



Skill: Inductive/iterative/recursive sequences

Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. For each of the following sequences write down u_2, u_3, u_4 and u_5 . Hence find a formula for u_n .

a. $u_{n+1} = u_n + 5$ ($u_1 = 3$) b. $u_{n+1} = \frac{1}{2}u_n$ ($u_1 = 6$)

c. $u_{n+1} = u_n - 4$ ($u_1 = 5$) d. $u_{n+1} = 3u_n$ ($u_1 = 4$)

2. Write down the first five terms in the following inductively defined sequences:

a. $u_1 = 0, u_2 = 1, u_n = 3u_{n-1} - u_{n-2}$ where $n \geq 3$.

b. $u_1 = 5, u_n = 5 + \frac{1}{10}u_{n-1}$ where $n \geq 2$.

c. $u_1 = 1, u_{n+1} = (n+1)u_n$ where $n \geq 2$.

d. $u_1 = 1, u_2 = 1, u_{n+2} = u_{n+1} + u_n$ where $n \geq 1$.

e. $u_1 = 3, u_2 = -1, u_{n+2} = u_{n+1} - u_n$ where $n \geq 1$.

f. $u_1 = 1, u_2 = 1, u_{n+2} = 2u_{n+1} + 3u_n$ where $n \geq 1$.

3. The sequence u_1, u_2, u_3, \dots where u_1 is a given real number, is given by

$$u_{n+1} = u_n^2 - 8$$

i. Given that $u_2 = u_1$, find the possible values of u_1 in exact form.

ii. Given instead that $u_3 = u_1$, show that $u_1^4 - 16u_1^2 - u_1 + 56 = 0$.

4. The sequence u_1, u_2, u_3, \dots where u_1 is a given real number is defined by

$$u_{n+1} = (u_n + 1)^2 - 6.$$

i. Given that $u_1 = u_2$, find exactly the two possible values of u_1 .

ii. Given instead that $u_3 = u_1$, show that $u_1^4 + 4u_1^3 - 4u_1^2 - 17u_1 + 10 = 0$.