THE SCIENCE CUBE

Energy & Forces in Rolling

1. Kinetic energy of a rolling object
2. Energy conservation in rolling motion
3. Friction and rolling
4. Static vs kinetic friction in rolling
5. Analysing rolling motion on an incline
6. Key formulas and equations
7. Common Mistakes and Misconceptions:

KINETIC ENERGY OF A ROLLING OBJECT

A rolling object possesses two forms of kinetic energy:

1. Rotational Kinetic Energy:

$$\text{Rotational KE} = 1/2\,(I\,\omega^2)$$

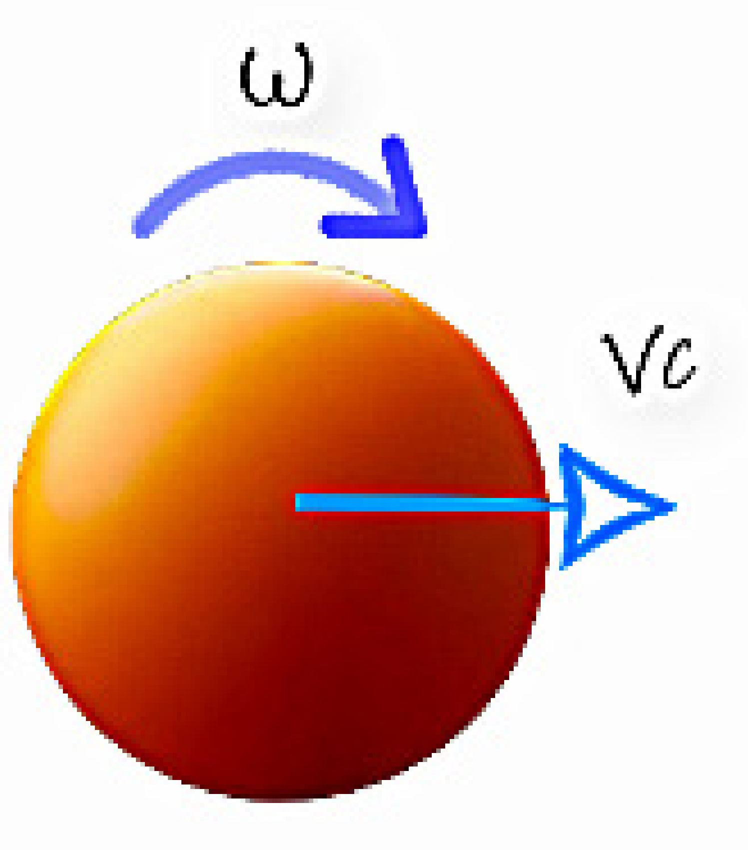
- I is the moment of inertia bout COM
- ω is the angular velocity.

2. Translational Kinetic Energy:

$$\text{Translational KE} = 1/2\,(M\,v_c^2)$$

- M is the mass of the object
- v_c is the velocity of the center of mass

$$\text{Total KE} = 1/2\,(I\,\omega^2) + 1/2\,(M\,v_c^2)$$



ENERGY CONSERVATION IN ROLLING MOTION

When an object rolls without slipping, its total mechanical energy is conserved.

1. $Mgh = 1/2\,(M v^2) + 1/2\,(I\,\omega^2)$
   - This is also a condition to verify if an object is rolling without slipping.

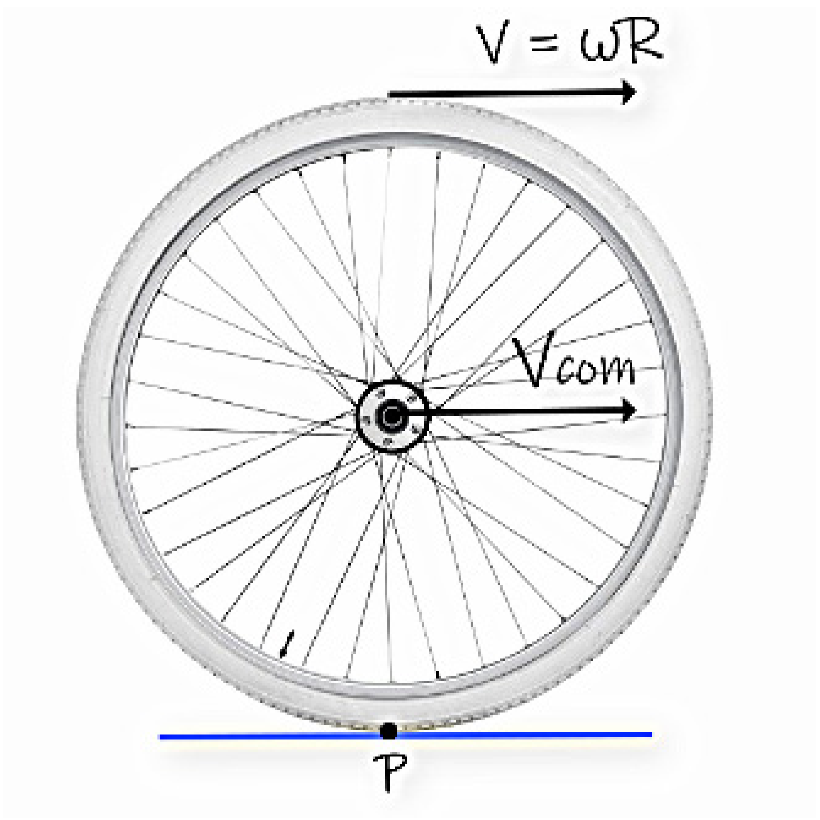
2. If sliding or slipping occurs, kinetic friction generates thermal energy, and the equation must be modified to include heat loss:
   - $Mgh = 1/2\,(M v^2) + 1/2\,(I\,\omega^2) + E_{thermal\ loss}$

FRICTION AND ROLLING

1. If a wheel rolls at constant speed, the tangential velocity equals the linear velocity

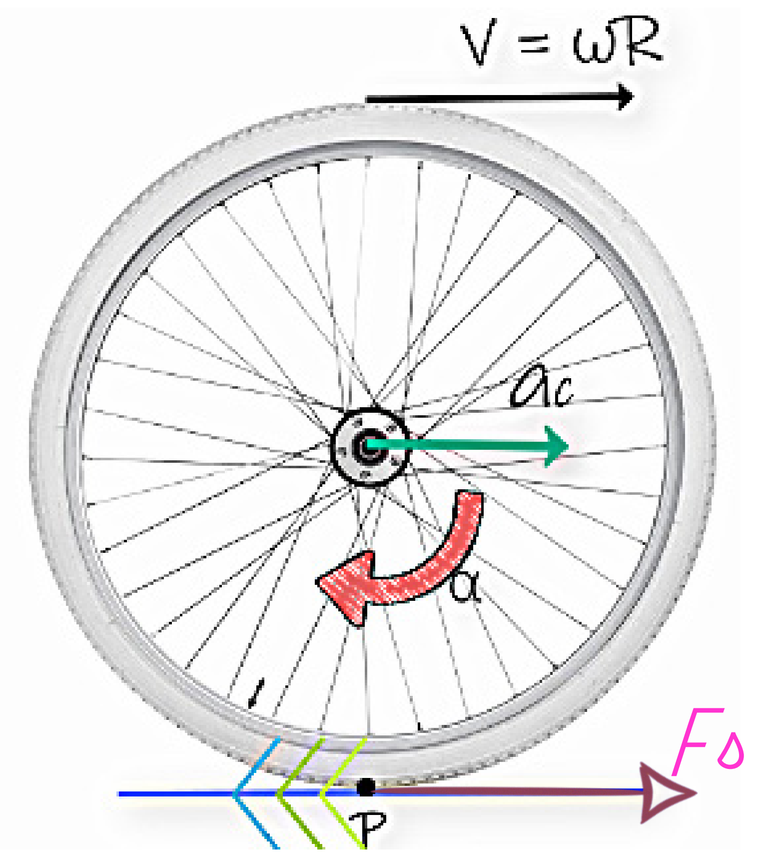
$$\omega R = v_c$$

- This means there is no relative motion between the wheel and the surface at the contact point (P), so no frictional force is required for motion



2. Acceleration and Friction
   - If a force acts on the wheel to increase its speed, it causes acceleration of the center of mass and angular acceleration of the wheel.

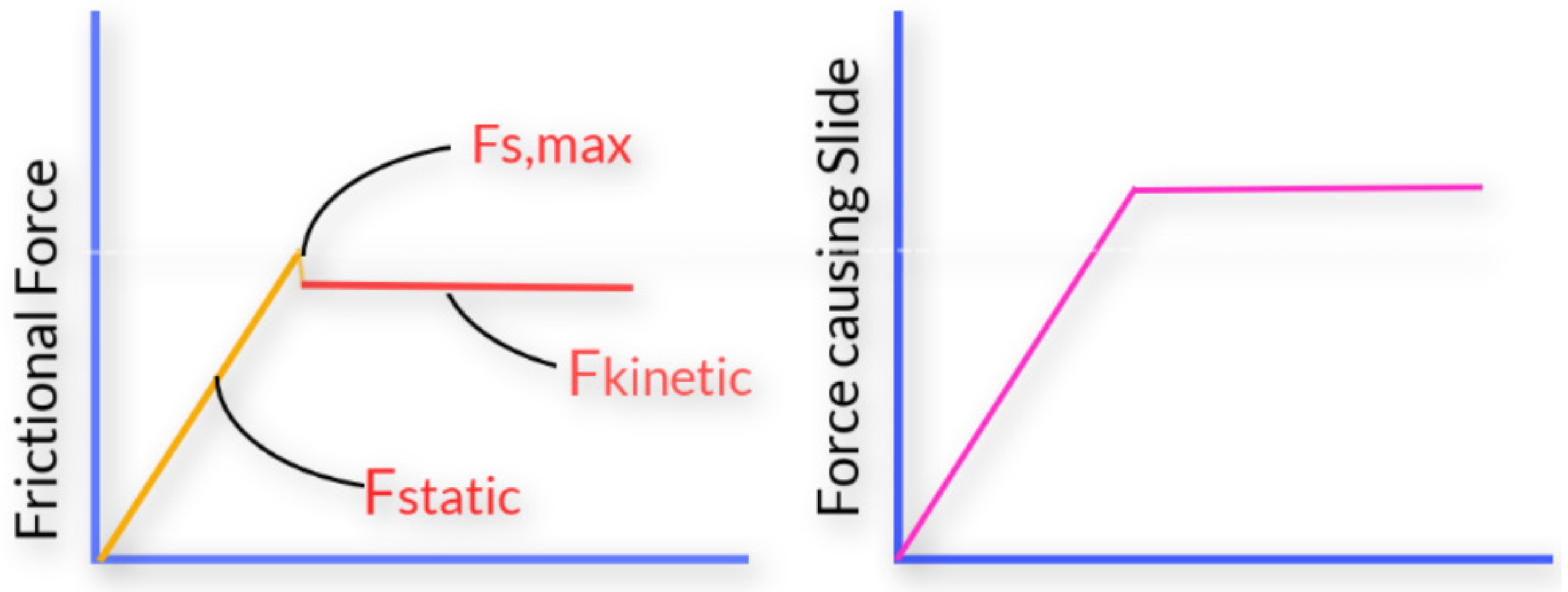
When $\omega R > v_c$, a static frictional force (F_s) acts at point P to prevent sliding



STATIC & KINETIC FRICTION IN ROLLING

1. Condition for rolling without slipping

$$\omega R = v_c$$

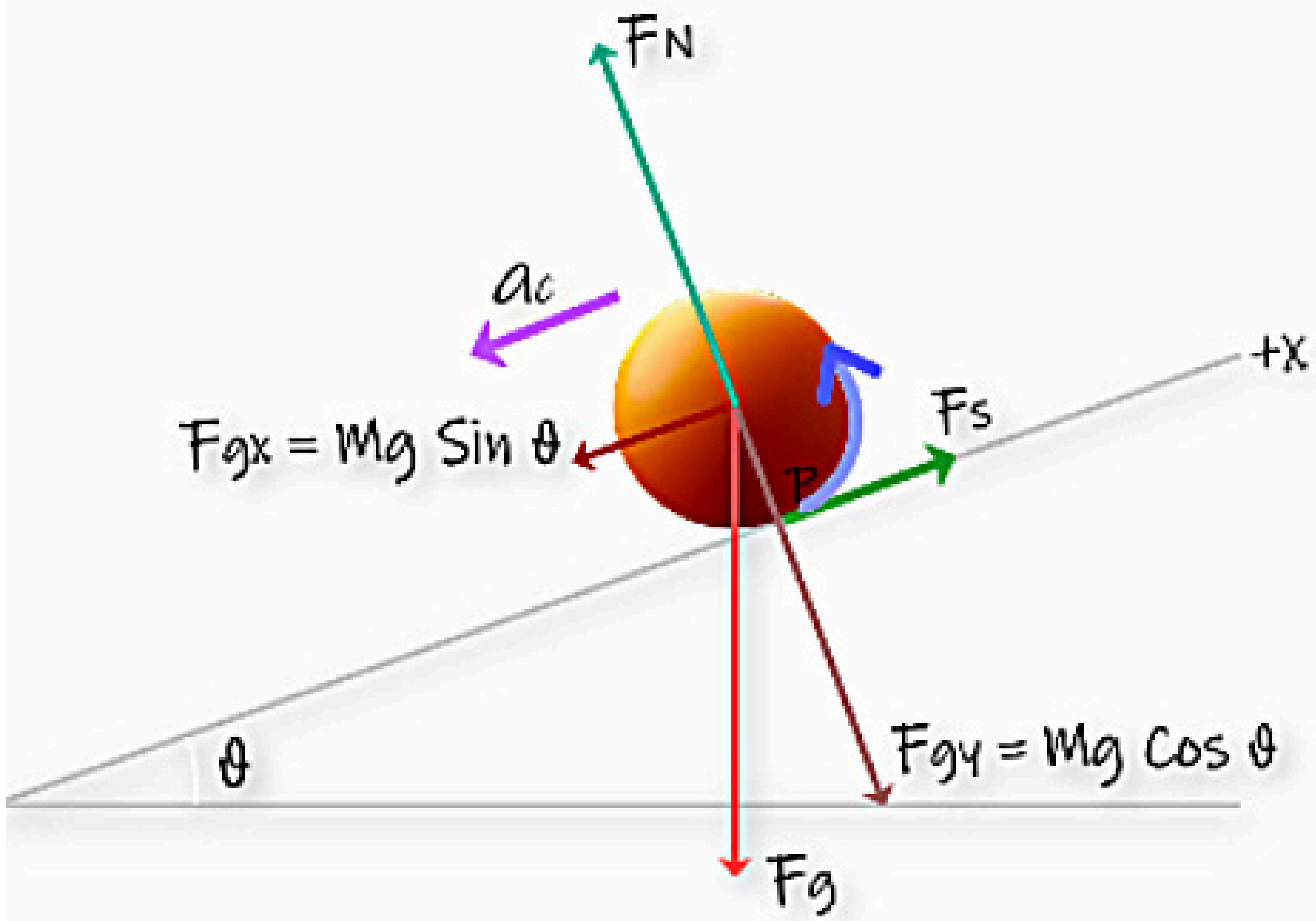


2. If an external force induces static friction, condition for rolling without slipping can also be given using the equation

$$a_c = R\,\alpha$$

3. If the force causing slip exceeds the maximum static friction ($F_{s,max}$),
   - the wheel slides, and kinetic friction takes over.
   - The above equations become invalid

DERIVATION: LINEAR ACCELERATION OF A ROLLING SPHERE



$$F_s - Mg\sin\theta = Ma_c$$

$$\tau = rF_s$$

$$\tau = I_c\,\alpha$$

$$RF_s = I_c\,\alpha$$

$$a_c = R\alpha$$

$$-a_c / R = \alpha$$

$$F_s = -\,I_c\,(a_c / R^2)$$

$$a_c = -g\sin\theta / (1 + I_c / MR^2)$$

$Mg\sin\theta > F_{s,max}$ (body will slide)

HOLLOW & SOLID SPHERE: WHICH ROLLS FASTER?

Rolling Ball

Consider a solid sphere and a hollow sphere, both with the same mass (2 kg) and radius (0.2 m), rolling down a ramp inclined at 30 degrees. Let's compare their linear accelerations and energies.

Energy Conservation:

At the top, both have same potential energy: mgh
As they roll, this potential energy converts into:

- Translational kinetic energy: $1/2\,(m\,v^2)$
- Rotational kinetic energy: $1/2\,(I\,\omega^2)$

The total mechanical energy is conserved, so:

$$mgh = 1/2(mv^2) + 1/2\,(I\,\omega^2)$$

HOLLOW & SOLID SPHERE: ACCELERATION CALCULATION

Moment of Inertia:

For a solid sphere: $I = (2/5) \times m \times R^2$.
For a hollow sphere: $I = (2/3) \times m \times R^2$.

The hollow sphere has a larger moment of inertia, meaning more of its energy is used for rotation, leaving less for translational motion.

Linear Acceleration: $a = (g \times \sin(\theta)) / (1 + (I / (m \times R^2)))$.

For the solid sphere:

$I = (2/5) \times m \times R^2$:
$a = (g \times \sin(\theta)) / (1 + 2/5)$
$a = 3.5\ m/s^2$.

For the hollow sphere:

$I = (2/3) \times m \times R^2$:
$a = (g \times \sin(\theta)) / (1 + 2/3)$
$a = 2.94\ m/s^2$.

HOLLOW & SOLID SPHERE: INFERENCES

Key Observations:

1. The solid sphere accelerates faster than the hollow sphere because it has a smaller moment of inertia. This means more energy is converted into translational motion.
2. The hollow sphere accelerates more slowly because a larger proportion of its energy is used for rotational motion.

Kinetic Energy Distribution at the bottom of the ramp:

1. For the solid sphere, a greater proportion of its total kinetic energy is in translational motion.
2. For the hollow sphere, a greater proportion of its total kinetic energy is in rotational motion.

KEY FORMULAS & EQUATIONS

Equation	When to Use	Notes
Total $KE = \frac{1}{2} M v_c^2 + \frac{1}{2} I \omega^2$	Calculating the total kinetic energy of a rolling object, combining translational and rotational components.	Ensure to use the correct moment of inertia (I) for the object's shape
$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$	Analyzing energy conservation when an object starts to roll without slipping from a height h.	Valid only in the absence of non-conservative forces like air resistance or kinetic friction.
$v_c = R\omega$	Relating linear velocity of the center of mass to angular velocity for pure rolling motion.	If $v_c \neq R\omega$, slipping occurs, and kinetic friction must be considered.
$a_c = R\alpha$	Connecting linear acceleration of the center of mass to angular acceleration. This can also be taken as a condition in rolling without slipping.	Applies only when there is no slipping; otherwise, this relationship doesn't hold.
$\tau = I_c\alpha$	Determining angular acceleration given the net torque and moment of inertia.	Torque (τ) should be calculated about the center of mass or the point of contact, depending on the problem.
$f_s = (I_c * a_c) / R^2$	Calculating the static friction force necessary to prevent slipping during rolling.	Static friction adjusts to prevent slipping; it does not always reach its maximum value.
$a_c = (g \sin\theta) / (1 + I_c / MR^2)$	Finding the linear acceleration of an object rolling down an incline at angle θ.	Depends on the object's moment of inertia; different shapes yield different accelerations.

COMMON MISTAKES AND MISCONCEPTIONS

1. Misconception: A continuous force is needed to keep an object moving.

Clarification: According to Newton's first law, an object in motion will remain in uniform motion unless acted upon by an external force. In the context of rolling motion, once an object is rolling without slipping on a frictionless surface, it doesn't require a continuous force to maintain its motion. Frictional forces, however, can cause the object to eventually come to a stop.

2. Misconception: Heavier objects roll down a slope faster than lighter ones.

Clarification: In the absence of air resistance and other dissipative forces, all objects, regardless of mass, experience the same acceleration due to gravity when rolling down an incline. The distribution of mass (moment of inertia) and the shape of the object influence the rolling motion, not the mass alone.

3. Misconception: Rolling motion always involves frictional force.

Clarification: While friction is necessary to initiate rolling without slipping, a rolling object moving at constant velocity on a perfectly rigid surface experiences no net frictional force. Friction acts only when there's relative motion or a tendency for such motion between surfaces.

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COMMON MISTAKES AND MISCONCEPTIONS

4. Misconception: The point of contact in rolling motion is stationary.

Clarification: In pure rolling motion without slipping, the point of contact between the rolling object and the surface is instantaneously at rest relative to the surface. However, due to the object's rotation and translation, this point changes continuously.

5. Misconception: An object's rotational and translational speeds are independent.

Clarification: For an object rolling without slipping, there's a direct relationship between its translational speed (v) and angular speed (ω), given by $v = \omega R$, where R is the radius. This relationship ensures synchronized motion between rotation and translation.

6. Misconception: Rolling objects do not experience energy loss.

Clarification: In real-world scenarios, rolling objects encounter rolling resistance due to deformations at the contact point, leading to energy dissipation and gradual slowing down. This is distinct from kinetic friction and is influenced by factors like material properties and surface texture.

COMMON MISTAKES AND MISCONCEPTIONS

7. Misconception: Static friction always acts to oppose motion.

Clarification: In rolling motion, static friction doesn't oppose motion but facilitates rolling without slipping by preventing relative motion at the point of contact. Its direction and magnitude adjust to maintain the rolling condition.

8. Misconception: The normal force and gravitational force are an action-reaction pair.

Clarification: According to Newton's third law, action-reaction pairs act on different objects. The normal force acts on the rolling object due to the surface, while the gravitational force acts on the object due to Earth's mass. These forces are not an action-reaction pair but rather two forces acting on the same object.

9. Objects of same shapes roll in the same way?

Clarification: Objects of same shape can have varying moments of inertia, which affect their rotational acceleration. For instance, a solid sphere and hollow sphere of the same mass and radius will reach the bottom of an incline at different times due to their differing moments of inertia. The object with the smallest moment of inertia will generally reach the bottom first.