $\Delta \theta$   $\Delta \theta = 238 \text{ rad}$ 

Then we can convert the angular displacement from radians to revolutions:

$$\frac{238 \text{ rad}}{2\pi \text{ rad}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 119/\pi \text{ rad}$$

### 13. Answer: $5\pi$ cm or 15.7 cm

First we can find the angular displacement:

$$\Delta\theta=\theta_{\rm f}-\theta_{\rm i}=(2\pi/3~{\rm rad})-(\pi/3~{\rm rad})=\pi/3~{\rm rad}$$

Then we can use the equation that relates tangential displacement and angular displacement:

$$\Delta s = r \Delta \theta = (15 \text{ cm})(\pi/3 \text{ rad}) = 5\pi \text{ cm}$$

#### 14. Answer: 80 m

The two people are on the same rotating object (the carousel) so they share the same angular motion and have the same angular displacement, but different tangential displacements. First we can find the angular displacement of person A using the radius of person A's circular path (5 m):

$$\Delta s_A = r \Delta \theta_A$$
 (50 m) = (5 m) $\Delta \theta_A$   $\Delta \theta_A = 10 \text{ rad} = \Delta \theta_B$ 

Then we can use that same angular displacement for person B to find their tangential displacement, using the radius of person B's circular path (8 m):

$$\Delta s_{\rm B} = r \Delta \theta_{\rm B} = (8 \text{ m})(10 \text{ rad}) = 80 \text{ m}$$

#### 15. Answer: 0.9 km

The question says both cars travel the same angular displacement so we can find the angular displacement of car A and use that for the angular displacement of car B. Using the unit of km:

$$\Delta s_A = r_A \Delta \theta_A$$
 (1.2 km) = (0.6 km) $\Delta \theta_A$   $\Delta \theta_A = 2 \text{ rad} = \Delta \theta_B$ 

Then we can use that same angular displacement for car B to find the radius of track B:

$$\Delta s_{\rm B} = r_{\rm B} \Delta \theta_{\rm B}$$
 (1.8 km) =  $r_{\rm B}$ (2 rad)  $r_{\rm B} = 0.9$  km

#### 16. Answer: 15 rev

The linear displacement of the belt must match the tangential displacement of points on the edges of the motor wheel and the platter. If the platter rotates once, the belt moves one circumference of the platter and the motor wheel turns so a point on its edge has a tangential displacement of one circumference of the platter. First we can find the circumference of the platter in mm:

$$C_{\text{platter}} = \pi d_{\text{platter}} = \pi (330 \text{ mm}) = 330\pi \text{ mm}$$

That is the tangential displacement of the belt and a point on the edge of the motor wheel when the platter rotates once. We can use that to find the angular displacement of the motor wheel when the platter rotates once:

$$\Delta s_{\text{motor}} = r_{\text{motor}} \Delta \theta_{\text{motor}}$$
 (330 $\pi$  mm) = (22 mm / 2) $\Delta \theta_{\text{motor}}$   $\Delta \theta_{\text{motor}} = 30\pi$  rad

Then we can convert the angular displacement of the motor wheel from radians to revolutions:

$$\frac{30\pi \text{ rad}}{2\pi \text{ rad}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 15 \text{ rev}$$

## **Answers - Velocity**

## 17. Answer: A

Any two points on the same rotating object have the same angular speed. The two points have the same tangential speed if the radius of their circular path is the same (the points are the same distance from the center of rotation). In this case the flies are the same distance from the edge of the record so they must be the same distance from the center, and the radius of their circular paths are the same. The relationship between tangential and angular speed is:  $\mathbf{v_t} = r \boldsymbol{\omega}$ 

#### 18. Answer: A

The relationship between tangential speed and angular speed is:  $v_t = r \omega$ . If the two cars are driving along the same circular track (their paths have the same radius) then the car with a greater tangential speed will have a greater angular speed.

#### 19. Answer: 20 rad/s

We can use the equation that relates tangential velocity and angular velocity:

$$v_t = r \omega$$
 (8 m/s) = (0.4 m) $\omega$   $\omega$  = 20 rad/s

#### 20. Answer: 1.8 m/s

We can use the equation that relates tangential velocity and angular velocity. The radius is half of the diameter.  $v_t = r \omega = (0.6 \text{ m} / 2)(6 \text{ rad/s}) = 1.8 \text{ m/s}$ 

### 21. Answer: 12 m

We can use the equation that relates tangential velocity and angular velocity:

$$v_t = r \omega$$
 (18 m/s) =  $r$ (1.5 rad/s)  $r = 12$  m

## 22. Answer: 202.9 rpm

First we can find the angular velocity of their hands from the tangential velocity:

$$v_t = r \omega$$
 (17 m/s) = (0.8 m) $\omega$   $\omega$  = 21.25 rad/s

Then we can convert the angular velocity from rad/s to rev/min (rpm):

$$\frac{21.25 \text{ rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 202.9 \text{ rev/min (rpm)}$$

### 23. Answer: $\pi/15$ mm/s or 0.21 mm/s

First we can determined the angular speed of the minute hand. The minute hand rotates 1 revolution each hour:

$$\omega = \frac{1 \text{ rev}}{h} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/1800 \text{ rad/s}$$

Then we can find the tangential speed of a point on the tip of the minute hand, using the unit of mm:

$$v_{\rm t} = r \, \omega = (120 \text{ mm})(\pi/1800 \text{ rad/s}) = \pi/15 \text{ mm/s}$$

## 24. Answer: $55\pi$ m or 173 m

First we can convert the angular velocity of the Ferris wheel from rpm to rad/s:

$$\omega = \frac{5 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/6 \text{ rad/s}$$

Then we can find the tangential velocity of the person on the Ferris wheel who is following a circular path with the same diameter and radius (half of the diameter) as the Ferris wheel:

$$v_t = r \omega = (22 \text{ m} / 2)(\pi/6 \text{ rad/s}) = 11 \pi/6 \text{ m/s}$$

Then we can find the tangential displacement of the person using the equation for tangential velocity:

$$v_t = \frac{\Delta s}{\Delta t}$$
  $(11\pi/6 \text{ m/s}) = \frac{\Delta s}{(30 \text{ s})}$   $\Delta s = 55\pi \text{ m}$ 

#### 25. Answer: 395 km

First we can find the angular velocity of the ISS around the earth in rad/s:

$$\omega = \frac{1 \text{ rev}}{92.5 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/2775 \text{ rad/s}$$

Then we can find the radius of the ISS path using the equation that relates tangential and angular velocity:

$$v_t = r \omega$$
 (7.66 km/s) =  $r(\pi/2775 \text{ rad/s})$   $r = 6766 \text{ km}$ 

The radius of that path is equal to the radius of the earth plus the height of the ISS above the ground, so the height of the ISS above the ground is:

$$(6766 \text{ km}) - (6371 \text{ km}) = 395 \text{ km}$$

#### 26. Answer: 231 cm

The flies are on the same rotating object so they have the same angular velocity but different tangential velocites. First we can find the tangential velocity of fly A using the equation for tangential velocity:

$$v_{tA} = \frac{\Delta s_A}{\Delta t_A} = \frac{(44 \text{ cm})}{(2 \text{ s})} = 22 \text{ cm/s}$$

Then we can find the angular velocity of fly A, which is also the angular velocity of fly B:

$$v_{tA} = r_A \omega_A$$
 (22 cm/s) = (4 cm) $\omega_A$   $\omega_A$  = 5.5 rad/s =  $\omega_B$ 

Then we can use the angular velocity of fly B to find the tangential velocity of fly B:

$$v_{tB} = r_B \omega_B = (14 \text{ cm})(5.5 \text{ rad/s}) = 77 \text{ cm/s}$$

Then we can find the tangential displacement of fly B using the equation for tangential velocity:

$$v_{tB} = \frac{\Delta s_B}{\Delta t_B}$$
 (77 cm/s) =  $\frac{\Delta s_B}{(3 \text{ s})}$   $\Delta s_B = 231 \text{ cm}$ 

### 27. Answer: 61.8 km/h

The cars remain side by side which means the cars have the same angular velocities (they complete 1 revolution, or any angular displacement, in the same amount of time). First we can find the angular velocity of car B which is also the angular velocity of car B, using the units of km and h:

$$v_{tA} = r_A \omega_A$$
 (65 km/h) = (0.1 km) $\omega_A$   $\omega_A$  = 650 rad/h =  $\omega_B$ 

Then we can use the angular velocity of car B to find the tangential velocity of car B, using the units of km and h:

$$v_{tB} = r_B \omega_B = (0.095 \text{ km})(650 \text{ rad/h}) = 61.8 \text{ km/h}$$

## **Answers - Acceleration**

#### 28. Answer: 0.1 m

We can find the radius of the fly's path using the equation that relates tangential and angular acceleration:

$$a_t = r \alpha$$
 (0.2 m/s<sup>2</sup>) = r(2 rad/s<sup>2</sup>)  $r = 0.1$  m

## 29. Answer: 6 m/s<sup>2</sup>

We can use the equation that relates tangential and angular acceleration:

$$a_t = r \alpha = (1.5 \text{ m})(4 \text{ rad/s}^2) = 6 \text{ m/s}^2$$

#### 30. Answer: 0.6 rad/s<sup>2</sup>

We can use the equation that relates tangential and angular acceleration:

$$a_t = r \alpha$$
 (0.2 m/s<sup>2</sup>) = (3.5 m) $\alpha$   $\alpha$  = 0.6 rad/s<sup>2</sup>

### 31. Answer: -0.8 rad/s<sup>2</sup>

We can use the equation that relates tangential and angular acceleration:

$$a_t = r \alpha$$
 (-7 m/s<sup>2</sup>) = (9 m) $\alpha$   $\alpha$  = -0.8 rad/s<sup>2</sup>

## 32. Answer: -0.3 rad/s<sup>2</sup>

First we can find the tangential acceleration of the person from the change in tangential velocity:

$$a_{\rm t} = \frac{\Delta v_{\rm t}}{\Delta t} = \frac{v_{\rm tf} - v_{\rm ti}}{\Delta t} = \frac{(5 \text{ m/s}) - (8 \text{ m/s})}{(2 \text{ s})} = -1.5 \text{ m/s}^2$$

Then we can use the equation that relates tangential and angular acceleration:

$$a_t = r \alpha$$
 (-1.5 m/s<sup>2</sup>) = (5 m) $\alpha$   $\alpha$  = -0.3 rad/s<sup>2</sup>

#### 33. **Answer: 2 m**

First we can find the tangential acceleration of the rock from the change in tangential velocity:

$$a_{t} = \frac{\Delta v_{t}}{\Delta t} = \frac{v_{tf} - v_{ti}}{\Delta t} = \frac{(16 \text{ m/s}) - (25 \text{ m/s})}{(3 \text{ s})} = -3 \text{ m/s}^{2}$$

Then we can use the equation that relates tangential and angular acceleration:

$$a_t = r \alpha$$
 (-3 m/s<sup>2</sup>) = r(-1.5 rad/s<sup>2</sup>)  $r = 2$  m

#### 34. Answer: $63\pi$ cm/s<sup>2</sup> or 198 cm/s<sup>2</sup>

First we can convert the final angular velocity of the wheel from rpm to rad/s:

$$\omega_{\rm f} = \frac{450 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 15\pi \text{ rad/s}$$

Then we can find the angular acceleration of the wheel from the change in angular velocity:

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_{\rm f} - \omega_{\rm i}}{\Delta t} = \frac{(15\pi \text{ rad/s}) - (0 \text{ rad/s})}{(5 \text{ s})} = 3\pi \text{ rad/s}^2$$

Then we can find the tangential acceleration from the angular acceleration:

$$a_{\rm t} = r \alpha = (21 \text{ cm})(3\pi \text{ rad/s}^2) = 63\pi \text{ cm/s}^2$$

#### 35. Answer: 3.9 s

First we can find the tangential acceleration of the car from the angular acceleration and radius so that we can use the tangential kinematic equations. The radius is half of the diameter.

$$a_t = r \alpha = (600 \text{ m} / 2)(0.01 \text{ rad/s}^2) = 3 \text{ m/s}^2$$

Then we can convert the initial tangential velocity (54 km/h) from km/h to m/s:

$$v_{ti} = \frac{54 \text{ km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 15 \text{ m/s}$$

Then we can use the kinematic equation below to find the amount of time it takes the car to travel a tangential displacement of 80 m while accelerating. Using 15 m/s for the initial velocity and 0 m for the initial position:

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_tt^2$$
 (80 m) = (0 m) + (15 m/s)t +  $\frac{1}{2}$ (3 m/s<sup>2</sup>)t<sup>2</sup> t = 3.9 s

(We can solve for t using the quadratic formula)

## 36. Answer: 218 rpm

First we can convert the angular displacement from revolutions to radians:

$$\Delta\theta = \frac{40 \text{ rev}}{1 \text{ rev}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 80\pi \text{ rad}$$

Then we can find the angular acceleration of the centrifuge using the tangential acceleration of the vial:

$$a_{\rm t} = r \alpha$$
 (2 m/s<sup>2</sup>) = (0.1 m) $\alpha$   $\alpha$  = 20 rad/s<sup>2</sup>

Then we can find the initial angular velocity using the kinematic equation below, using 0 for the initial position:

$$\theta_{\rm f} = \theta_{\rm i} + \omega_{\rm i} t + \frac{1}{2} \alpha t^2$$
 (80 \pi rad) = (0 rad) + \omega\_{\mathbf{i}}(4 s) + \frac{1}{2}(20 \text{ rad/s}^2)(4 s)^2 \omega\_{\mathbf{i}} = 22.83 \text{ rad/s}

Then we can convert the initial angular velocity from rad/s to rpm:

$$\frac{22.83 \text{ rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 218 \text{ rpm}$$

#### 37. Answer: 13.7 rad/s<sup>2</sup>

First we can find the angular displacement of the fly from the tangential displacement:

$$\Delta s = r \Delta \theta$$
 (15 m) = (0.75 m) $\Delta \theta$   $\Delta \theta = 20$  rad

Then we can convert the initial and final angular velocities from rpm to rad/s:

$$\omega_{\rm i} = \frac{200 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 20\pi/3 \text{ rad/s}$$

$$\omega_{\rm f} = \frac{300 \text{ rev}}{\min} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 10\pi \text{ rad/s}$$

Then we can use the kinematic equation below to find the angular acceleration of the fan:

$$\omega_{\rm f}^2 = \omega_{\rm i}^2 + 2\alpha(\theta_{\rm f} - \theta_{\rm i})$$
 (10 $\pi$  rad/s)<sup>2</sup> = (20 $\pi$ /3 rad/s)<sup>2</sup> + 2 $\alpha$ (20 rad)  $\alpha$  = 13.7 rad/s<sup>2</sup>