Tailored Tutors

CONTENTS

Tailored Tutors

TID

Issue	Detail	Tally
Didn't read the question	Didn't give units	
	Didn't give the right form	
	Radians/degrees mix up	
	Tangent/normal mix up	
	dp or sf or exact	
Graph	Missing info on graph	
	Forgot it should be upside down	
	No equation of asymptote	
Minus sign	When expanding brackets	
	When adding fractions	
	When rearranging	
Forgot	+c	
	Plus Minus	
Brackets	Used invisible brackets	
	Didn't use brackets at all	
Mis-Copy	Mistake Copying the Q	
	Mistake reading my own writing	
	Miscopy on a page turn	
Squashing	Mistake at the bottom of a page	
Algebra	Individually squared/rooted/logged	
Rules	Made up rules of indices	
	Made up rules of logs	
Trig	Wrong Trig Identity	
Derivatives	Wrong Derivative	
	Forgot the Chain Rule	
Integral	Didn't GDA, got it wrong	
In a test: could have checked but didn't :(		

MORE TID

Issue	Detail	Tally

CHECKING TECHNIQUES

KNOW YOURSELF

Always have your TID sheets to hand when you do maths
Do an extra check for your most common mistakes

Check a graph sketch

Sketch it on a graphical calc.
y intercept labelled?
Asymptote equations stated?
Does the Q ask you to find x intercepts or turning points?

Check a solution to an equation

Sub your answer back in to see if it works
Or, solve the equation on your calculator and compare answers

Check simultaneous equations eg where does $y = 3x$ cross $x^2 + y^2 = 8$

Sub the matched x and y coordinates into both equations to see if they work
Or, plot the graphs on a calculator and find the intersection

Check writing a number in a different form eg write $\dfrac{3}{2-\sqrt{5}}$ in the form $a + b\sqrt{5}$

Type the number into your calc then type the rearranged number in. Compare.

Check writing an expression in a different way eg write $\dfrac{2\sqrt{x}+1}{x}$ in the form $ax^n + +bx^m$

Type the original expression into your calc with any x you like subbed in. Sub the same x into your answer. Compare.

Check the equation of a line

Sub the original (known) point on the line into your equation to see if it works
Sketch your line on a graphical calc - see what its gradient is & whether it goes through the right point

Check the equation of the tangent/normal

Sketch the curve and your tangent/normal equation on your graphical calc and see if it is the tangent/normal at the right point.

Check a derivative or definite integral

Use the $\dfrac{\mathrm{d}}{\mathrm{d}x}$ button at any x value and sub the same value into your derivative. Compare.

Use the $\displaystyle\int_a^b$ button on the calculator

1a: ALGEBRAIC TECHNIQUES

Rules of Indices $a^m \times a^n = a^{m+n}$ $\quad \dfrac{a^m}{a^n} = a^{m-n}$ $\quad (a^m)^n = a^{mn}$

Useful Facts $a^0 = 1$ $\quad a^1 = a$ $\quad (ab)^n = a^n b^n$

Negative & Rational Indices $a^{\frac{1}{m}} = \sqrt[m]{a}$ $\quad a^{-m} = \dfrac{1}{a^m}$ $\quad a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Manipulating Surds $\sqrt{ab} = \sqrt{a}\sqrt{b}$ $\quad \sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Rationalising the Denominator $\dfrac{a}{b\sqrt{c}} \times \dfrac{\sqrt{c}}{\sqrt{c}}$ $\quad \dfrac{a}{b \pm c\sqrt{d}} \times \dfrac{b \mp c\sqrt{d}}{b \mp c\sqrt{d}}$

Difference of Two Squares (DOTS) $a^2 - b^2 = (a-b)(a+b)$

Completing the Square $x^2 + bx + c = \left(x + \dfrac{b}{2}\right) - \left(\dfrac{b}{2}\right)^2 + c$

Turning point of $y = A(x+B)^2 + C$ is $(-B,\ C)$

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The DISCRIMINANT of a Quadratic $(\dots)x^2 + (\dots)x + c$ is $b^2 - 4ac$

If $b^2 - 4ac$ is positive, the quadratic has 2 real distinct roots

If $b^2 - 4ac$ is **N**egative, the quadratic has **N**o real roots

If $b^2 - 4ac$ is zer**O**, the quadratic has **O**ne real repeated root

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Simultaneous Equations find Intersection of Graphs ie where $y = f(x)$ crosses $y = g(x)$

Make the simpler equation $y = \dots$ or $x = \dots$
Sub into the more complicated equation
Solve to find one coordinate & use the simpler equation to get the other coordinate

Quadratic Inequalities $ax^2 + bx + c < 0$ solution = $\{x : \text{root } 1 < x < \text{root } 2\}$

$a > 0$ $\quad ax^2 + bx + c > 0$ solution = $\{x : x < \text{root } 1\} \cup \{x : x > \text{root } 2\}$

Graphical Inequalities

$f(x) < g(x)$ is the set of values of x for which the graph of $f(x)$ is below the graph of $g(x)$
$f(x) < 0$ is the set of values of x for which the graph of $f(x)$ is below the x axis
$f(x) > 0$ is the set of values of x for which the graph of $f(x)$ is above the x axis

Sketching Inequalities

For the inequality $y < f(x)$ or $y > f(x)$ the line $y = f(x)$ is drawn as a dotted line
For the inequality $y \leq f(x)$ or $y \geq f(x)$ the line $y = f(x)$ is drawn as a solid line

1b: ALGEBRAIC TECHNIQUES

An IMPROPER FRACTION has a numerator with a degree equal to or larger than the denominator. These can be converted to mixed fractions using ALGEBRAIC or SYNTHETIC DIVISION (spec requires dividing by $ax + b$ only) or by using EQUATING COEFFICIENTS.

$$\frac{F(x)}{Q(x)} = \text{divisor} + \frac{\text{remainder}}{Q(x)}$$

$$F(x) = (\text{divisor})(Q(x)) + \text{remainder}$$

FACTOR THEOREM if $(ax + b)$ is a factor of $f(x)$ then $f\left(\frac{-b}{a}\right) = 0$

if $f\left(\frac{-b}{a}\right) = 0$ then $(ax + b)$ is a factor of $f(x)$

A fraction with two distinct linear factors in the denominator and a linear or constant numerator can be split into PARTIAL FRACTIONS e.g. $\frac{5}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$

e.g. $\frac{2x+9}{(x-5)(x+3)^2} = \frac{A}{x-5} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

A, B and C are found using EQUATING COEFFICIENTS

The same BINOMIAL EXPANSION works for $n \in \mathbb{N}$ & $n \notin \mathbb{N}$

$(a+b)^n = \ldots \; {}^nC_r(a)^{n-r}(b)^r \; \ldots$

Binomial Coefficient	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$
a powers ↓	$(a)^n$	$(a)^{n-1}$	$(a)^{n-2}$	$(a)^{n-3}$
b powers ↑	1	b	$(b)^2$	$(b)^3$

Complete the table then multiply down each column to get the terms of the expansion

If n is negative or fractional, the expansion only converges to $(a+b)^n$ as $n \to \infty$ if $\left|\frac{b}{a}\right| < 1$

2: SKETCHING GRAPHS

Functions have roots. ROOTS of $f(x)$ are values of x for which $f(x) = 0$

Equations have solutions. SOLUTIONS of $f(x) = 0$ are values of x for which $f(x) = 0$

GRAPH TRANSFORMATIONS

$f(x+a)$ is a translation of $f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

$f(x)+a$ is a translation of $f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$

$f(ax)$ is a stretch of $f(x)$, scale factor $\frac{1}{a}$, parallel to the x axis (about the y axis)

$af(x)$ is a stretch of $f(x)$, scale factor a, parallel to the y axis (about the x axis)

$f(-x)$ is a reflection of $f(x)$ over the y axis
$-f(x)$ is a reflection of $f(x)$ over the x axis

ASYMPTOTES

Graphs go towards their asymptote(s) like a plane landing that never actually touches down.

Vertical asymptotes occur when x cannot be a certain value (ie it would mean dividing by 0)

Horizontal asymptotes show long term behaviour (either for very big positive x or very big negative x or both)

$y = \frac{1}{x}$ & $y = \frac{1}{x^2}$	2 asymptotes, $y = 0$ and $x = 0$
$y = a^x$ & $y = e^x$	1 horizontal asymptote at $y = 0$
$y = \log_a(x)$ & $y = \ln(x)$	1 vertical asymptote at $x = 0$
$y = \frac{ax+b}{cx+d}$	2 asymptotes, one vertical at $x = -\frac{d}{c}$ and one horizontal at $y = \frac{a}{c}$

3: FUNCTIONS

The DOMAIN of a function: Set of allowed input values
A set of x values
Usually given in the question

The RANGE of a function: The set of output values corresponding to the domain
A set of y values or $f(x)$ values
To find the range you need to sketch the graph

VERTICAL LINE TEST: a mapping is a 'function' if all vertical lines cross the graph once

HORIZONTAL LINE TEST: if all horizontal lines cross once, a function is ONE-TO-ONE
if some horizontal lines cross more than once, it is MANY-TO-ONE

INVERSE FUNCTIONS: Only one-to-one functions have inverses

$f^{-1}(x)$ is the inverse of $f(x)$

$ff^{-1}(x) = x$ and $f^{-1}f(x) = x$

$y = f^{-1}(x)$ is a reflection of $y = f(x)$ in the line $y = x$

The domain of $f^{-1}(x)$ is the range of $f(x)$ and vice versa

Finding the inverse function:

1. Change $f(x)$ to y
2. Change all the x to y and vice versa
3. Rearrange to get $y = ...$ (for quadratics, complete the square)
4. Change y to $f^{-1}(x)$
5. Remember the domain of $f^{-1}(x)$ is the range of $f(x)$

COMPOSITE FUNCTIONS $fg(x)$ means apply g to x first, then apply f to the result

$fg(x) = f(g(x))$

The MODULUS FUNCTION $y = |f(x)|$ makes any value positive
When $f(x) \geq 0$, $|f(x)| = f(x)$.
When $f(x) < 0$, $|f(x)| = -f(x)$.

To sketch $y = |f(x)|$, sketch $y = f(x)$ then reflect the section below the x-axis to above it.

To sketch the graph of $y = f(|x|)$, sketch $y = f(x)$ for $x \geq 0$ then reflect in the y-axis.

PARAMETRIC EQUATIONS e.g. $x = p(t)$ and $y = q(t)$

Each value of t defines a point with coordinates $(p(t), q(t))$.

The domain of the cartesian equation is the range of $p(t)$

The range of the cartesian equation is the range of $q(t)$

4: LINES CIRCLES TRIANGLES SECTORS

Useful formulae: $$\text{gradient} = m = \frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{distance } d = \sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$$

$$\text{midpoint} = \left(\text{average } x \text{ coordinate, average } y \text{ coordinate}\right)$$

Line with gradient m through (a, b):

$m(x-a) = y - b$	used to construct the line
$ax + by + c = 0$,	$a, b, c \in \mathbb{Z}$ used for final ans.
$y = mx + c$	used to identify the gradient

PARALLEL lines have the same gradient. $\text{grad } 2 = \text{grad } 1$

PERPENDICULAR lines have negative reciprocal gradients: $\text{grad } 2 = \dfrac{-1}{\text{grad } 1}$

If x & y are DIRECTLY PROPORTIONAL ($y \propto x$) then $y = kx$ (a straight line through the origin)

Equation of circle centre (a, b), radius r $\quad (x-a)^2 + (y-b)^2 = r^2$

$x^2 + y^2 - fx - gy + h = 0 \rightarrow$ complete the sq.

Circle facts: The tangent to a circle is perpendicular to the radius

If A, B, C lie on a circle and $\angle ABC = 90°$, then AC is a diameter of the circle

The perpendicular bisectors of two chords intersect at the centre of the circle

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Cosine Rule (angle A is opposite side a) $\quad a^2 = b^2 + c^2 - 2bc \cos A$

Sine Rule (angle A is opposite side a etc) $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle (angle C is between sides $\mathbf{a}$ and $\mathbf{b}$) $\text{Area} = \dfrac{1}{2}|\mathbf{a}||\mathbf{b}|\sin C$

The ARC LENGTH subtended by the angle θ radians in a circle radius r is $r\theta$

The SECTOR AREA of a sector subtended by the angle θ radians in a circle radius r is $\dfrac{1}{2}r^2\theta$

5: VECTORS

Vectors have magnitude (length) and direction $\begin{pmatrix} a \\ b \end{pmatrix} = a\mathbf{i} + b\mathbf{j}$, $\mathbf{i}$ = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j}$ = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The ANGLE between two vectors is found using the cosine rule

The MAGNITUDE (length) of $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$ is found using Pythagoras: $|\mathbf{a}| = \sqrt{x^2 + y^2}$

The UNIT VECTOR (length = 1) parallel to $\mathbf{a}$ is found by dividing $\mathbf{a}$ by $|\mathbf{a}|$

The DISTANCE from point A to B is the magnitude of the vector $\overrightarrow{AB}$

If $\mathbf{a}$ and $\mathbf{b}$ are PARALLEL, then $\mathbf{a} = \lambda\mathbf{b}$, where λ is a constant.
$\mathbf{a}$ and $-\mathbf{a}$ are parallel and have the same length, but are in opposite directions.

The POSITION VECTOR of a point A is the vector from the origin O to A.

If the position vector of A is $\mathbf{a}$ and the position vector of B is $\mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

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3 dimensional vector geometry is the same as 2 dimensional vector geometry.

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k} \qquad \mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The ANGLE between two vectors is found using the cosine rule

The MAGNITUDE of $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is found using Pythagoras: $|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$

The angle θ_x between $\mathbf{a}$ and the x axis is the angle between $\mathbf{a}$ and $\mathbf{i}$ which is $\dfrac{a_1}{|\mathbf{a}|}$

The angle θ_y between $\mathbf{a}$ and the y axis is the angle between $\mathbf{a}$ and $\mathbf{j}$ which is $\dfrac{a_2}{|\mathbf{a}|}$

The angle θ_z between $\mathbf{a}$ and the z axis is the angle between $\mathbf{a}$ and $\mathbf{k}$ which is $\dfrac{a_3}{|\mathbf{a}|}$

If Q divides $\overrightarrow{AB}$ in the ratio $\lambda : \mu$, $\quad \overrightarrow{OQ} = \overrightarrow{OA} + \dfrac{\lambda}{\lambda + \mu}\overrightarrow{AB}$

for midpoint, use $\lambda = \mu = 1$

6: EXPONENTIALS & LOGS

If a and b are both positive:

$$a^x = b \quad \updownarrow \quad x = \log_a(b) \qquad\qquad e^x = b \quad \updownarrow \quad x = \ln(b)$$

If $y = a^x$, $a > 1$, then as x increases, y increases. This is EXPONENTIAL GROWTH.
If $0 < a < 1$, then as x increases, y decreases. This is EXPONENTIAL DECAY.

An EXPONENTIAL MODEL has the form $y = Ae^{kt} + B$ or $y = Ar^{kt} + B$

If $k > 0$ it models exponential growth, if $k < 0$ it models exponential decay.
As k (either positive or negative) gets closer to 0 the rate of change of y gets slower.
The further from zero k is (either positive or negative), the faster the rate of change of y.

Useful facts: $\log_x(x) = 1$ $\qquad$ $\log_x(1) = 0$

Graphs: The graph of $y = \ln x$ is a reflection of $y = e^x$ in the line $y = x$.
The graph of $y = \ln x$ has a vertical asymptote at $x = 0$
The graph of $y = e^x$ has a horizontal asymptote at $y = 0$

Laws of Logs:

$$\log_x(a) + \log_x(b) = \log_x(ab) \qquad \log_x(a) - \log_x(b) = \log_x\left(\frac{a}{b}\right)$$

$$\log_x(a)^k = k\log_x(a) \qquad \log_x\left(\frac{1}{a}\right) = \log_x(a)^{-1} = -\log_x a$$

Differentiating e^{kx}

$$f(x) = e^{kx} \implies f'(x) = ke^{kx} \qquad y = e^{kx} \implies \frac{\mathrm{d}y}{\mathrm{d}x} = ke^{kx}$$

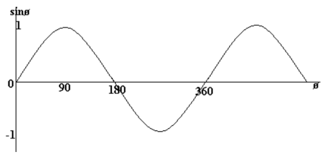
$$f'(x) = kf(x) \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = ky$$

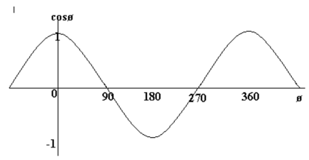
$$f'(x) \propto f(x) \qquad \frac{\mathrm{d}y}{\mathrm{d}x} \propto y$$

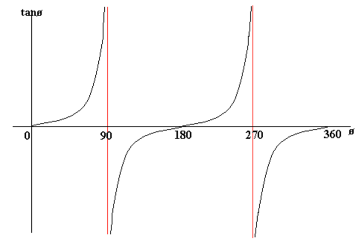
$y = ax^m$ can be rearranged to give $\log y = m\log x + \log a$.
If $y = ax^n$, the graph of $\log y$ against $\log x$ is a straight line: gradient $= m$
vertical intercept $= \log a$.

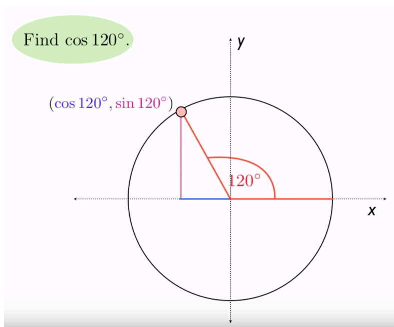
$y = ab^x$ can be rearranged to give $\log y = x\log b + \log a$.
If $y = ab^x$, the graph of $\log y$ against x is a straight line: gradient $= \log b$
vertical intercept $= \log a$.

7a: TRIGONOMETRY









For a point (x, y) on the unit circle with angle θ from the positive x axis:

$$\cos\theta = x \text{ coordinate}$$
$$\sin\theta = y \text{ coordinate}$$
$$\tan\theta = \frac{y}{x} = = \frac{\sin\theta}{\cos\theta} = \text{gradient}$$

Solutions to $\sin\theta = a$ and $\cos\theta = x$ only exist for $-1 \leq a \leq 1$.
Solutions to $\tan\theta = a$ exist for all $a \in \mathbb{R}$.

To solve the mini-trig equation $\sin\theta = a$

(1) Use the calculator to find the first solution $\theta_1 = \sin^{-1}(a)$
(2) The second solution is $\theta_2 = 180 - \theta_1$
(3) Now ±360 as many times as you like to θ_1 and θ_2 to get more solutions.

To solve $\sin\theta = a$: solution 1 is $\theta_1 = \sin^{-1}(a)$, solution 2 is $\theta_2 = 180 - \theta_1$ then ±360

To solve $\cos\theta = a$: solution 1 is $\theta_1 = \cos^{-1}(a)$, solution 2 is $\theta_2 = -\theta_1$ then ±360

To solve $\tan\theta = a$: solution 1 is $\theta_1 = \tan^{-1}(a)$, solution 2 is $\theta_2 = 180 + \theta_1$ then ±360

To solve the mini-trig equation $\sin(\ldots) = a$ (or cos or tan)

(1) Put $y = \ldots$
(2) Solve $\sin y = a$ as above and ±360 to get many y solutions
(3) Rearrange $y = \ldots$ to get the θ solutions in the required range

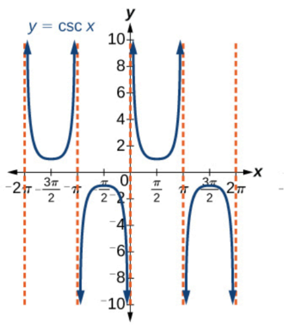
7b: TRIGONOMETRY

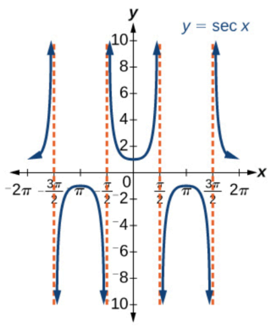
2π radians $= 360°$ ($\approx$ 6 rads) $\quad$ π radians $= 180°$ ($\approx$ 3 rads) $\quad$ 1 radian $\approx 57°$

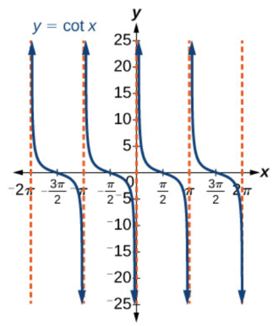
$30° = \dfrac{\pi}{6}$ radians $\quad$ $45° = \dfrac{\pi}{4}$ radians $\quad$ $60° = \dfrac{\pi}{3}$ radians

When θ is small and measured in radians: $\sin\theta \approx \theta$ $\quad$ $\tan\theta \approx \theta$ $\quad$ $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$

$$\text{cosec}\,x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$







3 PYTHAGOREAN IDENTITIES

$\sin^2 x + \cos^2 x \equiv 1$

$1 + \cot^2 x \equiv \text{cosec}^2 x$ $\quad$ + proof

$\tan^2 x + 1 \equiv \sec^2 x$ $\quad$ + proof

3 ADDITION FORMULAE

$\sin(A+B) \equiv \sin A \cos B + \cos A \sin B$

$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$

$\tan(A+B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ + proof

3 DOUBLE/HALF ANGLE

$\sin 2A \equiv 2 \sin A \cos A$ $\quad$ + proof

$\cos 2A \equiv \cos^2 A - \sin^2 A$ $\quad$ + proof

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$ $\quad$ + proof

4 BONUS DOUBLE/HALF ANGLE FORMULAE

$\cos 2A \equiv 2\cos^2 A - 1$ $\quad$ + proof $\quad$ $\cos 2A \equiv 1 - 2\sin^2 A$ $\quad$ + proof

$\cos^2 x \equiv \dfrac{1}{2}(1 + \cos 2x)$ $\quad$ + proof $\quad$ $\sin^2 x \equiv \dfrac{1}{2}(1 - \cos 2x)$ $\quad$ + proof

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$a \sin x + b \cos x$ can be written in the HARMONIC FORM $R\cos(x + a)$

- this helps to solve equations of the form $a \sin x + b \cos x = k$
- this also helps find the max and min of functions $y = a \sin x + b \cos x$

8a: DIFFERENTIATION

NOTATION

$$y = ax^n \quad \updownarrow \quad \frac{dy}{dx} = anx^{n-1} \quad \updownarrow \quad \frac{d^2y}{dx^2} = an(n-1)x^{n-2}$$

$$f(x) = ax^n \quad \updownarrow \quad f'(x) = anx^{n-1} \quad \updownarrow \quad f''(x) = an(n-1)x^{n-2}$$

Differentiation from FIRST PRINCIPLES: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

The TANGENT to the curve $y = f(x)$ at the point $(\alpha, f(\alpha))$: $f'(\alpha)(x - \alpha) = y - f(\alpha)$

The NORMAL to the curve $y = f(x)$ at the point $(\alpha, f(\alpha))$: $-\dfrac{1}{f'(\alpha)}(x - \alpha) = y - f(\alpha)$

$f'(a) > 0$ $f(x)$ is INCREASING at $x = a$ (going up/positive gradient)

$f'(a) < 0$ $f(x)$ is DECREASING at $x = a$ (going down/negative gradient)

$f'(a) = 0$ $x = a$ is a STATIONARY POINT. The gradient of the tangent to the function at $x = a$ is zero. It could be a maximum, a minimum or point of inflection.

$f(x)$ is CONCAVE for a given interval if $f''(x) \le 0$ for every value of x in the interval.

$f(x)$ is CONVEX for a given interval if $f''(x) \ge 0$ for every value of x in the interval.

At a MINIMUM point y is going down, then stops, then goes up

$\dfrac{dy}{dx}$ is negative, then $\dfrac{dy}{dx} = 0$, then $\dfrac{dy}{dx}$ is positive

$\dfrac{dy}{dx}$ is going up, so $\dfrac{d^2y}{dx^2}$ is positive

At a MAXIMUM point y is going up, then stops, then goes down

$\dfrac{dy}{dx}$ is positive, then $\dfrac{dy}{dx} = 0$, then $\dfrac{dy}{dx}$ is negative

$\dfrac{dy}{dx}$ is going down, so $\dfrac{d^2y}{dx^2}$ is negative

At a POINT OF INFLECTION, the graph changes from CONVEX to CONCAVE (or vice versa)

ie) $\dfrac{d^2y}{dx^2}$ just before the point is positive, $\dfrac{d^2y}{dx^2}$ just after the point is negative (or vice versa)

A point of inflection may or may not be a stationary point.

8b: DIFFERENTIATION

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$$

1st Principles

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$$

Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec x = -\sec x \tan x$$

Chain or Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

1st Principles

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\operatorname{cosec}^2 x$$

Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

Chain or Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{kx} = ke^{kx}$$

No proof needed

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$$

No proof needed

$$\frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln a \quad \text{for } a > 0$$

Implicit Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

1st Principles

CHAIN RULE

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

where y is a function of u & u is a function of x

$$\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$$

eg PARAMETRIC DIFFERENTIATION $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

eg IMPLICIT DIFFERENTIATION $$\frac{\mathrm{d}}{\mathrm{d}x}(f(y)) = f'(y)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(y^n) = ny^{n-1}\frac{\mathrm{d}y}{\mathrm{d}x}$$

PRODUCT RULE

$$\frac{\mathrm{d}}{\mathrm{d}x}(uv) = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

where u and v are functions of x.

$$f(x) = g(x)h(x) \implies f'(x) = g(x)h'(x) + h(x)g'(x)$$

eg IMPLICIT DIFFERENTIATION $$\frac{\mathrm{d}}{\mathrm{d}x}(xy) = x\frac{\mathrm{d}y}{\mathrm{d}x} + y$$

QUOTIENT RULE

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

where u and v are functions of x.

$$f(x) = \frac{g(x)}{h(x)} \implies f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2}$$

9: INTEGRATION +c

NOTATION

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^n \quad (n \neq -1) \qquad\qquad f'(x) = x^n \quad (n \neq -1)$$

$$y = \frac{1}{n+1}x^{n+1} + c \qquad\qquad f(x) = \frac{1}{n+1}x^{n+1} + c$$

Find c: Substitute both x and y values (coordinates of a point on the curve or the value of the function at a given point) into the integrated function. Solve the equation.

The AREA between the positive section of the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by $\int_a^b f'(x)\ \mathrm{d}x = [f(x)]_a^b = f(b) - f(a)$.

If the curve $y = f(x)$ is below the x-axis between $x = a$ and $x = b$, the integral $\int_a^b y\ \mathrm{d}x$ will be negative. Remember though, the <u>area</u> is positive.

If the graph of $y = f(x)$ is above the graph of $y = g(x)$ then the area between the graphs of $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ is given by $\int_a^b f(x) - g(x)\ \mathrm{d}x$.

GDA Guess the answer - Differentiate your guess (usually using the chain rule) - Adjust

POWERS $\int f'(x)(f(x))^n\ \mathrm{d}x$ Guess $(f(x))^{n+1}$

FRACTIONS $\int \frac{f'(x)}{f(x)}\ \mathrm{d}x$ Guess $\ln|f(x)|$

$\int \frac{\text{linear or higher}}{ax+b}\ \mathrm{d}x$ Use Algebraic Division

$\int \frac{\text{lower power}}{\text{factorised thing}}\ \mathrm{d}x$ Use Partial Fractions

TRIG

$\int \cos^2 x\ \mathrm{d}x$ Use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\int \tan^2 x\ \mathrm{d}x$ Use $\tan^2 x = \sec^2 x - 1$

$\int \sin^2 x\ \mathrm{d}x$ Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\int \cot^2 x\ \mathrm{d}x$ Use $\cot^2 x = \mathrm{cosec}^2 x - 1$

SUBSTITUTION Change $\mathrm{d}x$ to $\left(\frac{\mathrm{d}x}{\mathrm{d}u}\right)$ $\mathrm{d}u$ - Sub in - Change limits

PARTS $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\ \mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\ \mathrm{d}x$

SEPARATION OF VARIABLES

Multiply both sides by $\mathrm{d}x$ - Multiply/divide anything on the 'wrong' side - Put on integral signs
Integrate both sides & don't forget +c

10: SEQUENCES & SERIES

A SERIES is the sum of the terms of a sequence $S_n = \sum_{k=1}^{n} u_n$

RECURRENCE RELATIONS $u_{n+1} = f(u_n)$ define each term as a function of the previous term.

INCREASING SEQUENCE $u_{n+1} > u_n$ for all $n \in \mathbb{N}$

DECREASING SEQUENCE $u_{n+1} < u_n$ for all $n \in \mathbb{N}$

PERIODIC SEQUENCE

the terms repeat in a cycle

there is an integer k for which $u_{n+k} = u_n$ for all $n \in \mathbb{N}$

k = the ORDER of the sequence.

ARITHMETIC SEQUENCE

a constant difference between consecutive terms.

a = first term d = common difference

n^{th} term $= a + (n-1)d$

Sum of 1st n terms = $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

OR $S_n = \frac{n}{2}(a + l)$ l = the last (n^{th}) term $= a + (n-1)d$

GEOMETRIC SEQUENCE

a common ratio between consecutive terms.

a = first term r = common ratio

$u_n = ar^{n-1}$

Sum of 1st n terms = $S_n = \frac{a}{1-r}(1 - r^n)$, $r \neq 1$

If $|r| < 1$ the sequence is CONVERGENT (it converges to 0)

Sum of ∞ many terms = $S_\infty = \frac{a}{1-r}$ provided $|r| < 1$

To solve arithmetic & geometric sequences & series problems

1. Translate the information into equations
2. Use simultaneous equations

11: NUMERICAL METHODS

CHANGE OF SIGN (root)

$f(a) = \ldots$
$f(b) = \ldots$ (at least 3sf accuracy)

"The function $f(x)$ is continuous on the interval $[a, b]$ and changes sign between $f(a)$ and $f(b)$, therefore the function has at least one root for $a < x < b$"

CHANGE OF SIGN (turning point)

$f'(a) = \ldots$
$f'(b) = \ldots$ (at least 3sf accuracy)

"The function $f'(x)$ is continuous on the interval $[a, b]$ and changes sign between $f(a)$ and $f(b)$, therefore the function $f'(x)$ has at least one root for $a < x < b$ which means $f(x)$ has at least one turning point for $a < x < b$"

CHANGE OF SIGN (intersection)

$h(a) = f(a) - g(a) = \ldots$
$h(b) = f(b) - g(b) = \ldots$ (at least 3sf accuracy)

"The function $h(x) = f(x) - g(x)$ is continuous on the interval $[a, b]$ and changes sign between $h(a)$ and $h(b)$, therefore the function $h(x)$ has at least one root for $a < x < b$ which means $y = f(x)$ crosses $y = g(x)$ at least once for $a < x < b$"

ITERATIVE METHOD

Rearrange $f(x) = 0$ into the form $x = g(x)$
Use the iterative formula $x_{n+1} = g(x_n)$

NEWTON-RAPHSON

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Form the equation of the tangent to $y = f(x)$ at $x = x_n$

Sub in $y = 0$ and rearrange to $x_n = \ldots$

TRAPEZIUM RULE

$$\int_a^b y \, \mathrm{d}x = \lim_{\delta x \to 0} \sum_{x=a}^{x=b} f(x)\, \delta x$$
$$\approx \frac{1}{2} h \left[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n \right]$$

12: PROOF

$x \in \mathbb{N}$ x is a NATURAL number 0, 1, 2, 3, 4, … All natural numbers are integers: $\mathbb{N} \subset \mathbb{Z}$

$x \in \mathbb{Z}$ x is an INTEGER 0, ±1, ±2, ±3, ±4, … All integers are rational: $\mathbb{Z} \subset \mathbb{Q}$

$x \in \mathbb{Q}$ x is a RATIONAL number: $x = \frac{m}{n}$, $m, n \in \mathbb{Z}$ All rational numbers are real: $\mathbb{Q} \subset \mathbb{R}$

$x \in \mathbb{R}$ x is a REAL number (all numbers not involving $\sqrt{-1}$)

$\text{Thing } 1 \Rightarrow \text{Thing } 2$ If thing 1 is true, then thing 2 is definitely true

$\text{Thing } 1 \Leftarrow \text{Thing } 2$ If thing 2 is true, then thing 1 is definitely true

$\text{Thing } 1 \Leftrightarrow \text{Thing } 2$ Thing 1 and thing 2 are EQUIVALENT
If thing 1 is true then thing 2 is definitely true
and also if thing 2 is true then thing 1 is definitely t

$\text{Thing } 1 \equiv \text{Thing } 2$ Thing 1 and thing 2 are the same for all values of the unknown
eg $x^2 + 3x \equiv x(x+3)$ because it's always true
… but $x^2 + 3x = x(x+1)$ because it's not always true

Interval Notation $x \in [a, b]$ means $a \leq x \leq b$ $x \in [a, b)$ means $a \leq x < b$

$x \in (a, b)$ means $a < x < b$ $x \in (a, b]$ means $a < x \leq b$

Set Notation $\{x \in \mathbb{R} : x < a\} \cup \{x \in \mathbb{R} : x > b\}$ means $x < a$ OR $x > b$
$\{x \in \mathbb{R} : x < a\} \cap \{x \in \mathbb{R} : x > b\}$ means $x < a$ AND ALSO $x > b$

A mathematical proof involves
Stating any assumptions made
Showing every step clearly
Having every step follow on logically from the previous step
Covering all possible cases
Writing a statement of proof at the end

To prove an identity you should
Begin with one side of the identity
Manipulate it until it matches the other side
Show every step of your working

You can prove a mathematical statement by EXHAUSTION: breaking the statement into smaller cases and proving each case separately (eg prove for odds then evens)

You can disprove a mathematical statement by COUNTER-EXAMPLE: give one example that does not work for the given statement

PROOF BY CONTRADICTION

1. Start by assuming statement is not true in words
2. Write what this means *mathematically*
3. Using logical steps, show that this assumption leads to a contradiction
4. Conclude that the assumption is incorrect, so the original statement is true

1: LDS & SAMPLING

POPULATION:	whole set of items of interest.
CENSUS:	measures every individual in a population.
SAMPLE:	a selection of observations from a subset of the population, which is extrapolated to estimate information about the whole population.

SIMPLE RANDOM SAMPLE:	every individual in a population is equally likely to be selected.
SYSTEMATIC SAMPLING:	individuals are chosen at regular intervals from an ordered list.
STRATIFIED SAMPLING:	population is divided into strata/groups & random samples taken.
QUOTA SAMPLING:	sample that reflects the characteristics of the population is chosen.
OPPORTUNITY SAMPLING:	sample is chosen from suitable individuals available at the time.

Large Data Set Key Features (eg outliers, units of data)

2: DATA REPRESENTATION

The MIDPOINT is the average of the upper and lower class boundaries.

The CLASS WIDTH is the difference between the upper and lower class boundaries.

The RANGE measures spread. It is the difference between the largest and smallest values.

The IQR is a measure of spread. It is the difference between the upper and lower quartiles.

VARIANCE is a measure of spread. It is the average distance from each data point to the mean

$$\frac{\sum (x-\bar{x})^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2 \qquad \text{Standard Deviation} = \sqrt{\text{Variance}}$$

The MEAN can be calculated using the formula $\bar{x} = \dfrac{\sum x}{n}$

The MEDIAN is the middle value when the data values are put in ascending order.

Ungrouped data: For the lower quartile, calculate $\dfrac{n}{4}$. For the 7th decile, calculate $\dfrac{7n}{10}$ etc
If this is a whole number, the data point you need is halfway between this point and the point above. If not, round up.

Grouped data: Use LINEAR INTERPOLATION to find quantiles

An OUTLIER is any value greater than (eg) $Q_3 + \frac{3}{2}(\text{IQR})$ or less than $Q_1 - \frac{3}{2}(\text{IQR})$
Removing these values 'CLEANS' the data.

On a histogram: $\text{frequency density} = \dfrac{\text{frequency}}{\text{width}} \times k$

Joining the middle of the top of each bar on the histogram forms a FREQUENCY POLYGON.

3: CORRELATION & REGRESSION

BIVARIATE DATA has pairs of two variables, allowing scatter graphs to be drawn.

CORRELATION (and correlation coefficients) describe the relationship between two variables.
Weak/Strong - Positive/Negative - As …… increases, …….. increases/decreases

PMCC Product Moment Correlation Coefficient ρ

ρ describes the strength of the linear correlation (weak/strong, positive/negative)
$-1 \leq \rho < 0$ Negative correlation, from strong (near -1) to weak (near 0)
$0 < \rho \leq -1$ Positive correlation, from weak (near 0) to strong (near 1)

LINEARLY CORRELATED variables have scatter graphs with points lying close to a line

The REGRESSION LINE of p on t, in the form $p = a + bt$, is the line of best fit of a scatter graph.

If variables are linearly correlated, the regression line is a reliable way to estimate the RESPONSE/DEPENDENT variable using the EXPLANATORY/INDEPENDENT VARIABLE (not the other way round)

4: PROBABILITY

TREE DIAGRAMS shows the outcomes of two or more events occurring in succession.

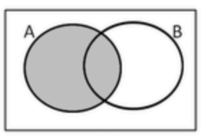
VENN DIAGRAMS shows a graphic representation of two or more events.

TWO WAY TABLES shows 2 sets of mutually exclusive evens such as eye colour (blue/brown/other) and hair colour (blonde/brown/other)

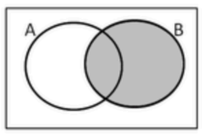
PROBABILITY DISTRIBUTION a table or formula showing all of the possible outcomes and their associated probabilities.

The sum of all probabilities is 1: $\sum P(X = x) = 1$

CUMULATIVE PROBABILITY the probability of obtaining up to <u>and including</u> the outcome.



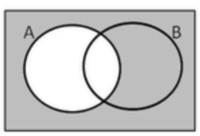
$P(A)$



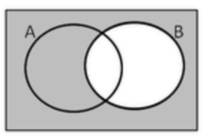
$P(B)$

Useful formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



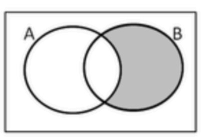
$P(A')$



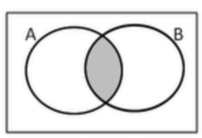
$P(B')$

If A and B are MUTUALLY EXCLUSIVE, $P(A \cap B) = 0$

Therefore, $P(A \cup B) = P(A) + P(B)$



$P(A' \cap B)$



$P(A \cap B)$

If A and B are INDEPENDENT, $P(A \cap B) = P(A) \times P(B)$

Therefore, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

The probability that B occurs <u>given that</u> A is known to have occurred is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Typical Questions: "Given that A, find the probability of B"
"Find the probability of B, given that A"
"A thing is chosen at random & found to be A. Find the probability it is B"

INDEPENDENT events

$$P(A \mid B) = P(A \mid B') = P(A)$$
$$P(B \mid A) = P(B \mid A') = P(B)$$
$$P(A \cap B) = P(A) \times P(B)$$

5: BINOMIAL DISTRIBUTION

X = The number of …. out of n

$X \sim \mathrm{B}(n, p)$

Conditions for a binomial distribution

- two possible outcomes
- a fixed number of trials, n
- fixed probability of success, p
- trials are independent

The probability of an individual outcome: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

The BINOMIAL COEFFICIENT $\binom{n}{x} = \dfrac{n!}{x!(n-r)!}$

= the no. of ways x things can be chosen from a list of n things

$P(X \leq x)$ is found using your calculator

$$P(X < x) = P\big(X \leq (x-1)\big)$$
$$P(X \geq x) = 1 - P(X < x)$$
$$P(X > x) = 1 - P(X \leq x)$$

6: THE NORMAL DISTRIBUTION

$X \sim \mathrm{N}(\mu, \sigma^2)$ μ = population mean, σ^2 = population variance

$Z \sim \mathrm{N}(0, 1)$ the STANDARD NORMAL distribution $Z = \dfrac{X-\mu}{\sigma}$

KEY FEATURES:

Used to model continuous data
Area under curve = 1
Bell shaped curve, asymptotic at each end
Symmetric about the mean ($\text{mean} = \text{median} = \text{mode}$
$P(X = a) = 0$ for any individual value a
Between $\mu \pm 1\sigma \approx 68\,\%$ of the data
Between $\mu \pm 2\sigma \approx 95\,\%$ of the data
Between $\mu \pm 3\sigma \approx 99.7\,\%$ of the data
The distribution has inflection points at $\mu \pm \sigma$

CALCULATOR INSTRUCTIONS:

TT TOP TIP: Always draw a little sketch and shade the part you want to know/do know
If you don't know μ and/or σ, use 2 horizontal axes. One for X and one for Z.

APPROXIMATING THE BINOMIAL Only works if n is large and $p \approx 0.5$

$$\mathrm{B}(n, p) \to \mathrm{N}(np, np(1-p))$$

The CONTINUITY CORRECTION (Edexcel and OCR only) must be applied - use a number line!

7: HYPOTHESIS TESTING

A hypothesis test determines whether or not there is sufficient evidence to claim that a population parameter is not what it was previously thought to be. If the probability of the result OR MORE EXTREME **is less** than the significance level, the result **is significant** and H_0 **is rejected**

NULL HYPOTHESIS	$\mathrm{H}_0 : \ \ldots$	what we currently think the parameter is
ALTERNATIVE HYPOTHESIS	$\mathrm{H}_1 : \ \ldots$	what someone is claiming (and we are testing)

CRITICAL VALUES	the first value(s) of the test statistic X to fall inside the critical region
CRITICAL REGION	values of X for which H_0 would be rejected
ACCEPTANCE REGION	values of X for which H_0 would not be rejected
SIGNIFICANCE LEVEL	probability of incorrectly rejecting H_0 (probability of critical region)

BINOMIAL DISTRIBUTION $X \sim B(n, p)$

Population parameter is p $\quad \mathrm{H}_0 : p = \ldots$

One-tailed test:	$\mathrm{H}_1 : p < \ldots$	find $P(X \leq a)$, compare with the significance level
	$\mathrm{H}_1 : p > \ldots$	find $P(X \geq a)$, compare with the significance level
Two-tailed test:	$\mathrm{H}_1 : p \neq \ldots$	find either $P(X \leq a)$ if a is 'weirdly small' or $P(X \geq a)$ if a is 'weirdly big'. Compare with half the significance level.

NORMAL DISTRIBUTION $X \sim \mathrm{N}(\mu, \sigma^2)$

Population parameter is μ $\quad \mathrm{H}_0 : \mu = \ldots$

Consider $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$

One-tailed test:	$\mathrm{H}_1 : \mu < \ldots$	find $P(\bar{X} \leq a)$, compare with the significance level
	$\mathrm{H}_1 : \mu > \ldots$	find $P(\bar{X} \geq a)$, compare with the significance level
Two-tailed test:	$\mathrm{H}_1 : \mu \neq \ldots$	find either $P(\bar{X} \leq a)$ if a is 'weirdly small' or $P(\bar{X} \geq a)$ if a is 'weirdly big'. Compare with half the significance level

LINEAR CORRELATION

Population parameter is ρ $\quad \mathrm{H}_0 : \rho = 0$

One-tailed test:	$\mathrm{H}_1 : \rho < \ldots$	find the critical value of ρ for the values of n and α. Compare with the PMCC. $\mathrm{PMCC} > \alpha$ is significant
	$\mathrm{H}_1 : p > \ldots$	find the critical value of ρ for the values of n and α. Compare NEGATIVE α with the PMCC. $\mathrm{PMCC} < -\alpha$ is a significant result
Two-tailed test:	$\mathrm{H}_1 : p \neq \ldots$	find the critical value of ρ for the values of n and 0.5α. Compare the PMCC with α if the PMCC is 'weirdly big positive', or with $-\alpha$ if the PMCC 'weirdly big negative'. A significant result is $\mathrm{PMCC} > \alpha$ or $\mathrm{PMCC} < -\alpha$

8: FORCE-ACCELERATION PROBLEMS

NEWTON'S 1st LAW: an object at rest will stay at rest, an object with constant velocity will move at that velocity unless a resultant force acts upon it.

NEWTON'S 2nd LAW: $\mathbf{F}$ is force, measured in Newtons (N), m is mass, measured in kg, $\mathbf{a}$ is acceleration, with magnitude measured in ms^{-2}

$\sum \mathbf{F} = m\mathbf{a}$ The sum of the forces in a particular direction is equal to the mass times the acceleration in that direction

NEWTON'S 3rd LAW: For every action there is an equal and opposite reaction.

RESOLVING FORCES: The unresolved force is between the perpendicular directions

A force of magnitude F at angle θ to the direction of motion

component $F\cos\theta$ in the direction of the motion

component $F\sin\theta$ perpendicular to this direction

RESULTANT FORCE the sum of all forces acting on an object.
If there is a resultant force acting on an object, it will accelerate in the direction of the resultant force.

If 3 forces act on an object in equilibrium, you can form a TRIANGLE OF FORCES and use the cosine rule to find missing information.

If a particle is in EQUILIBRIUM the resultant moment and the resultant force are both zero

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WEIGHT $\mathbf{W} = m\mathbf{g}$ Weight is equal to the mass m kg times the force of gravity $\mathbf{g}$
Weight *only* acts on the body with that mass
The magnitude of the weight is equal to the mass m kg times the acceleration $g = 9.81 \text{ ms}^{-2}$ (which is due to the force of gravity $\mathbf{g}$)

TENSION Occurs when a string or rod is in tension

THRUST Occurs when a rod is in compression

FRICTION Occurs between an object and a non-smooth surface

$F_{\max} = \mu R$ Opposes motion (or potential motion) and acts parallel to the surface

Starts at 0 and increases to prevent motion until it reaches $F_{\max}$

When friction reaches $F_{\max}$, the body is on the point of moving

While the body is moving, friction is $F_{\max}$

NORMAL REACTION Occurs between an object and a surface
It acts perpendicular to the surface

If particles are connected by an INEXTENSIBLE STRING, their accelerations are equal

If particles are connected over a SMOOTH PULLEY, the tensions in the string are equal

If a string is LIGHT you do not need to consider its weight

9: CONSTANT ACCELERATION

DISPLACEMENT — A vector. The magnitude of displacement is called DISTANCE
Displacement is the area under a velocity-time graph

VELOCITY — A vector. The magnitude of velocity is called SPEED
Velocity is the gradient of a displacement-time graph

ACCELERATION — A vector. The magnitude of acceleration is also called acceleration
Acceleration is the gradient of a velocity-time graph

SUVAT

(1) Decide which way is positive (up or down, left or right)

s	u	v	a	t

(2) Complete the table/tables.
- Be careful with **s**, **u**, **v**, **a** ± signs
- State which particle/section of motion each table is for
- Consider the links: t for one particle may be t-2 for the other
- For vertical motion under gravity, $a = 9.81$ downwards

(3) Write down the equation without the thing you're not interested in

$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad s = vt - \frac{1}{2}at^2$$

(4) Sub in clearly then solve/use simultaneous equations to solve

MISTAKES

(1) Not thinking about which way is positive (up or down, left or right)
(2) Getting the signs wrong on some of **s**, **u**, **v**, **a**
(3) Doing a suvat for AC when the acceleration is not constant on AC
(4) Putting in distance not displacement or speed not velocity

PROJECTILES

Horizontally — $u = v$, $a = 0$ the only equation is $s = ut$ (use right as positive)
Velocity $U \cos \alpha$, α measured from the positive horizontal

Vertically — $a = 9.81 \text{ ms}^{-2}$ downwards (use up as positive)
Initial velocity $U \sin \alpha$, α measured from the positive horizontal

Flight Equation — (I) Vertically use $s = y,\ u = U \sin \alpha,\ a = -9.81$
(II) Horizontally use $s = x,\ u = U \cos \alpha$
Rearrange (II) into $t = ...$ and sub into (I) carefully

$$y = x \tan \alpha - \frac{gx^2(1 + \tan^2 \alpha)}{2U^2}$$

Time of Flight From Ground ↕

s	u	v	a	t
0	$U \sin \alpha$		-9.81	T

$$T = \frac{2U \sin \alpha}{g}$$

Time to Greatest Height ↕

s	u	v	a	t
	$U \sin \alpha$	0	-9.81	T

$$T = \frac{U \sin \alpha}{g}$$

Horizontal Range From Ground ↔

s	u	v	a	t
	$U \cos \alpha$			$\frac{2U \sin \alpha}{g}$

$$s = \frac{U^2 \sin 2\alpha}{g}$$

use $2 \sin \alpha \cos \alpha = \sin 2\alpha$

Tailored Tutors

10: VARIABLE ACCELERATION

Differentiate ↓		Integrate + c ↑
	s or x (displacement)	
	v (velocity)	
	a (acceleration)	

The constant of integration, c, can be found by substituting in a known displacement/velocity.

The <u>change in displacement</u> from time t_1 to time t_2 is $\int_{t_1}^{t_2} v(t)\ \mathrm{d}t$.

If the particle doesn't change direction from time t_1 to time t_2, then the change in displacement is the same as the distance travelled.

The <u>change in velocity</u> from time t_1 to time t_2 is $\int_{t_1}^{t_2} a(t)\ \mathrm{d}t$.

Don't forget: displacement, velocity and acceleration are vectors, so can be positive or negative.

Differentiate ↓		Integrate $+\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ ↑
	$\mathbf{r}$ or $\mathbf{x}$ (displacement)	
	$\mathbf{v}$ or $\dot{\mathbf{r}}$ or $\dot{\mathbf{x}}$ (velocity)	
	$\mathbf{a}$ or $\ddot{\mathbf{r}}$ or $\ddot{\mathbf{x}}$ (acceleration)	

The constant of integration, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ is found by substituting in a known displacement/velocity.

The <u>change in displacement</u> from time t_1 to time t_2 is $\int_{t_1}^{t_2} \mathbf{v}(t)\ \mathrm{d}t$.

If the particle doesn't change direction from time t_1 to time t_2, then the change in displacement is the same as the distance travelled.

The <u>change in velocity</u> from time t_1 to time t_2 is $\int_{t_1}^{t_2} \mathbf{a}(t)\ \mathrm{d}t$.

Don't forget: displacement, velocity and acceleration are <u>vectors</u>, distance, speed and magnitude of acceleration are the <u>length</u> of the corresponding vectors.

$$\text{New Position} = \text{Original Position} + t(\text{Velocity})$$

$$\text{New Velocity} = \text{Original Velocity} + t(\text{Acceleration})$$

Velocity:	$\mathbf{i}$ component	$\mathbf{j}$ component	
"Traveling East"	positive	zero	
"Traveling West"	negative	zero	
"Traveling South"	zero	negative	
"Traveling North"	zero	positive	
"Traveling North East"	positive	positive	$\mathbf{i}$ component = $\mathbf{j}$ component
"Traveling South East"	positive	negative	$\mathbf{i}$ component = $-\mathbf{j}$ component
"Traveling North West"	negative	positive	$\mathbf{i}$ component = $-\mathbf{j}$ component
"Traveling South West"	negative	negative	$\mathbf{i}$ component = $\mathbf{j}$ component

12: MOMENTS

First step	Resolve forces, if needed
Equation 1	Forces Up = Forces Down
Equation 2	Forces Left = Forces Right
Equation 3	Moments Clockwise = Moments Anticlockwise

→ Simultaneous Equations

MOMENT

of $\mathbf{F}$ about point P = $|\mathbf{F}| \times d$
d is the direct/shortest distance from P to the line of action of $\mathbf{F}$
The moment measures the turning effect of a force about a point
Units are Nm

TILTING

When a body is on the point of tilting about a pivot, the reaction in any other supports and the tension in and supporting wires is zero

HINGES

At the hinge, the reaction is not perpendicular to the wall
Draw the reaction force as $\mathbf{X}$ horizontal and $\mathbf{Y}$ vertical
Left/right, up/down doesn't matter. If it comes out negative it was the other way
The magnitude of the reaction is $\sqrt{\mathbf{X}^2 + \mathbf{Y}^2}$ in direction $\tan\theta = \dfrac{|\mathbf{Y}|}{|\mathbf{X}|}$

LADDERS

Ladder type problems are not set on the AQA exams.
The same ideas apply to beams resting on the floor and a pivot etc

Where the ladder touches the floor:
- friction is parallel to the floor, usually towards the wall but might not be
- the normal reaction $\mathbf{R}$ is perpendicular to the floor

Where the ladder touches the wall:
- friction (on a non-smooth wall) is parallel to the wall, usually up
- the normal reaction $\mathbf{S}$ is perpendicular to the wall

TT TOP TIPS

Finding d for each moment is the most common problem
Make a separate diagram for each moment.
Opposite the angle is $d\sin\theta$, adjacent to the angle is $d\cos\theta$
Don't put too much information on one diagram