PRACTICE PROBLEMS

ORBITAL MOTION

Orbital Motion and Circular Orbits

1. An object is in orbit around a planet. Which of the following forces must be acting on the object? (Select all that apply)
   - A A force acting in the direction of its velocity
   - B A gravitational force
   - C A force acting away from the center of the planet
   - D None of the above

2. Which of the following explains why an object in a circular orbit does not "fall" and hit the planet it's orbiting?
   - A An object in orbit has no weight
   - B There is no force acting on the object which points towards the planet
   - C The object has inertia
   - D The object is in circular motion and the centripetal force acting on it has a greater magnitude than the gravitational force acting on it

3. A satellite is in a circular orbit around a planet. Which of the following could you determine if you only know the radius of the orbit and the mass of the planet? (Select all that apply)
   - A The speed of the satellite
   - B The kinetic energy of the satellite
   - C The period of the orbit
   - D The acceleration of the satellite

4. Which of the following describes the direction of the net force acting on an object in a circular orbit?
   - A In the same direction as the velocity
   - B Perpendicular to the velocity
   - C In the opposite direction as the velocity
   - D Cannot be determined

5. A rocket is launched from the surface of the earth. Which of the following is required for the rocket achieve a circular orbit around the earth?
   - A The rocket needs to have a small enough mass so it's not affect by the earth's gravity
   - B The rocket needs to travel directly upwards (away from the earth) very fast
   - C The rocket needs to travel directly upwards until it's no longer affected by the earth's gravity
   - D The rocket needs to travel sideways (parallel to the earth's surface) very fast

6. A satellite of mass m is in a circular orbit around a planet of mass M with an orbital speed of v and an orbital radius of r. Which of the following represent the net force acting on the satellite? (Select all that apply)
   - A $\frac{mv^2}{r}$
   - B Gm
   - C GM
   - D $\frac{GMm}{r^2}$

7. Which of the following is true about an object in orbit? (Select all that apply)
   - A It has no weight
   - B There are no forces acting on it
   - C It has no apparent weight
   - D There is no net force acting on it

8. A satellite of mass m is in a circular orbit with a speed of v around a planet of mass M. Which of the following is equal to the radius of the orbit?
   - A $\frac{Gm}{v^2}$
   - B $\frac{GM}{v^2}$
   - C $\frac{M}{v^2}$
   - D $\frac{GMm}{v^2}$

9. For a satellite in a circular orbit around the earth, which of the following is constant? (Select all that apply)
   - A The magnitude of the velocity
   - B The magnitude of the acceleration
   - C The radius of the orbit
   - D The magnitude of the net force on the satellite

10. Two satellites are in circular orbits around the earth. Satellite A has a greater mass and is farther from the surface of the earth, and satellite B has a smaller mass and is closer to the surface of the earth. Which satellite has a greater orbital speed?
    - A Satellite A
    - B Satellite B
    - C They have the same speed
    - D Cannot be determined

11. A moon is in a circular orbit around a planet. The moon has a mass of m and the planet has a mass of M. The distance between the center of the moon and the center of the planet is r. Which of the following would be equal to the amount of time it takes the moon to complete half of one orbit?
    - A $\pi\sqrt{\frac{r^3}{GM}}$
    - B $2\pi\sqrt{\frac{r^3}{Gm}}$
    - C $\sqrt{\frac{r^3}{Gm}}$
    - D $2\pi\sqrt{\frac{r^3}{GM}}$

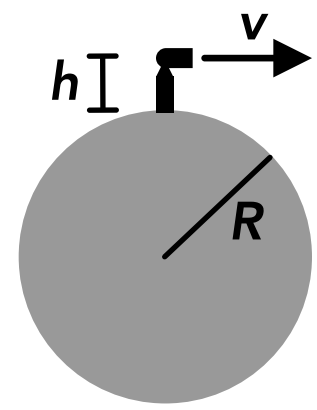
12. Two satellites are in orbit around a planet. Satellite A has twice the mass of satellite B, and satellite B has half of the orbital radius of satellite A. Which satellite has the greater kinetic energy?

A Satellite A
B Satellite B
C They have the same kinetic energy
D Cannot be determined

13. An object is in a circular orbit around planet A of mass M_A. If the same object were in a circular orbit around planet B which has a mass of $2M_A$, and the orbital radius was the same, how would the object's speed compare between the two orbits?

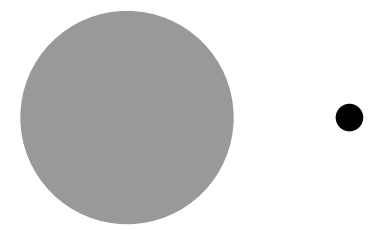
A The speed would be greater when orbiting planet A
B The speed would be greater when orbiting planet B
C The speed would be the same in both orbits
D Cannot be determined

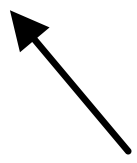
14. A cannon ball is shot out of a cannon with a speed of v from a height of h above the surface of a planet with a radius of R and a mass of M. The ball has a mass of m and is shot horizontally, parallel or tangent to the surface of the planet. In terms of the other variables, what speed does the ball need to be shot so that it travels all the way around the planet and hits the cannon?



A $\sqrt{\frac{GM}{R}}$
B $\sqrt{\frac{GM}{h}}$
C $\sqrt{\frac{Gm}{R}}$
D $\sqrt{\frac{GM}{h+R}}$

15. A satellite is in a circular orbit around a planet as shown on the right. The satellite is orbiting counterclockwise around the planet from this view. Which of the following shows the direction of the satellite's velocity when it's at the point shown?

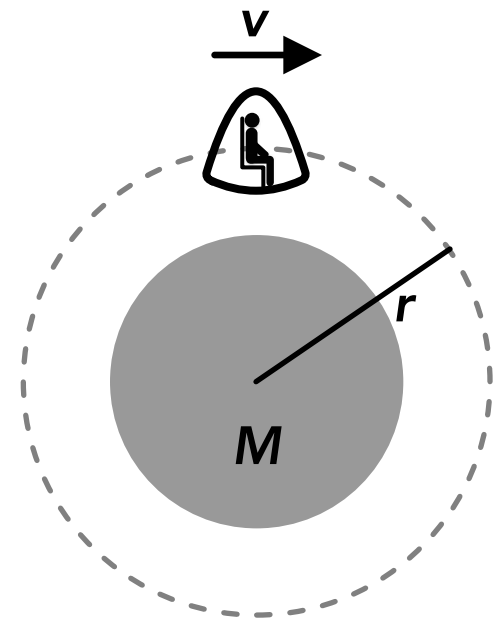


A

B

C

D


16. An astronaut with a mass of m is in a space capsule in a circular orbit around the earth which has a mass of M. The capsule has a speed of v and the radius of the circular orbit is r. Which of the following is equal to the apparent weight of the astronaut? (Select all that apply)



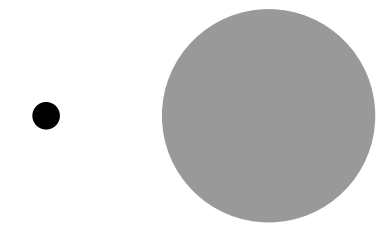
A $\frac{mv^2}{r}$

B mG

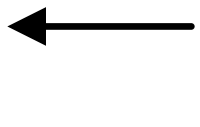
C $\frac{GMm}{r^2}$

D None of the above

17. A satellite is in a circular orbit around a planet as shown on the right. The satellite is orbiting clockwise around the planet from this view. Which of the following shows the direction of the satellite's acceleration when it's at the point shown?



A

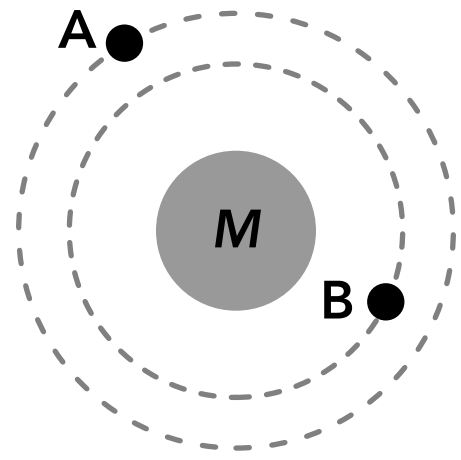


B

C

D

18. Two satellites are in orbit around a planet as shown on the right. Satellite A has a mass of m_A and satellite B has a mass of $3m_A$. Which satellite takes a longer time to complete 1 orbit?



A Satellite A
B Satellite B
C They take the same amount of time to complete 1 orbit
D Cannot be determined

19. A satellite with a mass of 500 kg is in orbit around the earth, 300 km above the earth's surface. What is the speed of the satellite? Use $\mathbf{6 \times 10^{24}\ kg}$ for the mass of the earth and $\mathbf{6.4 \times 10^{6}\ m}$ for the earth's radius.

20. A 200 kg object is in orbit around a planet with a mass of $\mathbf{2 \times 10^{24}\ kg}$. If the radius of the circular orbit is $\mathbf{3 \times 10^{6}\ m}$, what is the total energy of the object?

21. Astronomers discover a new planet in a distance solar system which has a small moon orbiting around it in a circular orbit. From their observations they are able to estimate that the distance between the center of the planet and the center of the moon is 8×10^5 **m**. They also observe that it takes 15 earth days for the moon to make one orbit around the planet. Based on those observations, what is the mass of the planet?

22. The International Space Station (ISS) is in orbit around the earth and takes 92.5 minutes to complete one orbit. If we assume the orbit is circular, how high is the ISS above the surface of the earth in km? Use 6×10^{24} **kg** for the mass of the earth and 6.4×10^6 **m** for the radius of the earth.

23. If we assumed that the moon orbited the earth in a circular orbit whose center is at the center of the earth, how long would it take the moon to complete one orbit around the earth, in days? Use 7.35×10^{22} **kg** for the mass of the moon, 6×10^{24} **kg** for the mass of the earth, and 3.84×10^8 **m** for the distance between the center of the moon and the center of the earth.

Answers

1. B
2. C
3. A, C, D
4. B
5. D
6. A, D
7. C
8. B
9. A, B, C, D
10. B
11. A
12. C
13. B
14. D
15. B
16. D
17. C
18. A
19. 7729 m/s
20. $\mathbf{4.45 \times 10^{9}\ J}$
21. $\mathbf{1.80 \times 10^{17}\ kg}$
22. 384 km
23. 27.4 days

Answers - Orbital Motion and Circular Orbits

1. **Answer: B**
   An object in orbit is in projectile motion or free fall, and the only force acting on it is the gravitational force from the planet that it's orbiting. There is no force required to keep it moving forward and there is no force acting away from the planet preventing it from moving towards the planet (Newton's 1st law).

2. **Answer: C**
   An object in orbit has a gravitational force or weight force acting on it which points towards the center of the planet. This gravitational force acts as the centripetal force which keeps the object in circular motion. The object does not fall out of orbit because it has inertia and would tend to move in a straight line tangent to the orbital path, away from the planet, if the gravitational force was not acting on it (Newton's 1st law of motion).

3. **Answer: A, C, D**
   The speed of the satellite, the period of the orbit, and the acceleration (g) of the satellite can be found using the equations below. The mass of the satellite would be needed to find the kinetic energy of the satellite.
   $$v = \sqrt{\frac{GM}{r}} \qquad T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad g = \frac{GM}{r^2}$$

4. **Answer: B**
   The net force acting on an object in a circular orbit is the centripetal force since the object is in circular motion. The net force acts towards the center of the orbit which is perpendicular to the velocity (which is tangent to the circular path).

5. **Answer: D**
   In order for the rocket to achieve a circular orbit around the earth, it must travel sideways (parallel to the earth's surface) very fast. If it does not travel sideways very fast its trajectory will eventually fall and hit the earth. No matter how small the rocket's mass is or how high above the earth's surface it travels it will still be affected by the earth's gravity, which is also required to achive a circular orbit (gravity acts as the centripetal force). If the rocket only travels directly upwards (away from the earth) it will not achieve an orbit, it will just move away from the earth or fall back down.

6. **Answer: A, D**
   The satellite is in uniform circular motion so the net force acting on it is the centripetal force which is given in the equation below. The net force is the gravitational force between the planet and the satellite which is also given:
   $$F_c = \frac{mv^2}{r} = F_g = \frac{GMm}{r^2}$$

7. **Answer: C**
   An object in orbit has a gravitational force acting on it and no other forces. Its weight is the gravitational force, but the object is in free fall and has no apparent weight.

8. **Answer: B**
   An equation for the orbital radius can be found by rearranging the equation for the orbital speed. Or we can start with the fact that the centripetal force is equal to the gravitational force and rearrange that equation to find the radius. The radius does not depend on the mass of the satellite.
   $$\frac{mv^2}{r} = \frac{GMm}{r^2} \qquad r = \frac{GM}{v^2}$$

9. **Answer: A, B, C, D**
   The magnitude of the velocity (the speed) is constant for an object in a circular orbit. The magnitude of the acceleration is constant, which is the gravitational acceleration and the centripetal acceleration for the circular motion. The radius of the orbit is constant because it's a circle. The magnitude of the net force on the satellite is just the gravitational force which is constant.

10. **Answer: B**
    The relationship between the orbital radius and orbital speed is given in the equation below. The mass of the satellite is not relevant to the question. Satellite B is closer to the surface of the earth so it has a smaller orbital radius, so it must have a greater orbital speed.
    $$v = \sqrt{\frac{GM}{r}}$$

11. **Answer: A**
    The equation for the orbital period is below, which depends on the orbital radius and the mass of the planet but not the mass of the moon. The question is asking for half of the period.
    $$T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad \frac{T}{2} = \pi\sqrt{\frac{r^3}{GM}}$$

12. **Answer: C**
    The equation for the kinetic energy of an object in a circular orbit is given below, which is the equation for kinetic energy with the variable for speed replaced with the terms for orbital speed. If satellite A has twice the mass and twice the radius of satellite B, they have the same kinetic energy.
    $$v = \sqrt{\frac{GM}{r}} \qquad K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

13. **Answer: B**
    The equation for the orbital speed is given below. If the orbital radius is the same, the orbital speed will be greater if the mass of the planet is greater.
    $$v = \sqrt{\frac{GM}{r}}$$

14. **Answer: D**
    If the cannon ball travels all the way around the planet and hits the cannon (it returns to its initial position) then the ball is in orbit around the planet. The equation for orbital speed is given below, and the radius of the orbit is the radius of the planet plus the height of the ball above the surface of the planet.
    $$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{h + R}}$$

15. **Answer: B**
    An object in a circular orbit is in uniform circular motion around the planet. The velocity is tangent to the circular path and perpendicular to a line connecting the center of the planet and the object. The satellite is orbiting counterclockwise, so it's velocity at the point shown is directly upwards.

16. **Answer: D**
    An object in orbit does not have an apparent weight because it's in free fall. Options A and C represent the gravitational force or weight force acting on the astronaut from the planet, but there is no other force acting on the astronaut for them to experience an apparent weight, such as a normal force. The astronaut and the space capsule are both falling together, similar to a person in an elevator which is in free fall.

17. **Answer: C**
    The acceleration of an object in a circular orbit points towards the center of the planet that it's orbiting. This is the direction of the net force acting on the object, which is just the gravitational force from the planet. The object is in uniform circular motion so this is the centripetal acceleration, which always points towards the center of the circular path.

18. **Answer: A**
    The equation for the orbital period is given below. Satellite A has a greater orbital radius so it has a greater orbital period and it takes a longer time to complete 1 orbit. The satellite mass does not affect the period.
    $$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

19. **Answer: 7729 m/s**
    $$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11})(6\times10^{24}\text{ kg})}{(6.4\times10^{6}\text{ m}) + (300{,}000\text{ m})}} = 7729\text{ m/s}$$

20. **Answer: 4.45×10^{9} J**
    $$E = K + U_g = -\frac{GMm}{2r} = \frac{(6.67\times10^{-11})(2\times10^{24}\text{ kg})(200\text{ kg})}{2(3\times10^{6}\text{ m})} = 4.45\times10^{9}\text{ J}$$

21. **Answer: 1.80×10^{17} kg**
    Period: $T = \frac{15\text{ days}}{}\times\frac{24\text{ hours}}{1\text{ day}}\times\frac{60\text{ min}}{1\text{ hour}}\times\frac{60\text{ s}}{1\text{ min}} = 1{,}296{,}000\text{ s}$
    $$T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad (1{,}296{,}000\text{ s}) = 2\pi\sqrt{\frac{(8\times10^{5}\text{ m})^3}{(6.67\times10^{-11})M}} \qquad M = 1.80\times10^{17}\text{ kg}$$

22. **Answer: 384 km**
    Period: $T = \frac{92.5\text{ min}}{}\times\frac{60\text{ s}}{1\text{ min}} = 5{,}550\text{ s}$
    $$T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad (5{,}550\text{ s}) = 2\pi\sqrt{\frac{(h + 6.4\times10^{6}\text{ m})^3}{(6.67\times10^{-11})(6\times10^{24}\text{ kg})}} \qquad h = 384\text{ km}$$

23. **Answer: 27.4 days**
    $$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(3.84\times10^{8}\text{ m})^3}{(6.67\times10^{-11})(6\times10^{24}\text{ kg})}} = 2{,}363{,}405\text{ s}$$
    $$\frac{2{,}363{,}405\text{ s}}{}\times\frac{1\text{ min}}{60\text{ s}}\times\frac{1\text{ h}}{60\text{ min}}\times\frac{1\text{ day}}{24\text{ h}} = 27.4\text{ days}$$