Topic 1: Primes, Highest Common Factors and Lowest Common Multiples

Notes:

Natural Numbers:

1. Natural numbers are the counting numbers (not consecutive) such as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 etc…

2. As the set of natural numbers is endless, we say that the set of natural numbers is **infinite**. There is no end to this set of natural numbers.

Whole Numbers:

3. Whole numbers take the set of natural numbers but **includes the number zero** (naught), ie: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 etc…

Test of Divisibility:

4.

A number divisible by	**If**	**Example(s)**
2	it is even	132 and 230 are even.
3	its sum of digits is divisible by 3	126 1+2+6 = 9, which is divisible by 3.

4	the last two digits of the number is divisible by 4	144 Last two digits form 44, which is divisible by 4.
5	the last two digit is either 0 or 5	140, 255 Last digit ends with 0 and 5.
6	number is divisible by 2 and 3 (2x3=6)	180. Divisible by 2 (even) Divisible by 3 (1+8+0=9, divisible by 3)
7	(a) Multiply last digit by 2 (b) Get the result from **(a)**. Subtract from remaining numbers of original number (c) Result from **(b)** should be divisible by 7	1029 (a) 9x2 = 18 (b) 102 – 18 = 84 (c) 84 is divisible by 7
8	last three digit of number is divisible by 8	2648 648 is divisible by 8.
9	the sum of its digits is divisible by 9	9018

		9+0+1+8 = 18, which is divisible by 9.
10	the last digit is a 0	1020 and 350 are divisible by 10.
11	the difference between the sum of digits in the odd places and the sum of digits in the even places is equal to either 0 or a multiple of 11	5698, **(5+9) – (6+8) = 14 – 14 = 0 (divisible by 11)** 1 908 192, **(9+8+9) – (1+0+1+2) = 22 (multiple of 11)**
12	it is divisible by 3 and 4	8748 is divisible by 3 **(8+7+4+8 = 27, divisible by 3)** and 4 **(48 is divisible by 4)**

Factors and Multiples:

5. The **factors** of a number are whole numbers that divide the number exactly without leaving any remainder behind. For an example:

   8 = 1 x 8
   = 2 x 4
   = 1.6 x 5

   The factors of 8 are 1, 2, 4 and 8. Although 1.6 x 5 is a correct statement, 1.6 and 5 are **not** factors of 8. (Since they are not whole numbers.)

6. The multiple of a number is any number that can be obtained by multiplying the number by a positive integer.
   For an example:

   Multiples of 3 are 3, 6, 9, 12, 15, 18, 21…

Prime Numbers and Composite Numbers:

7. A **prime number** or a **prime** for short is a natural number that only has 2 factors, which are 1 and itself.
   For an example:

   Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23…

   13 is a prime as its only pair of factors is 1 and 13.
   (1x13=13)

8. A **composite number** is a natural number which has more than two different factors. Basically, a composite number is **not** a prime number.
   For an example:

   Composite numbers: 4, 6, 8, 9, 10, 12, 14, 15, 16 etc…

   12 is a composite number as its factors are 1, 2, 3, 4, 6, and 12 (more than two.)

9. 1 is **neither a composite number nor a prime**, as it only has **one factor** which is itself (aka. 1)

Prime Factorization:

10. A composite number can be expressed as the product of multiple prime numbers. (2 or more.)

11. The process of expressing a composite number of its prime factors is called prime factorization, which can be done with either the use of a **factor tree** or **repeated division**. (Repeated division more efficient.)

Index Notation:

12. When the number is being multiplied by itself more than once, we can use the index notation to represent it.

For an example:

16 = 2 x 2 x 2 x 2 can be expressed as 16 = 2^4, where it is read as “2 to the power of 4”

In general, *a x a x a … x a* has *n* factors, so it is written as a^n, and read as “*a* to the power of *n*”

Highest Common Factor (HCF) and Lowest Common Multiple (LCM):

13. The **highest common factor (HCF)** is the **largest** number, that is the common factor of two or more numbers.

14. The **lowest common multiple (LCM)** is the **smallest** number, that is the common multiple of two or more numbers.

Square and Square Roots:

15. The square of the number is the number multiplied by itself two times, denoted by $a\ x\ a$ = a^2.
For an example:

The square of 4 is 4^2 which is 16 (4 x 4 = 16)

16. A **perfect square** (or **square number** for short) is a number whose square root is a natural number. For example, 16 is a perfect square since √16 = 4.
Important notes:
   i. Square of a positive number is always **positive**. For example, 5^2 = 25.
   ii. Square of a negative number is always **positive**. For example, $(-2)^2$ = 4.
   iii. Positive square root of a positive number is always **positive**.
   iv. The negative square root of a positive number is always **negative**.
   v. Square roots of a negative number are **undefined**.

Cubes and Cube Roots:

17. The cube of the number is the number multiplied by itself thrice, denoted by $a\ x\ a\ x\ a$ = a^3.
For an example:

The cube of 4 is 4^3 = 64. (4 x 4 x 4 = 64)

18. A perfect cube (or cube number for short) is a number whose cube root is a natural number. For example, 64 is a perfect cube $since$ ^{3}√64 = 4.