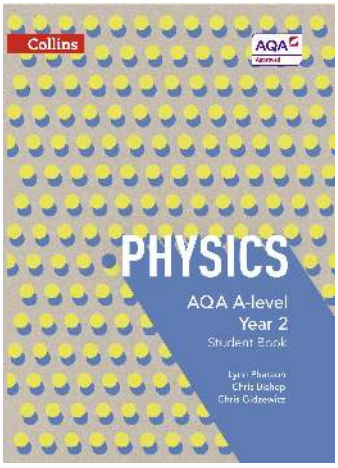
AQA A-level Physics Student Book Answers

Relativistic Waves and Quantum Fields (Queen Mary University of London)

Collins AQA A-level Physics

Year 2 Student Book Answers



Optional Units – Answers

AQA A-Level Physics Year 2
Student Book Answers

Chapter 1

ASSIGNMENT 1

A1 The total lateral friction on the tyres to just avoid the car going off at a tangent is equal to the required centripetal force:

$$F = \frac{mv^2}{r} = \frac{690 \times (120 \times 10^3 \div 3600)^2}{70} = 10952 = 11000 \text{ N}$$

A2 The asymmetric curved shape and angle of a racing car aerofoil makes moving air flow faster below it than above, so the air pressure is lower below the aerofoil. This creates a downward force on the aerofoil pushing the car downwards.

A3 At the Abbey bend, *g*-force $= \frac{\text{centripetal acceleration}}{\text{acceleration due to gravity}} = \frac{v^2/r}{g}$

$$= \frac{(120 \times 10^3 \div 3600)^2 \div 70}{9.81} = 1.6$$

So the *g*-force typically experienced by a driver at the Abbey bend is 1.6*g*.

ASSIGNMENT 2

A1 **a.** Estimate from the diagram of the beam length from the centre to the back of the chair ≈ 5 m.

Centripetal acceleration $a = r\omega^2 = r \times \left(\frac{2\pi}{T}\right)^2 = 5 \times \frac{4\pi^2}{100} \approx 2 \text{ m s}^{-2}$

b. The normal contact force exerted by the seat on the rider's back provides the centripetal force = mass of rider × centripetal acceleration. Estimate of rider's mass ≈ 60 kg.

Normal contact force $\approx 60 \times 2 = 120 \text{ N}$

A2 **a.** There is nothing to provide a centripetal force on them, so they tend to move tangentially until they hit the side of the drum.

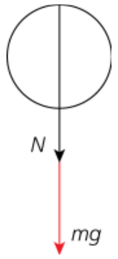
b. Centripetal acceleration $a = r\omega^2 = r \times (2\pi f)^2 = 0.3 \times \left(2\pi \times \frac{3000}{60}\right)^2 = 30000 \text{ m s}^{-2}$

c. Using the estimate of the mass of a swimming costume ≈ 50 g, the normal contact force, *N*, on the inside surface on the swimming costume is given by

$N = ma \approx 0.05 \times 30000 = 1500 \text{ N}$

ASSIGNMENT 3

A1



A2 a. $mg + N = \frac{mv^2}{r}$

b. $N = \frac{60 \times 8.8^2}{5} - 60 \times 9.81 = 341$ N

A3 $N - mg = \frac{mv^2}{r}$

$N = \frac{60 \times 15^2}{9} + (60 \times 9.81) = 2090$ N

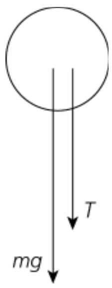
A4 At the top of the loop $mg + N = \frac{mv^2}{r}$

If $N = 0$, $mg = \frac{mv^2}{r}$, which gives

$v = \sqrt{rg} = \sqrt{5 \times 9.81} = 7.0$ m s^{-1}

PRACTICE QUESTIONS

1a. i.



1a. ii. $mg + T = \frac{mv^2}{r}$

Hence

$T = \frac{0.05 \times 3^2}{0.6} - (0.05 \times 9.81) = 0.26$ N

1b. i.

1b. ii. $T - mg = \frac{mv^2}{r}$

1c. The string is most likely to snap when the mass is at the bottom of its path since the tension in the string is at its greatest at this point.

2a. Although the aircraft has a constant speed, its direction is changing. Since velocity is a vector quantity, if the direction changes the velocity must also be changing. If there is a velocity change there must be some acceleration since acceleration is equal to rate of change of velocity.

2b. When the aircraft is banked, it rotates about its centre of mass so that the lift force on the wings is no longer acting in the vertical direction. The horizontal component of the lift creates a resultant force towards the centre of the circle which provides the centripetal force. (See figure 16 in the Student Book.)

2c. i. The banking angle θ can be found from $\tan\theta = \frac{v^2}{rg} = \frac{(280\times10^3 \div 3600)^2}{3000\times9.81} = 0.2056$, which gives the banking angle, $\theta = 12°$.

2c. ii. centripetal acceleration $a = \frac{v^2}{r} = \frac{(280\times10^3 \div 3600)^2}{3000} = 2.0 \text{ m s}^{-2}$

3a. The centripetal force on the electron is given by $F = \frac{mv^2}{r} = 8.2\times10^{-8}$ N. Rearranging gives the electron's speed $v = \sqrt{\frac{r}{m}\times8.2\times10^{-8}} = \sqrt{\frac{0.053\times10^{-9}}{9.11\times10^{-31}}\times8.2\times10^{-8}} = 2.2\times10^{6} \text{ m s}^{-1}$

3b. The angular speed $\omega = \frac{v}{r} = \frac{2.2\times10^6}{0.053\times10^{-9}} = 4.151\times10^{16} = 4.2\times10^{16} \text{ rad s}^{-1}$

3c. The time for one orbit $T = \frac{2\pi}{\omega} = \frac{2\pi}{4.151\times10^{16}} = 1.5\times10^{-16}$ s

4. C (Centripetal force: $F = mr\omega^2 = m\frac{D}{2}(2\pi f)^2 = 2mD\pi^2 f^2$)

5. D (Speed $v = \frac{2\pi r}{T} = \frac{2\times\pi\times1.5\times10^{11}}{365\times24\times3600} = 3.0\times10^4 \text{ m s}^{-1}$)

6. A (The speed of the object is uniform so the kinetic energy $\left(\frac{1}{2}mv^2\right)$ must not change)

7. A (The speed of the man, $v = r\omega = r\times\frac{2\pi}{T} = \frac{15\times2\times\pi}{50\times60} = \frac{\pi}{100} \text{ m s}^{-1}$)

8. C (When the mass is about to slip, the centripetal force $mr\omega^2$ is equal to the maximum frictional force $\frac{mg}{2}$. Hence $mr\omega^2 = \frac{mg}{2}$. Rearranging gives $= \sqrt{\frac{g}{2r}}$)

Chapter 2

REQUIRED PRACTICAL

P1 Human reaction time in switching the stopwatch on and off to correspond with the mass passing the fiducial marker will probably vary and therefore is best described as a random error. The repeat measurements will help to reduce the effects of this error.

P2 **a.** **i.** Range of $20T$ data $= 0.22$ s. Uncertainty in the average time for $20T$ is half this, ± 0.11 s.

ii. The uncertainty in $20T$ determined from the range of data is less than the typical reaction time error of ± 0.2 s.

b. Uncertainty in the average $20T$ value is best estimated as ± 0.2 s (as the value in part **a.i.** is based on only three repeats). This gives the uncertainty in time period T as $\frac{0.2}{20} = \pm 0.01$ s. So the time period $T = 0.49 \pm 0.01$ s.

c. **i.** % uncertainty in T is $\frac{0.01}{0.49} \times 100\% = \pm 2\%$

% uncertainty in $T^2 = 2 \times 2\% = \pm 4\%$

$T^2 = (0.49)^2 = 0.24\ \text{s}^2$

Uncertainty in $T^2 = \frac{4}{100} \times 0.24 = \pm 0.01\ \text{s}^2$

ii. The % uncertainty in the values of m could be $\frac{2}{50} \times 100\% = 4\%$ if 50 g masses, which is comparable to the % uncertainty in T^2. If 100 g masses were used the % uncertainty in the values of m could be $\frac{2}{100} \times 100\% = 2\%$. The student has a range of masses to choose from. If she uses masses greater than 50 g this reduces the % uncertainty in the mass, making T the greater source of uncertainty. However, if she uses masses less than 50 g this increases the % uncertainty in the mass, making m the greater source of uncertainty.

iii. If 10 oscillations were counted the uncertainty in T would be $\frac{0.2}{10} = \pm 0.02$ s, giving a % uncertainty of 4% in T and hence 8% in T^2. So this would be a source of error considerably greater than that of the mass values if masses of 50 g or more were to be used, but if masses of 20 g were used then % uncertainty in the values of m could be $\frac{2}{20} \times 100\% = 10\%$, so the greater uncertainty would still be in the mass.

P3 The uncertainty due to reaction time would be eliminated and the uncertainty in the time measurement itself is likely to be similar to or less than that of a digital stopwatch. If the same 50 g masses are used with an uncertainty of up to 4%, the uncertainty in the analysis and accuracy in the value of k will be improved by approximately a factor of 2 compared with the method involved with the timing of 20 oscillations.

P4 The pendulum should be set in motion and after a few swings, when it is moving with small amplitude, 20 oscillations should be timed with a digital stopwatch, counting the number of times that the pendulum bob passes the fiduciary marker going in the same direction. Two repeat measurements should be made and an average time for 20 oscillations calculated. The time period T is then found by dividing this by 20. This is repeated for a range of pendulum lengths l, each time measuring the length as stated in the Technique section.

A graph of T^2 against l is plotted. If a best-fit line can be drawn through the points and through the origin, the proportional relationship is confirmed.

A value for g can be obtained by measuring the gradient of the best-fit line, using as large a triangle as possible: gradient $= \frac{4\pi^2}{g}$ so $g = \frac{4\pi^2}{\text{gradient}}$.

P5 Draw the line of best fit and calculate the best gradient from this. Then draw lines of maximum gradient and minimum gradient that are still a reasonable fit to the points plotted. Whichever of these two gradients is furthest from the line of best fit is called the 'worst gradient' and the percentage uncertainty in the best gradient is found from

$$\text{percentage uncertainty} = \frac{\text{best gradient - worst gradient}}{\text{best gradient}} \times 100\%$$

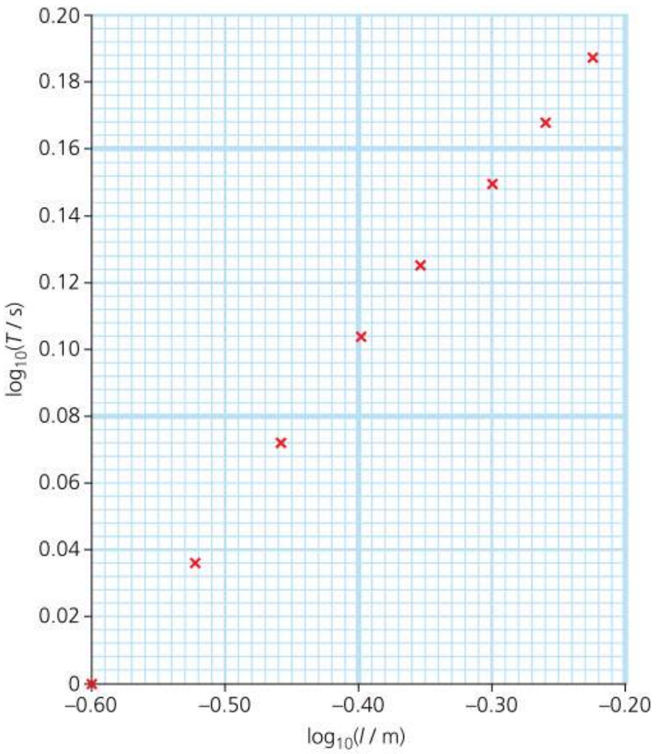
The percentage uncertainty in the gradient is equal to the percentage uncertainty in the value for *g*. The uncertainty in *g* can be found from: $\frac{\text{percentage uncertainty}}{100} \times g$

ASSIGNMENT 1

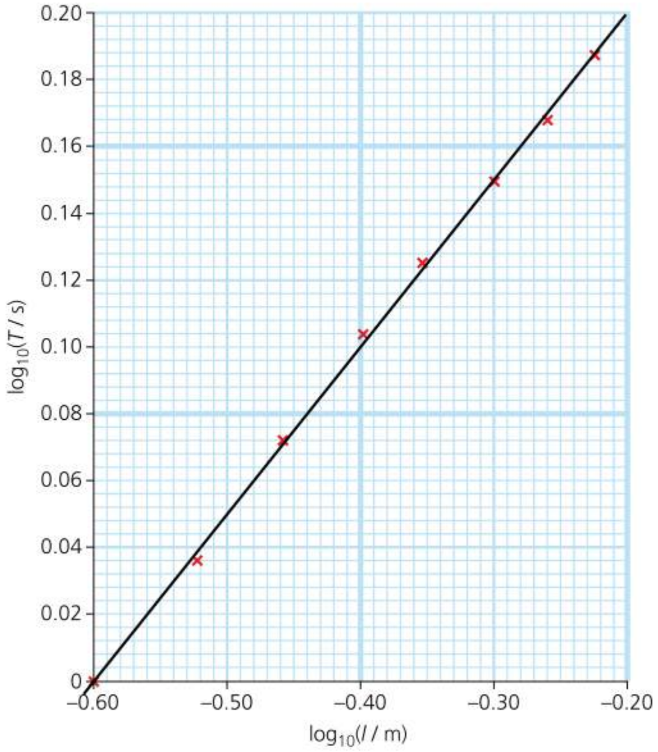
A1 [no answer required]

A2 a.

$\frac{l}{\text{m}}$	$\frac{T}{\text{s}}$	$\log\left(\frac{l}{\text{m}}\right)$	$\log\left(\frac{T}{\text{s}}\right)$
0.250	1.00	−0.602	0.000
0.300	1.09	−0.523	0.037
0.350	1.18	−0.456	0.072
0.400	1.27	−0.398	0.104
0.450	1.34	−0.347	0.127
0.500	1.42	−0.301	0.152
0.550	1.48	−0.260	0.170
0.600	1.55	−0.222	0.190



b.



From the graph, gradient = $\frac{0.21 - 0}{(-0.20) - (-0.60)} = \frac{0.21}{0.40} = 0.53$. This is within 6% of the theoretical prediction for the gradient, and that 6% difference could easily be accounted for by uncertainty in the measurements and by lack of precision in plotting the points and drawing the line of best fit.

PRACTICE QUESTIONS

1a. The student should take at least three measurements of the time for 20 cycles and calculate an average of the three values before dividing by 20 to get the time period.

1b. The equilibrium position is the preferred location for the fiducial marker since it is easier to see the pendulum pass the equilibrium position than reach the extreme position because the amplitude of the oscillation continuously decreases during the 20 oscillations.

2a. Time period of the mass-spring system $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{50}} = 0.628$ s

2b. Percentage uncertainty in the mass: $\%U_m = \frac{10}{500} \times 100 = \pm 2\%$

Percentage uncertainty in the spring constant: $\%U_k = \frac{2}{50} \times 100 = \pm 4\%$

Therefore the percentage uncertainty in the predicted value for the time period is

$\frac{1}{2} \times (\%U_m + \%U_k) = \frac{1}{2} \times (2 + 4) = \pm 3\%$

The uncertainty in the predicted time period = $\frac{3}{100} \times 0.628 = 1.9 \times 10^{-2} = 0.02$ s

Therefore the predicted time period $T = 0.63 \pm 0.02$ s

2c. i. A sample rate of 20 Hz means that a measurement is made every $\frac{1}{20}$ s = 0.05 s = 50 ms.

2c. ii. The number of measurements made in 8 cycles = $\frac{8 \times 0.628}{0.05} = 100$.

3a. i. The force exerted on the trolley by the spring on the left has decreased by $F = k\Delta l = 30 \times 0.06 = 1.8$ N and the force exerted by the spring on the right has increased by 1.8 N creating a resultant force of 3.6 N to the right.

3a. ii. Acceleration $a = \frac{F}{m} = \frac{3.6}{0.8} = 4.5\ \text{m s}^{-2}$ to the right.

3b. i. For SHM, the acceleration is directly proportional to the displacement but in the opposite direction.

3b. ii. Frequency $f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2 \times 30}{0.8}} = 1.378\ \text{Hz}$ so time period $T = \frac{1}{f} = \frac{1}{1.378} = 0.73\ \text{s}$

3c. i. Vibration frequency of copper ion $f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2 \times 200}{1 \times 10^{-25}}} = 1.0 \times 10^{13}\ \text{Hz}$

3c. ii. Maximum speed $v_{max} = \omega A = 2\pi f A = 2\pi \times 1 \times 10^{13} \times 1 \times 10^{-11} = 628\ \text{m s}^{-1}$

3c. iii. Maximum kinetic energy $E_k = \frac{1}{2} m v^2_{max} = \frac{1}{2} \times 1 \times 10^{-25} \times (628)^2 = 2.0 \times 10^{-20}\ \text{J}$

4a. When the bob is displaced to one side it gains height so on release the energy of the bob changes from gravitational potential energy to kinetic energy and back to gravitational potential energy on reaching the other extreme.

4b. Time period $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.6}{9.81}} = 1.55\ \text{s}$

4c. i. During the swing of the bob from the equilibrium position to one extreme the bob describes an arc of a circle. When the bob is first displaced, the angle between the string and the vertical is 10°, which is equal to $\frac{10}{360} \times 2\pi$ rad.

The length of the arc is equal to the amplitude and is equal to $r\theta = 0.6 \times \frac{10}{360} \times 2\pi = 0.105$ m.

4c. ii. The maximum speed is reached as the bob passes through the equilibrium position and is given by

$v_{max} = \omega A = \left(\frac{2\pi}{T}\right) A = \frac{2\pi \times 0.105}{1.55} = 0.426 = 0.43\ \text{m s}^{-1}$

4c. iii. As the bob passes through the equilibrium position, the tension exceeds the bob's weight by an amount equal to the required centripetal force since the bob is describing the arc of a circle. The magnitude of the tension at the equilibrium position is given by

$mg + \frac{mv^2}{r} = (0.02 \times 9.81) + \frac{0.02 \times 0.426^2}{0.6} = 0.202\ \text{N}$

5. D (Since the defining formula for SHM is $a = -\omega^2 x$, the graph of a versus x is a straight line with a negative slope since angular frequency, ω is constant.)

6. D (The maximum speed is given by $v_{max} = \omega A = 2\pi f A = 2\pi \times 320 \times 0.5 = 320\pi\ \text{mm s}^{-1}$)

7. B

8. A (Speed $v = 2\pi f\sqrt{A^2 - x^2} = v = 2\pi\sqrt{0.2^2 - 0.1^2} = 1\ \text{m s}^{-1}$)

Chapter 3

ASSIGNMENT 1

A1 a.

Quantity	Value of quantity with uncertainty	Percentage uncertainty
Current	2.5 ± 0.1 A	± 4.0%
Voltage	11.9 ± 0.1 V	± 0.84%
Time	300.0 ± 0.2 s	± 0.07%
Temperature rise	22 ± 1 °C	± 4.5%
Mass	1.000 ± 0.002 kg	± 0.2%

b. Specific heat capacity of copper $c = \frac{Q}{m\,\Delta\theta} = \frac{11.9 \times 2.5 \times 300}{1 \times 22} = 406\ \text{J kg}^{-1}\,°\text{C}^{-1}$

c. Percentage uncertainty in the specific heat capacity of copper = 4 + 0.84 + 0.07 + 4.5 + 0.2 = ±9.6%

d. Absolute value of uncertainty = $\frac{9.6}{100} \times 406 = \pm 39\ \text{J kg}^{-1}\,°\text{C}^{-1}$.

A2 The experiment gives specific heat capacity of copper $= 410 \pm 40\ \text{J kg}^{-1}\,°\text{C}^{-1}$. This is higher than the actual value of $385\ \text{J kg}^{-1}\,°\text{C}^{-1}$ but agrees within the range of experimental uncertainties. It is probably high because of heat losses: the value of *Q* that is transferred to the block would be lower than that calculated.

A3 All the insulation around the block could be made thicker and an addition layer of insulation in the form of a polystyrene block, for example, could be put beneath the block to further reduce energy transfer to the bench.

The % uncertainties in the current reading and in the temperature difference could be reduced. An ammeter with better resolution or a digital joulemeter could be used; a digital thermometer with better resolution could be used.

Assuming the insulation is very effective, heating the block for a longer time, resulting in a greater temperature rise, would reduce the % uncertainty in $\Delta\theta$ and so improve the accuracy of the experiment.

REQUIRED PRACTICAL

P1 a. The scale should be viewed directly, front-on at eye level.

b. The anti-parallax mirror is contained within the pressure gauge and aligned with the scale. The pointer is in the form of a narrow needle and the correct viewing of the gauge is achieved when the reflection of the needle is hidden behind the needle itself.

P2 a. The straight line shows that the relationship is of the form $V = kp^n$, giving log *V* = log *k* + *n* log *p*.

Intercept on *y*-axis = log *k* = 0.917

So *k* = $\log^{-1}$ 0.917 = 8.26

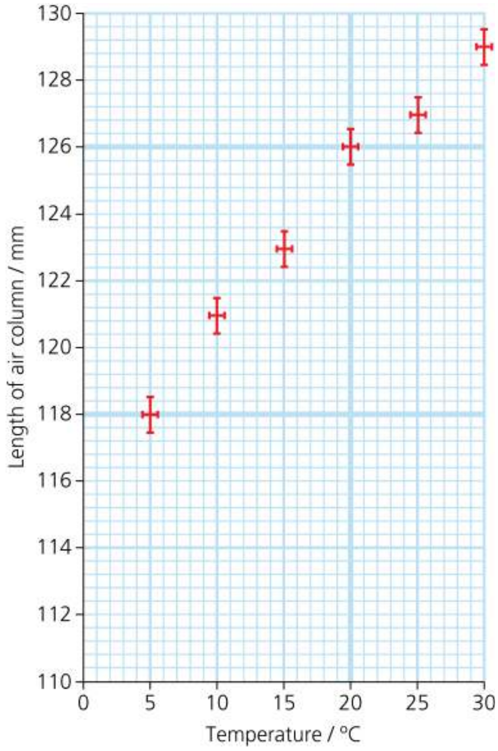
Gradient = *n* = −1.00

So $V = 8.26p^{-1}$ or $\frac{8.26}{p}$

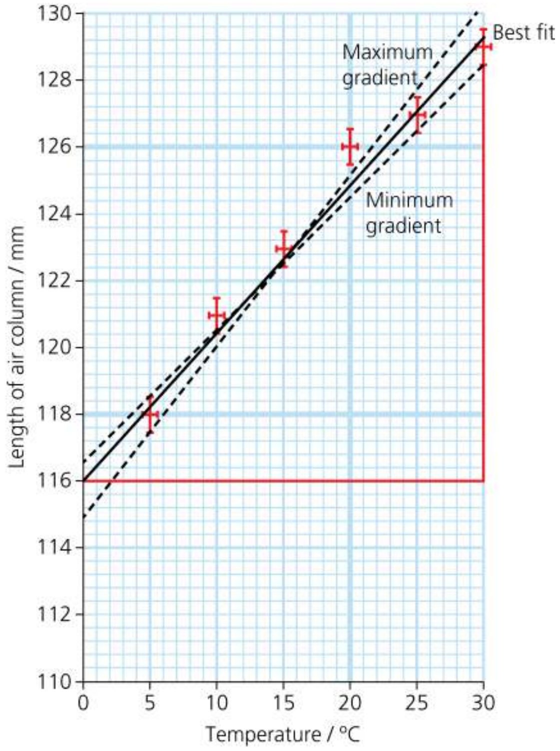
b. For *p* = 300 kPa: $V = \frac{8.26}{p} = \frac{8.26}{300 \times 10^3} = 2.75 \times 10^{-5}\ \text{m}^3 = 27.5\ \text{cm}^3$

P3 A thermometer with a smaller range and therefore greater resolution could be used. The capillary tube could be viewed with a magnifying glass to enable more accurate measurement of the length possibly to ±0.5mm.

P4 a.



b.



The gradient is $\frac{129 - 116}{30} \pm \frac{1.6}{30}$ = (0.43 ± 0.05) mm °C^{-1} and the *y* intercept is (116 ±1) mm

c. $l = 0.43\theta + 116$

$\therefore \theta = \frac{l - 116}{0.43}$

d. When $I = 0$, $\theta = \frac{-116}{0.43} = -270$°C (absolute zero)

e. Percentage uncertainty in $m = 100\left(\frac{0.05}{0.43}\right) = 12\%$

percentage uncertainty in $c = 100\left(\frac{1.6}{113}\right) = 1\%$

∴ percentage uncertainty in absolute zero = 1 + 12 = 13%

PRACTICE QUESTIONS

1a. An ideal gas is a gas that obeys Boyle's law, Charles' law and the pressure law at all pressures and temperatures and the effect of attractive forces between its molecules can always be regarded as negligible.

1b. i. The ideal gas equation can be rearranged to

$$n = \frac{pV}{RT} = \frac{1.6 \times 10^6 \times 0.20}{8.31 \times (273 + 22)} = 1.305 \times 10^2 = 131 \text{ mol}$$

1b. ii. $\rho = \frac{M}{V} = \frac{\text{molar mass} \times \text{number of moles}}{V} = \frac{4.3 \times 10^{-2} \times 1.305 \times 10^2}{0.20} = 28.06 = 28 \text{ kg m}^{-3}$

1b. iii. The new number of moles in the cylinder $n = \frac{pV}{RT} = \frac{3.6 \times 10^4 \times 0.20}{8.31 \times (273 - 50)} = 3.885 \text{ mol}$

Mass remaining in cylinder $= 3.885 \times 4.3 \times 10^{-2} = 0.167 = 0.17$ kg

2a. The Avogadro constant is the number or atoms in 12 g of carbon 12 and also the number of molecules in 1 mole of substance.

2b. i. The mean molecular kinetic energy

$$= \frac{3kT}{2} = \frac{3}{2} \times 1.38 \times 10^{-23} \times (22 + 273) = 6.107 \times 10^{-21} = 6.1 \times 10^{-21} \text{J}$$

2b. ii. Mass of a krypton atom $= \frac{0.084}{6.02 \times 10^{23}} = 1.395 \times 10^{-25}$ kg

$\frac{1}{2}m(c_{\text{rms}})^2 = \frac{3kT}{2}$ rearranges to give $(c_{\text{rms}})^2 = 2 \times \frac{\text{mean molecular kinetic energy}}{\text{mass}}$

Therefore $(c_{\text{rms}})^2 = 2 \times \frac{6.107 \times 10^{-21}}{1.395 \times 10^{-25}} = 8.756 \times 10^4 = 8.8 \times 10^4 \text{ m}^2 \text{ s}^{-2}$

2c. Since the gases are at the same temperature they have the same value for $\frac{1}{2}m(c_{\text{rms}})^2$

Therefore since a krypton atom has a large mass than an argon atom, its mean square speed must be smaller.

3a. $Q = mc\Delta\theta$ rearranges to give $\Delta\theta = \frac{Q}{mc} = \frac{8500}{0.12 \times 4200} = 16.87 = 17\,^\circ\text{C}$

3b. First, use $Q = mc\Delta\theta$ to determine the amount of energy required to raise the temperature of the water up to 100 °C.

$Q = mc\Delta\theta = 0.41 \times 4200 \times (100 - 26) = 1.274 \times 10^5 \text{J}$

Then determine the time taken from

$$t = \frac{\text{amount of energy supplied}}{\text{energy supplied per second}} = \frac{1.274 \times 10^5}{8500} = 14.99 = 15 \text{ s}$$

4a. Rearranging $Q = mc\Delta\theta$ gets the specific heat capacity

$$c = \frac{Q}{m\Delta\theta} = \frac{98 \times 30}{0.1 \times 14} = 2100 \text{ J kg}^{-1}\,^{\circ}\text{C}^{-1}$$

4b. Energy supplied by the heater in 500 s is $98 \times 500 = 49\ 000$ J

Energy needed to melt the ice $= mL = 0.1 \times 3.3 \times 10^{5} = 33\ 000$ J

Energy supplied to raise the temperature of water $= 49\ 000 - 33\ 000 = 16\ 000$ J

Water temperature $\Delta\theta = \frac{Q}{mc} = \frac{16000}{0.1 \times 4200} = 38\,^{\circ}\text{C}$

4c. The final temperature of the water would be higher when the procedure is repeated because for majority of the time, the ice/water mixture will have a temperature less than the room temperature of 25 °C, which means that energy will be transferred from the room into the ice/water mixture in addition to the energy from the heater.

5. C

6. C

7. C

8. D

9. B (Mean molecular kinetic energy $= \frac{3kT}{2} = \frac{3}{2} \times 1.38 \times 10^{-23} \times (50 + 273) = 6.7 \times 10^{-21}$ J)

10. D (Rearranging $pV = nRT$ gives $V = \frac{nRT}{p}$. The new volume $V_{\text{new}} = \frac{nR\,4T}{2p} = 2V$)

Chapter 4

ASSIGNMENT 1

A1 Centripetal acceleration $= r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = 3.84 \times 10^8 \times \left(\frac{2\pi}{27.3 \times 24 \times 3600}\right)^2 = 0.00272\ \text{m s}^{-2}$

A2 $\frac{9.81}{\text{Acceleration at Moon's orbit}} = \frac{9.81}{0.00272} = 3607$, or 3610 to 3 s.f.

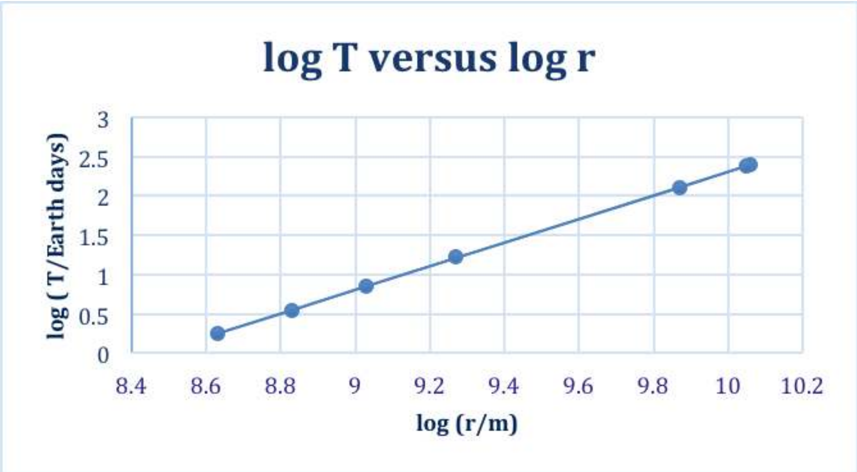
A3 $\frac{\text{Earth to Moon distance}}{\text{Distance from the centre to the Earth's surface}} = \frac{3.84 \times 10^8}{6.38 \times 10^6} = 60.2$

A4 The answers to **A2** and **A3** show that acceleration due to the Earth's gravitation decreases by a factor of $60^2 = 3600$ (2 s.f.) when the distance is increased by a factor of 60 (2 s.f.), demonstrating an inverse square relationship.

ASSIGNMENT 2

A1

log *T*	log *r*
2.40	10.06
2.38	10.05
2.11	9.87
1.22	9.27
0.85	9.03
0.55	8.83
0.25	8.63



A2 The gradient of the line produced when the data is plotted on Excel is 1.50, which confirms Newton's prediction. The percentage difference between the predicted value and the gradient of the graph drawn on graph paper can be calculated from

$$\% \text{ difference} = \frac{1.5 - \text{gradient of graph}}{1.5} \times 100$$

ASSIGNMENT 3

Students' own answers.

PRACTICE QUESTIONS

1a. i. Gravitational field strength is the force per unit mass and is a vector quantity.

1a. ii. Newton's law of gravity:

$$F = \frac{Gm_1m_2}{r^2}$$

For mass *m* on the Earth (mass *M*)

$$F = \frac{GMm}{(R+h)^2}$$

Gravitational field strength $g = \frac{F}{m} = \frac{GM}{(R+h)^2}$

1b. i. Force on the satellite

$$F = \frac{GMm}{(R+h)^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times2520}{(6.37\times10^6+1.39\times10^7)^2} = 2.44\times10^3\ \text{N}$$

1b. ii. Equating Newton's law with centripetal force:

$$F = \frac{GMm}{(R+h)^2} = m(R+h)\omega^2$$

Substituting $\omega = \frac{2\pi}{T}$ gives $\frac{GM}{(R+h)^2} = \frac{4\pi^2(R+h)}{T^2}$ and rearranging gives

$$T^2 = \frac{4\pi^2(R+h)^3}{GM} = \frac{4\pi^2(6.37\times10^6+1.39\times10^7)^3}{6.67\times10^{-11}\times5.98\times10^{24}} = 8.26\times10^8$$

which gives $T = 2.87\times10^4$ s.

Number of orbits in one day $= \frac{24\times3600}{2.87\times10^4} = 3.0$

1c. A satellite in a polar orbit scans the whole of the Earth making it suitable for weather mapping / military surveillance.

2a. Newton's law is

$$F = \frac{Gm_1m_2}{r^2}$$

Where m_1 and m_2 are point masses separated by distance *r*. The attractive force between the masses is *F* and *G* is the gravitational constant.

2b. i. Mass of the larger sphere $\rho V = \rho\times\frac{4}{3}\pi r^3 = 11.3\times10^3\times\frac{4}{3}\pi(0.1)^3 = 47.3\text{kg}$

2b. ii. Gravitational force $F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times47.3\times0.74}{(0.125)^2} = 1.5\times10^{-7}\ \text{N}$

2c. Since mass of sphere $= \rho\times\frac{4}{3}\pi r^3$ doubling the radius increases the mass of one sphere by a factor of 2^3.

Doubling the radius increases the separation by a factor of 2, so r^2 increases by a factor of 4

Gravitational force $F = \frac{Gm_1m_2}{r^2}$ so F increases by a factor of $8\times8\div4 = 16$.

3a. i. Equating Newton's law with centripetal force:

$$\frac{GMm}{r^2} = mr\omega^2$$

However, $\omega = \frac{2\pi}{T}$ so $\frac{GM}{r^2} = r\left(\frac{2\pi}{T}\right)^2$, which rearranges to give

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times(24\times3600)^2}{4\pi^2} = 7.29\times10^{22}$$

which gives radius if orbit $r = 4.23\times10^7$ m.

Altitude $4.2210^7 - 6.37\times10^6 = 3.58\times10^7\,\text{m} = 35\,800\,\text{km}$

3a. ii. Centripetal force $F = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2 = 650\times7370\times10^3\times\left(\frac{2\pi}{105\times60}\right)^2 = 4760\,\text{N}$

3b. A satellite in a low polar orbit has an orbital time period of a few hours and so can orbit the Earth several times in one day. It can have a range of orbits with different altitudes and time periods, but being relatively close to the Earth, it requires a greater speed than a more distant satellite. The satellite scans the whole of the Earth's surface making it ideal for mapping the Earth and underta king military surveillance, and monitoring weather and environmental changes over a period of time. A geosynchronous (or geostationary) satellite has an orbital time period equal to the Earth's rotational time period and travels in the same direction as the Earth's rotation. It travels more slowly than satellites closer to the Earth. The satellite orbits in the equatorial plane so that it maintains the same position above the Earth's surface. This ensures that the receiving dishes can have fixed positions pointing to the same spot in the sky and can maintain continuous contact with the satellite. Although the satellite can only communicate with a restricted area of the Earth's surface, its very high altitude ensures that this area is still very large. A satellite in this type of orbit is used for communications, for example, sending television, telephone and radio transmissions.

4. C (Equating gravitational field strengths: $\frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$, which can be rearranged: $\frac{r_1^2}{r_2^2} = \frac{m_1}{m_2}$ so $\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}$)

5. A (Using $-g = \frac{\Delta V}{\Delta r} = \frac{8.0}{10} = 0.80\,\text{N kg}^{-1}$)

6. D ($\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times3600} = 7.3\times10^{-5}\,\text{rad s}^{-1}$)

Chapter 5

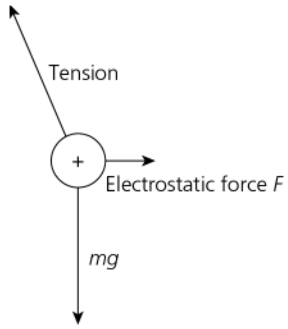
ASSIGNMENT 1

A1 A graph of d on the y axis versus $\frac{1}{r^2}$ on the x-axis generates a straight line through the origin showing that $d \propto \frac{1}{r^2}$ and therefore, since $F \propto d$, we can conclude that $F \propto \frac{1}{r^2}$, confirming Coulomb's law.

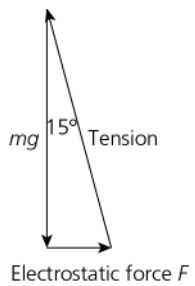
A2 $\sin\theta = \frac{d}{L}$ and therefore $d = L \times \sin\theta$

For $\theta = 10°$, $d = 40 \times \sin 10 = 6.95$ cm. Since all the values of d in the table are less than 6.95 cm, we can conclude that all the angles of $\theta < 10°$ and it is therefore acceptable to apply the small-angle approximation.

A3 **a.** **i.**



ii.



b. $\tan 15 = \frac{F}{mg}$

which rearranges to give

$F = mg \tan 15 = 1.2 \times 10^{-3} \times 9.81 \times \tan 15 = 3.2 \times 10^{-3}$ N

ASSIGNMENT 2

A1 Taking logs of $y = kx^2$:

$\log y = 2 \log x + \log k$

so a graph of log y against log x should be a straight line with gradient 2.0.

A2 **a.** The first two measurements of x are likely to be unreliable because they are small and therefore difficult to measure accurately.

b. A graph of y versus x^2 produces a straight line through the origin showing that $y \propto x^2$, confirming the parabolic shape of the electron path.

c. The log–log graph is a better test. With the y versus x^2 graph, if the data had quite a wide scatter, it might be difficult to tell whether the plotted points could be considered to fall on a straight line through the origin. But with the log–log graph, we know that the data will fit a straight line, whatever the power of x in the relationship. So a straight line can be confidently drawn, and the nearness of its gradient to 2.0 provides the test.

PRACTICE QUESTIONS

1a. The electric potential *V* at a point is defined as the work done (or energy needed) per unit charge in bringing a small positive test charge from infinity to the point.

1b. i. Electric potential $V = \frac{Q}{4\pi\varepsilon_0 r}$

which rearranges to $Q = V \times 4\pi\varepsilon_0 r = 3 \times 4\pi \times 8.85 \times 10^{-12} \times 0.3 = 1.0 \times 10^{-10}$ C

1b. ii. $V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{1.0 \times 10^{-10}}{4\pi \times 8.85 \times 10^{-12} \times 0.9} = 1.0$ V

1b. iii. $E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{1.0 \times 10^{-10}}{4\pi \times 8.85 \times 10^{-12} \times 0.6^2} = 2.50$ V m^{-1}

1c. i. Uniformly spaced parallel lines from the top to the bottom plate each with an arrow pointing downwards.

1c. ii. Uniform field: $E = \frac{V}{d} = \frac{2}{0.6} = 3.3$ V m^{-1}

1c. iii. The magnitudes of the electric field strengths in (b) and (c) are different because (b) is a radial field and (c) is a uniform field

2a. i. Electric field $E = \frac{V}{d} = \frac{600}{0.08} = 7500$ V m^{-1}

2a. ii. Electrostatic force $F = EQ = 7500 \times 0.17 \times 10^{-6} = 1.275 \times 10^{-3} = 1.3 \times 10^{-3}$ N

2b. i. Electrostatic force horizontally to the left; weight *mg* downwards; tension along the line of the thread towards the point of support.

2b. ii. $\tan\theta = \frac{F}{mg} = \frac{1.275 \times 10^{-3}}{4.8 \times 10^{-4} \times 9.81} = 0.2708$, which gives $\theta = 15°$

3a. Energy gained = work done, so $QV = 1.6 \times 10^{-19} \times 2000 = 3.2 \times 10^{-16}$ J

3b. Energy gained = $\frac{1}{2}mv^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times v^2 = 3.2 \times 10^{-16}$

Therefore $v = \sqrt{\frac{3.2 \times 10^{-16} \times 2}{9.11 \times 10^{-31}}} = 2.65 \times 10^7$ m s^{-1}

3c. i. The path will be a parabola curving upwards towards the top plate.

3c. ii. Electric field strength $E = \frac{V}{d} = \frac{3000}{0.05} = 6.0 \times 10^4$ V m^{-1}

3c. iii. The vertical acceleration $a = \frac{F}{m} = \frac{EQ}{m} = \frac{6.0 \times 10^4 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 1.1 \times 10^{16}$ m s^{-2}

4. D (Work is only done when a charge moves through a difference of potential)

5. A (The electron would be attracted to the more positive plate. Since it is moving in the opposite direction to the force, the electron will slow down as its kinetic energy is changed to potential energy.)

6. B (The +*Q* and +3*Q* change to –*Q* and +*Q* respectively. Since $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ the value of the numerator decreases to a third of its former value.)

Chapter 6

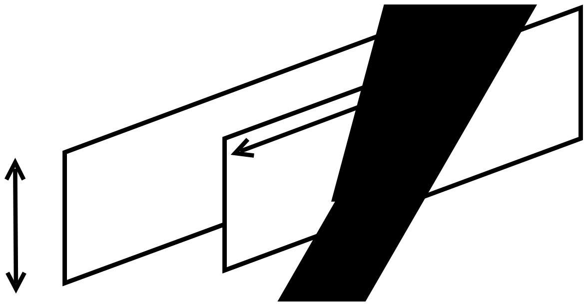
ASSIGNMENT 1

A1 Dependent variable: capacitance. Independent variable: plate separation. Control variable: area of overlap of the plates.

Adjust the callipers so that the internal jaws have the chosen separation and lock them in place using the locking screw. The jaws of the callipers can be positioned between the plates and the plates moved so that they fit snugly against the jaws. It would be preferable to have two sets of callipers which can be used simultaneously at opposite ends of the plates. The capacitance measurements should be plotted on the *y*-axis, with the plate separation *d* on the *x*-axis as this is the independent variable.

A2 Dependent variable: capacitance. Independent variable: area of overlap of the plates. Control variable: plate separation.

The area of overlap *A*, is equal to $L \times H$, as indicated below

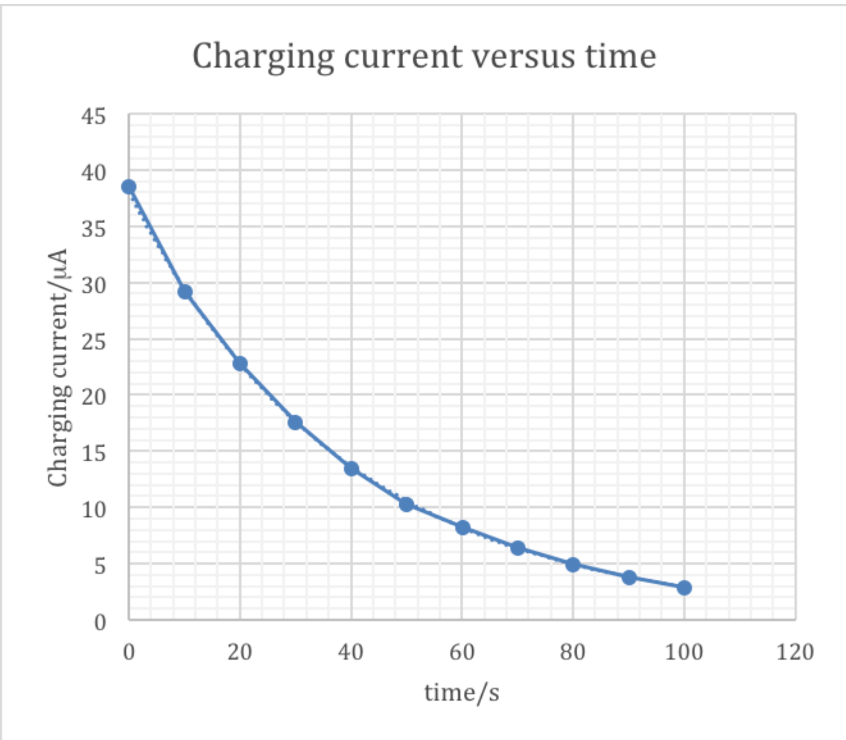


A3 Since the dimensions were measured with a 30 cm rule they would be 12.6 ± 0.1 cm and 9.1 ± 0.1 cm. The percentage uncertainties are 0.8% and 1.1% respectively. The area of overlap is 115 cm^2 with a percentage uncertainty of 1.9%. Therefore the area of overlap is $115 \pm 2\,cm^2$.

A4 It is safer to use a capacitance meter since a voltage does not need to be applied to the plates. In the method involving the coulombmeter a high-voltage power supply unit is required.

REQUIRED PRACTICAL

P1 The graph of charging current versus time is shown below.



To show that the curve is exponential it is necessary to demonstrate that the graph has a constant ratio property. This means that a constant increase in time will result in the charging current decreasing by a constant fraction. To establish this pattern, the times for the current to halve are taken from three different parts of the graph as shown below:

For the current to decrease from 30 μA to 15 μA is 36 – 9 = 27 s;

For the current to decrease from 20 μA to 10 μA is 51 – 25 = 26 s;

For the current to decrease from 10 μA to 5 μA is 79 – 51 = 28 s;

The times are sufficiently close to conclude that the constant ratio property has been demonstrated and also that the half-life for the circuit of 27 ±1 s.

P2 It is considered good practice to list the independent variable values in the left-hand column of a results table.

P3 The graph of $\log_e \frac{I}{\mu A}$ versus time is a straight line confirming that the current I decreases exponentially with time. The gradient is equal to -2.57×10^{-2} s^{-1}, which gives a time constant of 39 s.

P4 **a.** The charging current would drop more slowly.

b. $I = I_0 e^{-t/RC}$ so the time for the current to fall to $\frac{1}{e}$ occurs at time $t = CR$, therefore the time constant $= RC$. A larger resistor would make the time constant bigger.

P5 **a.** Time constant $= -\frac{1}{\text{gradient}}$ of $\log_e V$ vs t graph $= \frac{1}{0.893} = 1.12$ s

b. Total initial charge Q_0 = area between the curve and the time axis of I–t graph = 0.0326 C

Capacitance $C = \frac{Q_0}{V_0} = \frac{0.0326}{5.97} = 5.461 \times 10^{-3}$ F = 5460 μF

c. Since the time constant = RC = 1.12 s, resistance $R = \frac{1.12}{5.461 \times 10^{-3}} = 205\ \Omega$

P6 The nominal value of the capacitance is 4700 μF ± 20%. This gives a capacitance range of 5640 μF to 3760 μF. The measured value of 5460 μF fits within the manufacturer's tolerance percentage.

The nominal value of the resistance is 200 Ω ± 5%. This give a resistance range of 210 Ω to 190 Ω. The measured value of 205Ω fits within the manufacturer's tolerance percentage.

P7 Since the time constant of the circuit is 1.12 s, and the capacitor will be fully discharged in about 5 time constants ≈ 6 s, a measurement every 0.1 s would be suitable, which gives a sampling rate of 10 Hz.

P8 **a.** The graph of $\log_e V$ vs time of the data given in Table P2 gives a straight line with a gradient of -0.0324 s^{-1}. The time constant $= -\frac{1}{\text{gradient}} = \frac{1}{0.0324} = 30.9$ s.

b. Taking pd measurements at times indicated by a stopwatch leads to an uncertainty in the measurements as a consequence of trying to make simultaneous observations. The data logging equipment has no difficulty in making simultaneous measurements. A human observer is limited to probably about one set of measurements every 5 seconds, which is a sampling rate of 0.2 Hz, whereas a typical data logger can operate at a maximum sampling rate in excess of 1000 Hz.

PRACTICE QUESTIONS

1a. Capacitance is the charge stored per unit potential difference.

1b. i. Capacitance $C = \frac{Q}{V} = \frac{13.2 \times 10^{-6}}{6} = 2.2 \times 10^{-6}$ F

1b. ii. 63% of $13.2 \times 10^{-6} = 8.3 \times 10^{-6}$ C. This charge corresponds to a time of 15 ms.

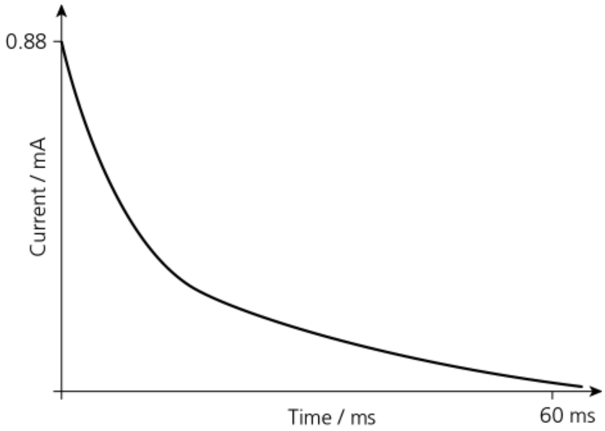
1b. iii. Time constant = $RC = 15 \times 10^{-3}$, which gives resistance $R = \frac{15 \times 10^{-3}}{2.2 \times 10^{-6}} = 6818 = 6820\ \Omega$

1b. iv. The gradient of a charge versus time graph gives the rate of flow of charge, which is the current.

1c. i. The maximum current occurs at the start of the charging process.

$$I_0 = \frac{\varepsilon}{R} = \frac{6}{6818} = 8.8 \times 10^{-4} \text{ A}$$

1c. ii.



2a. In a discharging circuit the time constant is the time for the charge to fall to $\frac{1}{e}$ of its original value which is equal to $\frac{1}{e} \times 4 = 1.47\ \mu\text{C}$, which corresponds to 216 ± 10 ms.

2b. If the resistance is smaller then the current will be larger assuming the capacitor is charged to the same pd. Since current is the rate of flow of charge, a bigger current would mean that the capacitor will discharge at a greater rate.

3a. i. Charge $Q = CV = 200 \times 10^{-6} \times 200 = 4.0 \times 10^{-2}$ C

3a. ii. Energy stored $= \frac{1}{2}CV^2 = 0.5 \times 200 \times 10^{-6} \times 200^2 = 4.0$ J

3b. Power output $= \frac{\text{energy output}}{\text{time}} = \frac{4}{1 \times 10^{-3}} = 4000\,W$

4. B (Half-life is 36 ms. Energy stored $E = \frac{1}{2}CV^2$ so for the energy to fall to $\frac{E}{16}$ would require the pd to fall to $\frac{1}{4}$ so 2 half-lives must pass, which is 72 ms.)

5. C (The charge is directly proportional to the pd)

6. D ($Q = CV$, so $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$)

Chapter 7

ASSIGNMENT 1

A1 Equating magnetic force to centripetal force gives $BQv = \frac{mv^2}{r}$ and therefore magnetic flux density $B = \frac{mv}{Qr}$, which shows that $B \propto \frac{\text{mass}}{\text{charge}}$ for a specific speed *v* and specific radius of curvature *r*.

A2 Magnetic flux density $B = \frac{mv}{Qr} = \frac{6.64 \times 10^{-27} \times 1.2 \times 10^5}{1.6 \times 10^{-19} \times 0.05} = 0.10\text{ T}$

A3 Equating $BQv = \frac{mv^2}{r}$ and rearranging gives $r = \frac{mv}{BQ}$, which shows that for a specific speed and flux density, the radius of curvature is directly proportional to the mass/charge ratio. The neon ion has almost double the mass of the boron ion but also double the charge and therefore both ions have very similar mass/charge ratio values.

REQUIRED PRACTICAL

P1 Independent variable: current. Dependent variable: magnetic force on the wire. Control variables: length of wire in the field, position of wire in the field.

P2 Repeat measurements should be taken.

P3 The table requires four columns as shown below. The current values should be expressed to 0.01 A since this matches the specified multimeter.

Current/A	Mass reading/g	Repeat mass reading/g	Force on the wire/N
1.00			
1.50			
2.00			
2.50			
3.00			
3.50			
4.00			
4.50			
5.00			
5.50			
6.00			

P4 Average mass reading is 2.55 ± 0.02 g. The percentage uncertainty in the mass is $= \frac{0.02}{2.55} \times 100 = 0.78\%$.

The force on the wire $F = \frac{0.55}{1000} \times 9.81 = 2.502 \times 10^{-2}$ N

Uncertainty on the force value is $\frac{0.78}{100} \times 2.502 \times 10^{-2} = 2 \times 10^{-4}$ N

Therefore the magnetic force on the wire is $2.50 \pm 0.02 \times 10^{-2}$ N

P5 To demonstrate that $F \propto I$, the graph of F versus I should be a straight line passing through the origin.

P6 Percentage uncertainty for a gradient $= \frac{\text{best gradient - worst gradient}}{\text{best gradient}} \times 100$

The worst gradient in this case is the shallowest.

The % uncertainty in the gradient $= \frac{0.0251 - 0.0244}{0.0251} \times 100 = \pm 2.8\%$

P7 Independent variable: length of wire. Dependent variable: magnetic force on the wire. Control variables: current, position of wire in the field.

P8 The stands attached to the clamps supporting the wire could be taped to the desk to ensure they don't move so that each wire always has the same position in the magnetic field. Alternatively a line could be drawn along the centre of the yoke parallel to its sides to which each wire is aligned.

P9 **a.** **i.** The graph suggests a systematic error.

ii. A possible source is regarding the length measurement. If the length of the wire was measured as shown in Figure P2, *within* the U-shape, it could be less than the effective length of the wire in the field since the wire is a few mm thick.

b. **i.** The magnetic force per unit current per unit length can be found by dividing the gradient of the line by the current flowing in the wire:

force per unit current per unit length $= 1.26 \div 5.00 = 0.25$ N m^{-1} A^{-1}

ii. The force per unit current per unit length is found using the gradient of the line, which is not affected by the systematic error.

P10 Referring to Part 3 of the required practical:

Independent variable: Magnetic flux density;

Dependent variable: Force on the wire;

Control variables: Electric current in the wire; length of the wire

P11 Referring to Part 3 of the required practical: If the student only had access to two different values of flux density for the experiment then only two data sets can be generated. It would not be appropriate to plot a graph containing only two points since valid conclusions could not be drawn from such a graph.

PRACTICE QUESTIONS

1a. *F*: magnetic force measured in newton. *B*: magnetic flux density measured in tesla; *I*: electric current measured in amps. *l*: length of wire in the field measured in metres.

1b. i. Weight of the bar $= mg = \rho Vg = 8900 \times 0.02 \times 0.03 \times 0.14 \times 9.81 = 7.334 = 7.3$ N

1b. ii. To support the copper bar the force due to the magnetic field must equal the weight of the bar:

$BIl = mg$

Therefore, $B = \frac{mg}{Il} = \frac{7.334}{60 \times 0.14} = 0.87$ T

2a. i. A charge particle at rest or moving parallel to the field experiences no magnetic force.

2a. ii. Equating magnetic force to centripetal force gives

$BQv = \frac{mv^2}{r}$

which rearranges to give $r = \frac{mv}{BQ}$, showing that $r \propto mv$ provided *B* and *Q* are constant.

2b. i. The magnetic field direction is out of the page.

2b. ii. Proton speed $v = \frac{BQr}{m} = \frac{0.48 \times 1.6 \times 10^{-19} \times 190 \times 10^{-3}}{1.67 \times 10^{-27}} = 8.738 \times 10^6 = 8.7 \times 10^6$ m s^{-1}

2b. iii. Time taken $t = \frac{\pi r}{v} = \frac{\pi \times 190 \times 10^{-3}}{8.738 \times 10^6} = 6.831 \times 10^{-8} = 6.8 \times 10^{-8}$ s

2b. iv. Time taken $t = \frac{r}{v} = \frac{\pi r}{\left(\frac{BQr}{m}\right)} = \frac{m\pi}{BQ}$ showing that *t* does not depend on radius *r*.

2c. Proton speed $v = \frac{BQr}{m} = \frac{0.48 \times 1.6 \times 10^{-19} \times 470 \times 10^{-3}}{1.67 \times 10^{-27}} = 2.161 \times 10^{7}$

Proton energy $\frac{1}{2}mv^2 = 0.5 \times 1.67 \times 10^{-27} \times (2.161 \times 10^{7})^2 = 3.901 \times 10^{-13}$ J = 2.4 MeV

3a. i. The direction of the magnetic field must be at right angles to the velocity.

3a. ii. *F*: magnetic force on a charged particle. *B*: magnetic flux density of the magnetic field. *Q*: charge on the particle. *v*: speed of the particle.

3b. i. Into the plane of the diagram.

3b. ii. The magnetic force on the ions is the same size as the electric force but in the opposite direction.

3b. iii. Since the magnetic force and the electric force are the same size:

$BQv = EQ$

Therefore $B = \frac{E}{v} = \frac{V}{dv} = \frac{48}{0.065 \times 1.7 \times 10^{5}} = 4.3 \times 10^{-3}$ T

3c. The force on the ions due to the electric field will be unchanged but the force due to the magnetic field would increase. Therefore, there would be a resultant upward force and the ions would be deflected upwards.

4. B ($BQv = \frac{mv^2}{r}$ which can be rearranged to $r = \frac{mv}{BQ} = \frac{p}{BQ}$. Doubling both *p* and *Q* means that *r* does not change.)

5. A (Reversing and doubling the current, doubles and reverses the force since $F = BIl$)

6. B (Fleming's left hand rule shows that the particles are deflected downwards)

Chapter 8

ASSIGNMENT 1

A1 **a.** The presence of both a negative and positive peak illustrates Lenz's law. As the N pole of the magnet enters the solenoid the change in flux induces an emf. The direction of this emf is such that the current that it drives makes the top of the solenoid act as an N pole to try to repel the incoming N pole of the magnet. The emf induced as the N pole of the magnet enters the solenoid corresponds to the negative peak on the voltage–time graph. As the magnet exits from the solenoid, the receding S pole induces an emf in the solenoid that drives a current in the opposite direction. This current would make the lower end of the solenoid act as an N pole to try to attract the receding S pole. Since the current and emf have changed direction, the emf corresponds to the positive peak on the voltage versus time graph.

b. The very high input resistance of the voltage sensor means that the current driven by the induced emf is extremely small. Therefore the forces exerted on the falling magnet, as a result of the current coil cutting the magnetic flux, which is proportional to the current (see section 7.2 of Chapter 7), are negligible and the magnet's acceleration will be that due to gravity.

A2 The size of the maximum induced emf as the magnet enters the solenoid is 0.09 V, but on leaving the solenoid, the magnet induces a larger maximum emf of 0.15 V. The force of gravity accelerates the magnet as it falls, so it exits the solenoid in a shorter time than it entered, causing a larger rate of change of flux linkage $\left(N\dfrac{\Delta\Phi}{\Delta t}\right)$ and hence a larger maximum induced emf, illustrating Faraday's law.

A3 Rearranging Faraday's law gives $\mathcal{E}\Delta t = N\Delta\Phi$, showing that the enclosed areas of the negative and positive peaks correspond to the change in flux linkage as the magnet enters and leaves the solenoid respectively. Since the change in flux linkage is the same as the magnet enters and leaves the solenoid, these two areas are equal in size.

A4 As the magnet enters and exits the solenoid, the induced emf drives a much larger current because the resistance of the ammeter is very small, so the only resistance in the circuit is due to the windings of the coil. The induced current will be in a direction such as to oppose the N pole entering the solenoid. As the magnet exits, the direction of the induced current will reverse in order to attract the receding S pole. Consequently the acceleration of the falling magnet will be reduced both on entering and exiting the solenoid.

ASSIGNMENT 2

A1 The primary voltage V_p is the independent variable.

A2 The transformer equation $\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p}$ can be rearranged to give $V_s = \dfrac{N_s}{N_p} \times V_p$. This predicts that a graph of V_s versus V_p would be a straight line through the origin with a gradient equal to $\dfrac{N_s}{N_p} = \dfrac{200}{400} = 0.5$

A3 Transformer efficiency = $\dfrac{I_s V_s}{I_p V_p} = \dfrac{0.112 \times 2.01}{0.071 \times 4.00} = 0.79$

The percentage uncertainty in the efficiency is the sum of the percentage uncertainties in: I_s, V_s, I_p, V_p

The percentage uncertainty in the efficiency = $\left(\dfrac{1}{112} + \dfrac{0.01}{2.01} + \dfrac{1}{71} + \dfrac{0.01}{4.00}\right) \times 100 = 3.0\%$

The uncertainty in the value for efficiency is therefore $\pm\dfrac{3}{100} \times 0.79 = \pm 0.02$

A4 **a.** Maximum value of $V_s = \dfrac{N_s}{N_p} \times V_p = 2 \times 12 = 24$ V

b. If the student had been using 200-turn and 1200-turn coils and had mistakenly connected them as a step-up transformer, the maximum output voltage, V_S, that the student could have been exposed to during the experiment would have been $V_s = \frac{N_s}{N_p} \times V_p = \frac{1200}{200} \times 12 = 72V$, which is much more dangerous than 24 V.

REQUIRED PRACTICAL

P1 Independent variable: angle θ.

Dependent variable: peak value of the induced emf.

Control variables: frequency of the signal generator; peak value of the alternating current in the large circular coil; position of the centre of the search coil relative to the centre of the large circular coil.

P2 Number of divisions from peak to peak is 2.9 ± 0.2 divisions, which has a percentage uncertainty of

$$\pm\frac{0.2}{2.9} \times 100 = \pm 6.9\%$$

The *y*-gain setting is $5\ \frac{\text{mV}}{\text{div}}$, which gives the peak-to-peak induced emf a value of 14.5 mV. The emf uncertainty is $\frac{\pm 6.9 \times 14.5}{100} = \pm 1\,\text{mV}$.

P3 **a.** A set square can be used to align the end of the search coil's plastic handle with the metre rule to help make measurement of *x* as accurate as possible. A point on the table should be marked to indicate exactly where the centre of the search coil should be positioned.

b. Angle $\theta = \cos^{-1}\frac{21.2}{22.8} = 21.6°$

The uncertainty in the distance from the centre of the search coil to the end of the plastic handle is ± 1 mm. The uncertainty in length *x* is ± 2 mm because alignment with the end of the search coil is required, which reduces the accuracy with which the measurement can be made. The percentage uncertainty in θ is equal to

$$\left(\frac{1}{22.8} \times 100\right) + \left(\frac{2}{21.2} \times 100\right) = \pm 1.4\%$$

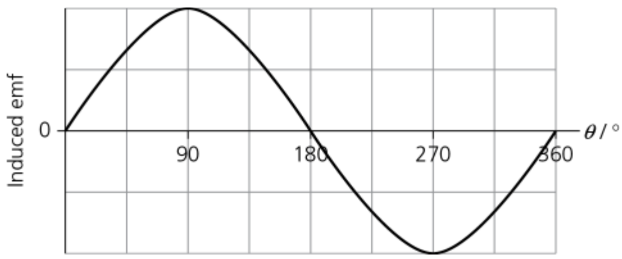
Therefore, angle $\theta = 21.6° \pm 0.3°$

c. Measuring angle θ with the protractor would be to ± 1°, so using length measurements with trigonometry is the more accurate of the two methods.

PRACTICE QUESTIONS

1a. Flux linkage $N\Phi = BAN \cos\theta = 2.8 \times 10^{-2} \times 1.9 \times 10^{-3} \times 50 \times \cos 35 = 2.2 \times 10^{-3}$ Wb turns.

1b. i.



1b. ii. In these positions the flux linkage is zero.

1b. iii. As the coil is rotated, the flux linkage through the coil changes. The induced emf in the coil is equal to the rate of change of flux linkage. Therefore the maximum induced emf occurs when the flux linkage is changing at its greatest rate and this occurs the when the plane of the coil is parallel to the magnetic field direction and $\theta = 90°$ or 270°.

2a. i. 128 V

2a. ii. 64 V

2b. Connect the alternating voltage to the *y*-input of the oscilloscope. Since two cycles takes 20 ms, set the time base to 2.0 ms/div. Adjust the *y*-gain to 20 V/divisions.

3a. i. A step-down transformer has a primary coil and a secondary coil wrapped around an iron core. The primary coil has more turns than the secondary coil. An alternating voltage is applied to the primary coil resulting in a smaller alternating voltage output from the secondary.

3a. ii. When a transformer is in operation, the ac currents flowing in the primary and secondary coils have a heating effect which results in a loss of energy. To minimise this energy loss, the coil windings are made from a thick wire of low resistivity typically copper.

The alternating current flowing in the primary coil creates a magnetic flux through the iron core that keeps changing. This changing flux induces emfs in the iron core which drive electric currents known as eddy currents. The flow of eddy currents generates heat which results in a loss of energy. To minimise this loss of energy, instead of having a solid block of iron as the core, the core is laminated and consists of layers of iron and insulator. The high resistance insulator prevents the flow of the eddy currents.

Since the current in the primary is an alternating current, the iron core is repeatedly magnetised in one direction, then demagnetised, then magnetised in the other direction. Magnetising then demagnetising the iron core generates heat in the core which results in a loss of energy. This energy loss is minimised by using a core made from a magnetically soft material such as soft (or annealed) iron which can be magnetised and demagnetised easily.

For a transformer to be efficient, as much of the magnetic flux created be the primary coil must pass through the secondary coil. To achieve this the transformer is designed to keep the two coils as close together as possible. For example, the coils can be wound one top of each other around the same section of the iron core.

(Any two of the above paragraphs are required)

3b. i. Power wasted in the internal circuits: $P = IV = 0.30 \times 9.0 = 2.7$ W

3b. ii. Efficiency $= \dfrac{\text{output power}}{\text{input power}} = 0.90$

Which gives the input power $= \dfrac{2.7}{0.90} = 3.0$ W

Since input power: $I_p V_p = 3.0$ then $I_p = \dfrac{3.0}{V_p} = \dfrac{3}{230} = 1.30 \times 10^{-2}$ A

3b. iii. Energy wasted in 1 year $Pt = 3.0 \times 0.80 \times 3.15 \times 10^7 = 7.56 \times 10^7$ J

3b. iv. The amount of energy equal to 1 kWh is equal to a power of 1 kW being delivered for 1 hour $=$ $1000 \times 3600 = 3.6 \times 10^6$ J

Number of kWh wasted $= \dfrac{7.56 \times 10^7}{3.6 \times 10^6} = 21$ kWh

Cost of energy wasted $= 21 \times 20 = 420$ p $=$ £4.20

3c. Advantages of switching off the computer is that this would save money and save essential fuel resources. Disadvantages of switching off the computer are that the energy saving from rebooting the computer may be greater than the energy saved by switching off and there is inconvenience of waiting for the computer to reboot.

4. D (Induced emf $= \varepsilon = N \dfrac{\Delta\Phi}{\Delta t} = 5 \times \dfrac{(15 \times 10^{-3} - 7.0 \times 10^{-3})}{0.5} = 8 \times 10^{-2}$ V $= 80$ mV)

5. A (Flux linkage $=$ *BAN cos θ* so the maximum value of flux linkage $=$ *BAN*)

Induced emf $= \varepsilon = BAN\omega \sin\omega t$ so the maximum value of the emf: $\varepsilon_0 = BAN\omega$.

So maximum flux linkage $= \dfrac{\varepsilon_0}{\omega} = \dfrac{\varepsilon_0}{2\pi f}$

6. B (The induced emf is equal to the rate of change of flux linkage. The emf is therefore equal to the gradient of the graph which is constant until t_0, then becomes zero.)

Chapter 9

ASSIGNMENT 1

A1 Average number of throws $= \frac{3.7+3.9+3.8+3.8}{4} = 3.8$ with a percentage uncertainty of 2.6%.

A2 A 20-sided dice would have a probability of $\frac{1}{20}$ for showing a '1' compared with $\frac{1}{6}$ for a six-sided dice. Therefore, with a 20-sided dice, fewer dice would be removed after each throw. It would therefore take a larger number of throws to halve the number of dice remaining.

A3 If the student removes those dice showing an even number, and the probability of throwing an even number is $\frac{1}{2}$, then half of the dice thrown would be removed after each throw, requiring 4 throws to reduce the number remaining to less than 50.

A4 The decay of a sample of radioactive isotope is a continuous process whereas the decay of the dice occurs at the instant of the throw and is therefore not continuous. (Also the decay of the dice is a geometric progression whereas radioactive decay is exponential.)

REQUIRED PRACTICAL

P1 Dependent variable: corrected count rate. Independent variable: distance between source and GM tube. Control variable: source activity.

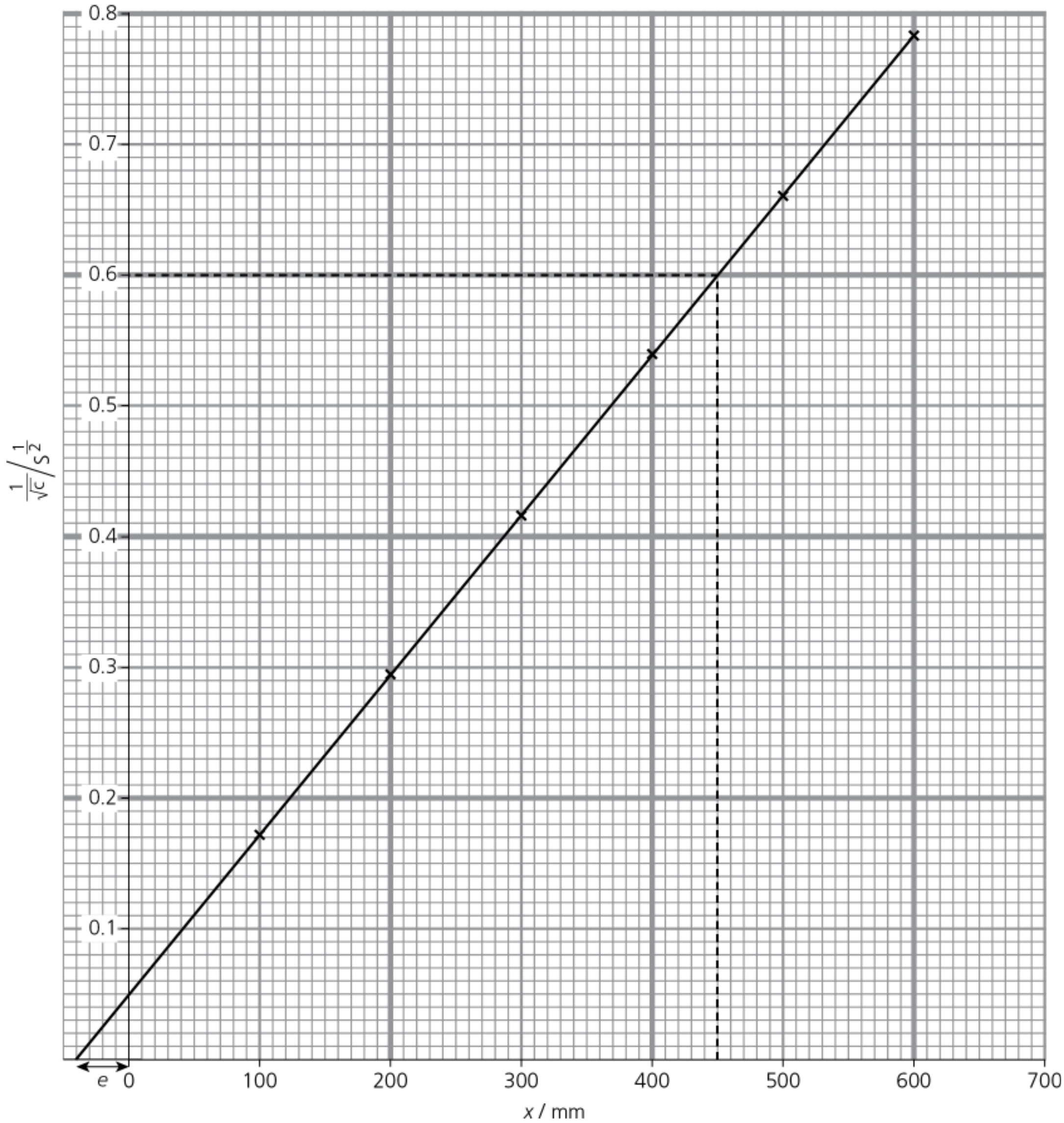
P2 Background count rate $= \frac{480}{20 \times 60} = 0.40$ counts s^{-1}

P3 **a.** Percentage uncertainty $= \pm \frac{\sqrt{2076}}{2076} \times 100 = \pm 2.2\%$

b.

Distance x/mm	Number of counts in 60 s	Count rate/counts s^{-1}	Corrected count rate C/counts s^{-1}	added column of values of $\frac{1}{\sqrt{C}}$
100	2101	35.02	34.62	0.17
200	725	12.08	11.68	0.29
300	378	6.30	5.90	0.41
400	238	3.97	3.57	0.53
500	169	2.82	2.42	0.64
600	125	2.08	1.68	0.77

c. i. Graph of $\frac{1}{\sqrt{C}}$ on the *y*-axis versus distance *x* on the *x*-axis:



ii. Systematic error *e* is 40 mm

d. When $x = 450$ mm, $\frac{1}{\sqrt{C}} = 0.590\ s^{\frac{1}{2}}$, which gives the corrected count rate $C = 2.87$ counts s^{-1}.

Therefore, the measured count rate is = 2.87 + 0.40 = 3.27 counts s^{-1}

PRACTICE QUESTIONS

1a. i. Uncertainty = $\pm\sqrt{13\,065} = \pm 114$

1a. ii. Uncertainty = $\pm\sqrt{12\,874} = \pm 113$

1a. iii. The distance of 10 cm between the source and the GM tube means that if alpha radiation was being emitted from the source, it wouldn't reach the GM tube so the change in the count reading cannot be due to the presence of alpha radiation. The count without the absorber could range from 12 951 to 13 179 and the count with the absorber could range from 12 761 to 12 987. Since the two counts overlap within their uncertainties, the apparent fall in the number of counts is insignificant and it can be concluded that the absorber had no effect on the radiation emitted from the source. It is therefore highly unlikely that beta radiation is being emitted from the source. The count reading with the absorber present is well above background levels, therefore, it can be concluded that the source is emitting gamma radiation.

1b. The source should be held with long tongs and should be returned to its lead lined box as soon as is possible to minimise exposure time.

2a. Sources of background radiation include cosmic rays from the Sun and from beyond the Solar System and radon gas in the atmosphere. (Could also include nuclear waste, nuclear fallout from weapons testing.)

2b. i. $\frac{\text{number of } \gamma \text{ photons incident on detector}}{\text{number of } \gamma \text{ photons produced by source}} = \frac{\text{detector area}}{\text{area of a sphere of radius 0.18 m}}$

$= \frac{1.5 \times 10^{-3}}{4\pi \times 0.18^2} = 3.684 \times 10^{-3} = 3.7 \times 10^{-3}$

$= \frac{1.5 \times 10^{-3}}{4\pi \times 0.18^2} = 3.684 \times 10^{-3} = 3.7 \times 10^{-3}$

2b. ii. Number of gammas entering the detector each second = $0.62 \times 400 = 248$.

Source activity = $248 \div 3.684 \times 10^{-3} = 6.732 \times 10^4 = 6.7 \times 10^4$ Bq

2c. Inverse square law for gamma: $I = \frac{k}{x^2}$, therefore the corrected count rate $C = \frac{\text{constant}}{x^2}$

Therefore, $0.62 \times 0.18^2 = C \times (0.1 + 0.18)^2$, which gives the new count rate, 0.26 s^{-1}.

3a. i. Decay constant $\lambda = \frac{\ln 2}{T_{½}} = \frac{\ln 2}{5740} = 1.208 \times 10^{-4} = 1.21 \times 10^{-4}$ years^{-1}

3a. ii. Number of atoms: $N = N_0 e^{-\lambda t}$, which rearranges to give $\frac{N}{N_0} = e^{-\lambda t}$

Therefore, $0.375 = e^{-1.208 \times 10^{-4} \times t}$. Taking natural logs gives $t = 8120$ years.

3b. i. If the sample is less than 200 years old, the change in the ratio of carbon-14 atoms to carbon-12 atoms compared with living wood would be too small to detect.

3b. ii. If the sample was over 60 000 years old, there are too few carbon-14 atoms present to be measured accurately.

4. C

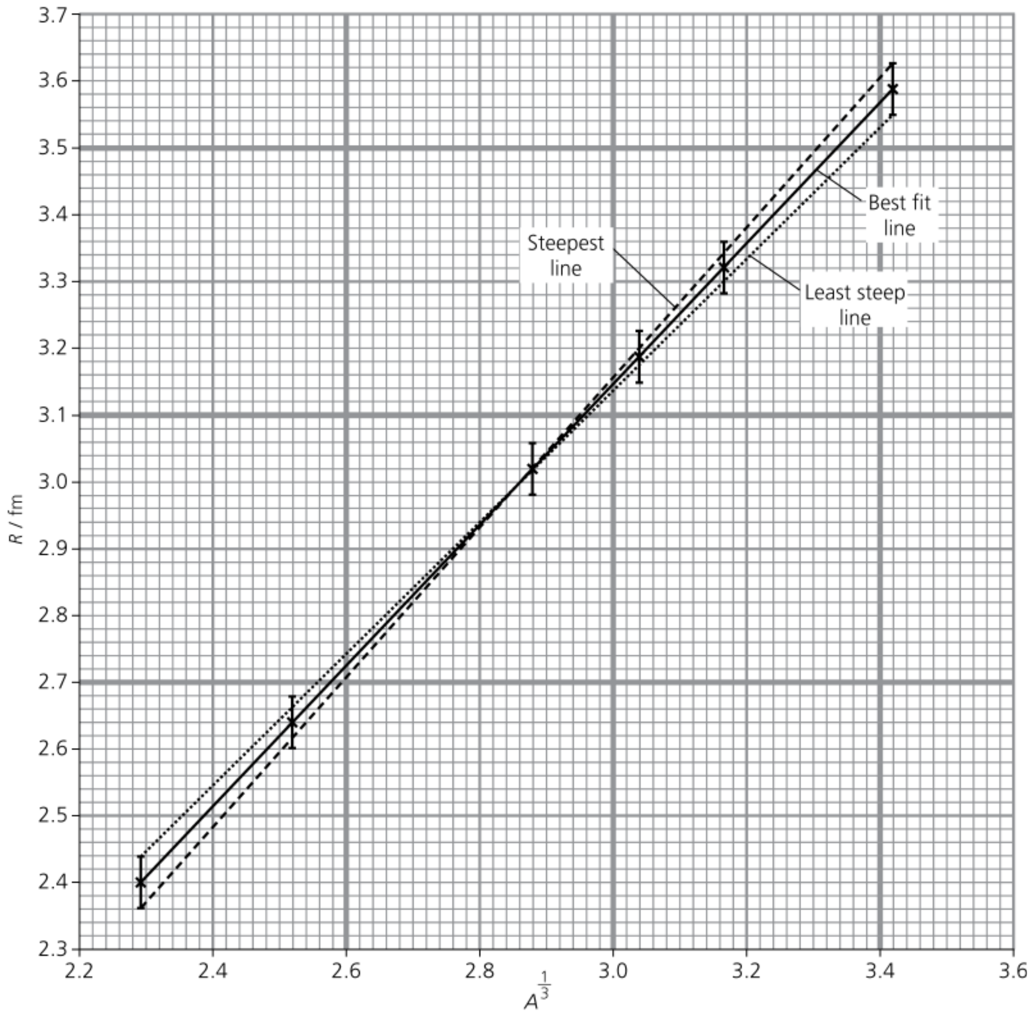
5. C (A drop in activity to $\frac{1}{128}$ correspond to 7 half-lives. So 1 half-life = $\frac{100}{7}$ = 14.3 days)

6. D (Gamma photons per second = $\frac{2500}{4\pi \times 2^2} = 50$)

Chapter 10

ASSIGNMENT 1

A1



A2 **a.** The gradient of the best fit line (in black) $= \frac{3.59 - 2.40}{3.42 - 2.29} = 1.053$ fm

Therefore, $R_0 = 1.05$ fm

b. **i.** The gradient of the steepest line (in dashed line) $= \frac{3.64 - 2.37}{3.42 - 2.29} = 1.124$ fm

Gradient of the least steep line (in dotted line) $= \frac{3.55 - 2.43}{3.42 - 2.29} = 0.9911$

ii. The worst gradient corresponds to the steepest line; therefore, the uncertainty in the gradient is equal to $1.124 - 1.053 = \pm 0.071$ fm.

The constant of proportionality $R_0 = 1.05 \pm 0.07$ fm

ASSIGNMENT 2

A1 Students' own answers.

A2 Students' own answers referring to data: e.g. 7.7% less coal was used in 2015; nuclear energy accounted for a very similar percentage – 22.2% in 2014 versus a slightly lower 20.5% in 2015; in 2015 8.6% more renewables were used than in 2014

a. Relevance to climate change: e.g. renewable energy makes up a greater proportion of electricity generation resources in 2015 than 2014, less carbon is released into the atmosphere than by burning fossil fuels, this should have a positive impact on climate change.

b. Support the argument for nuclear power: e.g. small amounts of material are required, carbon is not released into the atmosphere unlike burning fossil fuels, a constant reliable source of energy, whereas renewable sources often require things like the wind to be blowing, sunlight, tides etc.

Support the argument against nuclear power: e.g. nuclear power stations are expensive to operate, the fuel is expensive to obtain and carbon is released in the mining and transport of it, nuclear power stations have short lifespans and have to be decommissioned carefully and at great cost, any accidents can be very costly in terms of finance, lives and impact on the environment.

A3 Students' own answers.

A4 Students' own answers – some generally accepted arguments for/against and further information to request could be given in an answer here. See some of the ideas in **A2**.

PRACTICE QUESTIONS

1a. Control rods are lowered into the core of the reactor where they absorb neutrons reducing the number of fission reactions taking place.

1b. The fission fragments produced by the fission of uranium-235 are the main source of high level radioactive waste.

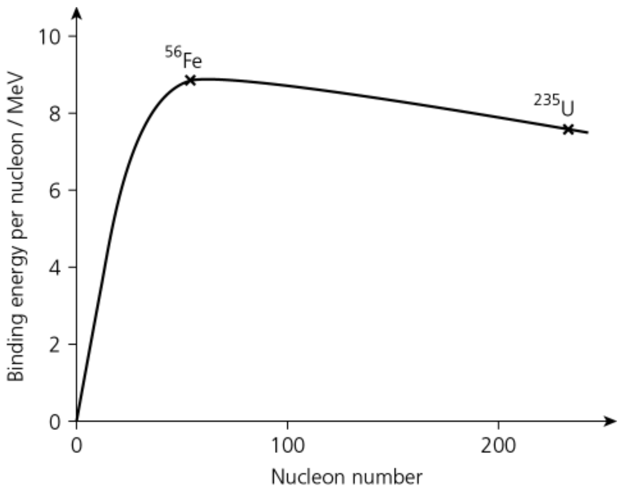
1c. i. If a moderator nucleus is excited it is in a nuclear energy level that is above its ground state. The excited nucleus undergoes a transition to its ground state emitting a gamma photon.

1c. ii. When the neutron undergoes an elastic collision with a moderator nucleus, it transfers some of its kinetic energy to the moderator nucleus. After a series of collisions, the neutron is travelling much more slowly and is much more likely to cause the fission of a uranium-235 nucleus.

2a. i. The binding energy of a nucleus is defined as the work that would need to be done to separate all of its nucleons.

2a. ii. An iron-56 nucleus has 26 protons and 30 neutrons. Therefore the mass difference is equal to $(30 \times 1.00867) + (26 \times 1.00728) - 55.92067 = 0.52871$ u. The binding energy is given by: $0.52871 \times 931.5 = 492.5$ MeV.

2a. iii.



The graph shows that the splitting of a heavy nucleus such as uranium-235 results in an overall increase in binding energy per nucleon and therefore a corresponding loss of mass, which results in a transfer of energy to the surroundings.

2b. i. Mass difference $= (2 \times 1.00728) - 0.00055 - 2.01355 = 0.00046$ u

2b. ii. Energy release $= 0.00046 \times 931.5 = 0.4285$ MeV

2b. iii. Fusion of two protons can take place inside stars because of the very high temperature which means that the protons are moving fast enough to overcome the electrostatic repulsion between them.

3a. $^{27}_{13}\text{Al} + \alpha \rightarrow \, ^{30}_{15}\text{P} + \, ^{1}_{0}\text{n}$

3b. Initial kinetic energy of the alpha particle is equal to the electrostatic potential energy at distance *d* from the centre of the aluminium nucleus. Therefore: $2.18\times10^{-12}=\frac{Q_1 Q_2}{4\pi\varepsilon_0 d}$, which rearranges to give distance

$$d=\frac{13\times1.6\times10^{-19}\times2\times1.6\times10^{-19}}{2.18\times10^{-12}\times4\pi\times8.85\times10^{-12}}=2.75\times10^{-15}\,\text{m}$$

4. B

5. C

6. A (Decay energy $=0.00254\times931.5=2.366$ MeV $=2.366\times10^{6}\times1.6\times10^{-19}$ J $=3.8\times10^{-13}$ J)

AQA A-Level Physics Year 2
Electronics Answers

Chapter 1

PRACTICE QUESTIONS

1a. i. The MOSFET has a high input resistance which means it has little or virtually no energy consumption in the ON and OFF states. The operation is by the potential difference on the gate only.

1b. Prevents static charge building up on the gate–source, reducing the capacitance and makes the gate voltage 0 V when no water/nothing between probes.

1c. A potential divider is formed by the copper strip sensor and the 1 MΩ resistor.

$$2.4\text{ V} = \frac{12 \times 1\,\text{M}\Omega}{(R_{\text{probes}} + 1)\,\text{M}\Omega} \text{ leading to } R_{\text{probes}} = \frac{9.6}{2.4} = 4\text{ M}\Omega$$

2a. The voltage across the divider network will not remain constant but will vary with the variation in the input voltage across it, therefore the voltages across the two resistors would also vary and so would not provide a steady voltage to the USB device.

2b. Maximum current through R is 80 mA + 15 ma = 95 mA

Minimum voltage is 10.5 V. Voltage across the resistor is (10.5 – 4.7) V = 5.8 V

$$\text{Value of } R = \frac{5.8\text{ V}}{95 \times 10^{-3}\text{ A}} = 61\ \Omega$$

2c. i. New voltage = 14.7 – 4.7 = 10 V

2c. ii. Current through R is $\frac{10\text{ V}}{61\Omega} = 164\text{ mA}$

2c. iii. Power dissipated = $(0.163)^2 \times 61\ \Omega = 1.6\text{ W}$

3a. i. About 750 nm

3a. ii. At 650nm the spectral responsivity is 0.38 A/W. For an incident power of 10 μW the photocurrent generated is $0.38 \times 10 \times 10^{-6} = 3.8\ \mu\text{A}$.

3b. i. Total energy of photons produced in scintillator = $3.3 \times 1.6 \times 10^{-19} \times 44 \times 10^3 = 2.3 \times 10^{-14}$ J

$$\text{Scintillator efficiency} = \frac{2.3 \times 10^{-14}\text{ J}}{6.8 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}} \times 100\% = 1.8\%$$

3b. ii. $$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34}\text{ Js} \times 3.00 \times 10^8\text{ m s}^{-1}}{458 \times 10^{-9}\text{ m}} = 4.3 \times 10^{-19}\text{ J} = 2.7\text{ eV}$$

Chapter 2

PRACTICE QUESTIONS

1a. i. A bit is a single binary value being either 0 or 1. A byte is a group of 8 bits.

1a. ii. $\frac{32}{8} = 4$ bytes

1b. i. Analogue signals are continuous and have any state value within a range. Digital signals only have two states: ‘high’ and ‘low’, ‘on’ and ‘off’ or 1 and 0.

1b. ii. In an analogue signal the signal information is encoded in the shape of the signal, which is prone to distortion by noise on the signal. In digital signals the information is encoded in the pattern of states and although the digital waveform can be distorted with noise, the state pattern can still be easily recognised.

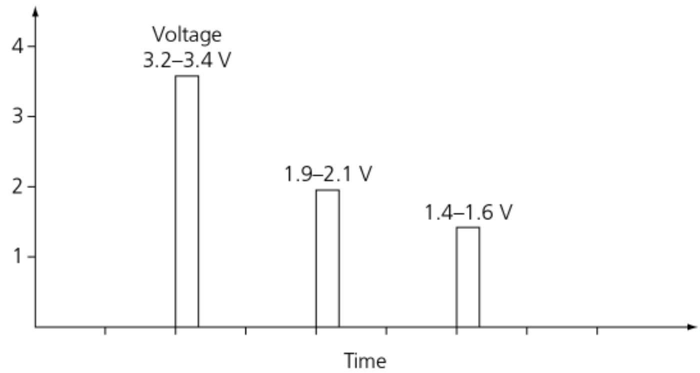
1c. i. The signal-to-noise ratio (SNR) is defined as the ratio of signal power to the noise power, often expressed in decibels.

1c. ii. $\text{SNR} = 20 \log \left(\frac{5}{0.2}\right) = 28 \text{ dB}$

2a. Pulse code modulation is the process by which the amplitude of an analogue signal is sampled and converted to a binary value represented as a series of pulses.

2b. i. Bandwidth = (20 000 – 20) Hz = 19 980 Hz

2b. ii.



2b. iii. 2×20 kHz = 40 kHz

2b. iv. The quantisation of the ADC is $2^{16} = 65\,536$ unique values. Therefore the voltage range of the ADC can be resolved into $\frac{10}{2^{16}}$ V = 152.6 μV.

2b. v. Increase the sampling rate by increasing the clock frequency into the sampling gate and increasing the number of bits used by the ADC.

3a. $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $C = \frac{1}{(f_0^2 \times 4\pi^2 \times L)} = \frac{1}{(50^2 \times 4\pi^2 \times 0.1)} = 5.1 \mu\text{F}$

3b. Q factor = $\frac{f_0}{f_B} = \frac{50}{2.5} = 20$

3c. $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8.0 \times 5.1 \times 10^{-6}\text{F}}} = 25\,\text{HZ}.$ If the value of *r* is lower then more current will flow in the LC circuit and the resonant peak at 25 Hz will be higher.

Chapter 3

ASSIGNMENT 1

A1 **a.** The resistance will increase, because l increases and A decreases.

b. The resistance will decrease.

A2 $\Delta R = R \times \text{GF} \times \text{strain} = 120 \times 2.0 \times 500 \times 10^{-6} = 0.12\,\Omega$

A3 **a.** They are the same.

b. Since the potentials at X and Y are the same no current will flow between X and Y and the voltage will read zero.

c. The potentials at X and Y will no longer be equal and a current will flow through the voltmeter and a voltage will be measured.

A4 **a.** The op-amp is configured as a difference amplifier.

b. Voltage gain $= \frac{500}{10} = 50$

c. **i.** $V_{out} = (V_+ - V_-) \times 50$. With $V_+ = V_-$ the output will be zero.

ii. $\Delta R = R \times \text{GF} \times \text{strain} = 150 \times 3.0 \times 750 \times 10^{-6} = 0.3375\,\Omega$

iii. Voltage at the inverting input will be $\frac{150}{300} \times 12 = 6\text{ V}$

Voltage at the non-inverting input will be $\frac{150 + 0.3375}{300} \times 12 = 6.0135\text{ V}$

Voltage difference $= 0.0135\text{ V}$

iv. Output $= 0.0135 \times 50 = 0.675\text{ V}$

v. Voltage at the non-inverting input will be $\frac{150 + 0.675}{300} \times 12 = 6.027\text{ V}$

Voltage difference $= 0.027\text{ V}$

Output $= 0.027 \times 50 = 1.35\text{ V}$, which is double that in part (iv).

The op-amp output voltage is proportional to the resistance change of the strain gauge, so gives linear amplification.

PRACTICE QUESTIONS

1a. **i.** Voltage gain $= -\frac{R_2}{R_1} = \frac{470000}{4700} = -100$

1a. **ii.** Point X is very close to zero volts due to the large open gain of the amplifier but it is not actually connected to 0 V.

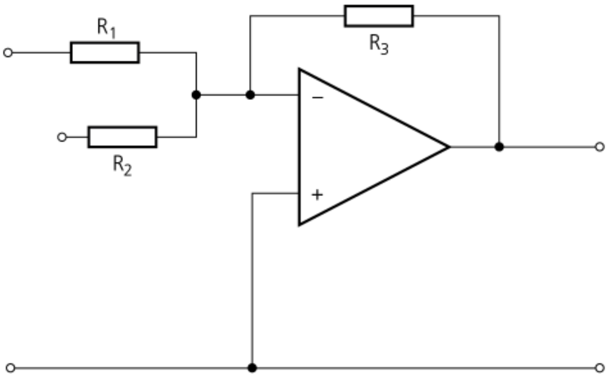
1b. **i.** The op-amp is connected as a difference amplifier.

1b. **ii.** T is a thermistor.

1b. **iii.** R_5 and R_6 form a potential divider into the non-inverting input of the op-amp. T and the variable resistor form another potential divider into the inverting input. Initially, the variable resistor is adjusted so the voltages at A and B are equal and there is 0 V at the output of the op-amp. If the temperature changes so that the resistance of the thermistor causes the voltage at A to become less than the voltage at B, then there will be a voltage at the output of the op-amp, causing the LED to light.

1b. **iv.** R_4 is a current-limiting resistor that ensures that the current through the LED does not exceed its maximum value.

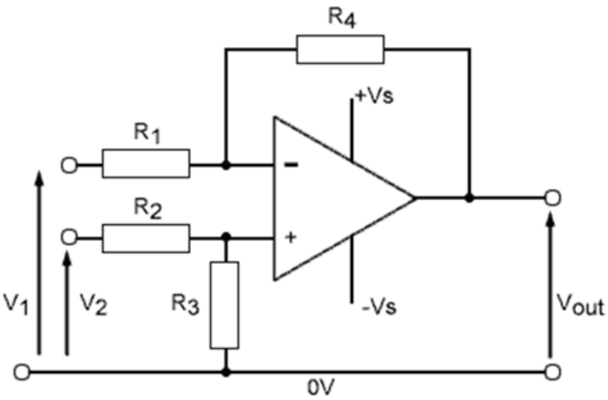
2a.



$V_{out} = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)R_3$

2b. For a voltage gain of 1, $R_1 = R_2 = R_3 = 10$ kΩ

2c.

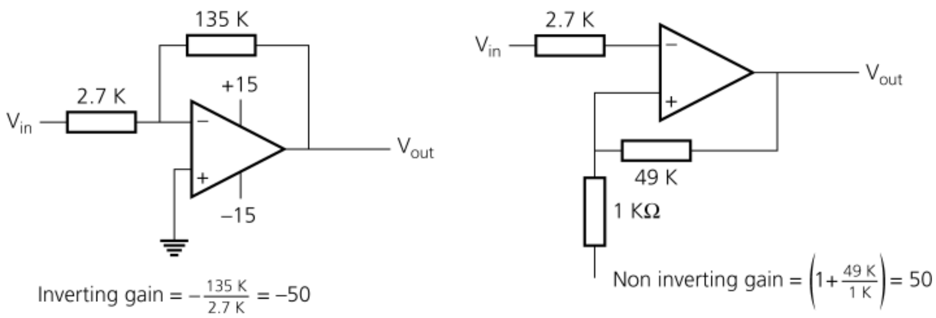


2d. $V_{out} = (V_1 - V_2) \times \left(\frac{R_4}{R_1}\right)$

2e. If the L + R and L – R signals are fed into the summing circuit then the output will be –((L + R) + (L – R)) = –2L

If the L + R and L – R signals are fed into the difference circuit then the output will be ((L + R) – (L – R)) = 2R, thus the left and right signal can be separated.

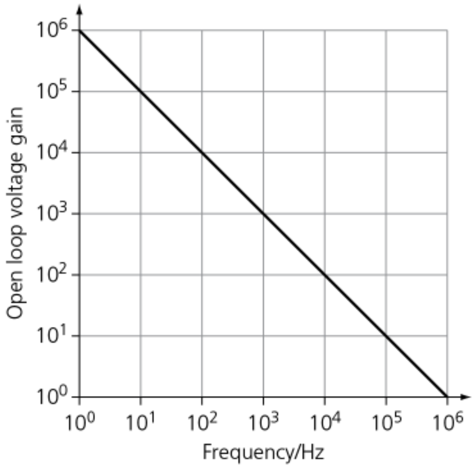
3a. **i.** and **ii.**



3b. Voltage gain = $\frac{V_{out}}{V_{in}}$ = 50 so max input = $\frac{15}{50}$ = 0.3 V = 300m V

3c. The gain–bandwidth product is defined as open-loop gain × bandwidth.

3d.



3e. Gain × bandwidth = 10^6 Hz so frequency at which the voltage gain of the amplifier becomes less than

$50 = \dfrac{10^6 \text{Hz}}{50} = 20$ kHz.

3f. The voltage gain remains at 50 up to 20 kHz so the preamplifier has sufficient gain for the audio frequency bandwidth.

4a. This is a potential divider network. Voltage at B = $\dfrac{20}{20+30} \times 12 = 8$ V

4b. R_1 has to be half the value of R_2. Possible values are 1.8 kΩ and 3.6 kΩ, which will give 8 V.

4c. i. When the temperature is 20 °C, the voltage at B will exceed 8V and the comparator will give a high (12 V) output.

4c. ii. When the temperature rises to 30 °C, the voltage at B will be less than 8 V and the comparator will give a low (0 V) output.

Chapter 4

ASSIGNMENT 1

A1 An oscillating system is said to resonate when there is an increase in amplitude of its oscillations while it is acted on by an external force whose frequency is equal or close to the natural frequency of a system.

A2 The frequency value 32 768 is a power of 2 (2^{15}).

A3 The binary divider divides the 32 768 pulses from the oscillator to down to 1 Hz.

A4 Fifteen separate stages of divide-by-two type flip flops are needed, because 32 768 = 2^{15} (see table).

STAGE	ACTION	FREQUENCY
0	Crystal out	32 768 Hz
1	Divide by 2	16 384 Hz
2	Divide by 2	8 192 Hz
3	Divide by 2	4 096 Hz
4	Divide by 2	2 048 Hz
5	Divide by 2	1 024 Hz
6	Divide by 2	512 Hz
7	Divide by 2	256 Hz
8	Divide by 2	128 Hz
9	Divide by 2	64 Hz
10	Divide by 2	32 Hz
11	Divide by 2	16 Hz
12	Divide by 2	8 Hz
13	Divide by 2	4 Hz
14	Divide by 2	2 Hz
15	Divide by 2	1 Hz

A5 A modulo-*n* counter is a counter that counts up to a chosen number and then resets.

A6 BCD (binary coded decimal) is a system of counting in binary from 0 to 9 using 4 bits.

A7 The binary number 0111

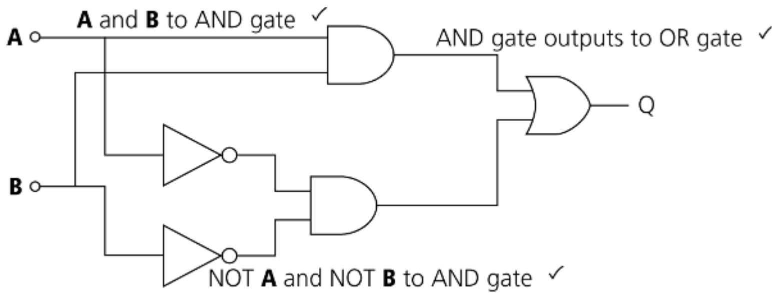
A8 By adding two displays together, a full range of numbers from 00 to 99 can be displayed using eight data bits (a byte).

PRACTICE QUESTIONS

1a.

A	B	$\overline{A}$	$\overline{B}$	A.B	$\overline{A} \cdot \overline{B}$	Q
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

1b.



2a. i. D = $\overline{A}$

2a. ii. E = $\overline{B}.\overline{C}$

2b. Q = $\overline{D} + \overline{E}$

2c.

A	B	C	D	E	Q
0	0	0	**1**	**1**	**0**
0	0	1	**1**	**1**	**0**
0	1	0	**1**	**1**	**0**
0	1	1	**1**	**0**	**0**
1	0	0	**0**	**1**	**0**
1	0	1	**0**	**1**	**0**
1	1	0	**0**	**1**	**0**
1	1	1	**0**	**0**	**1**

3a.

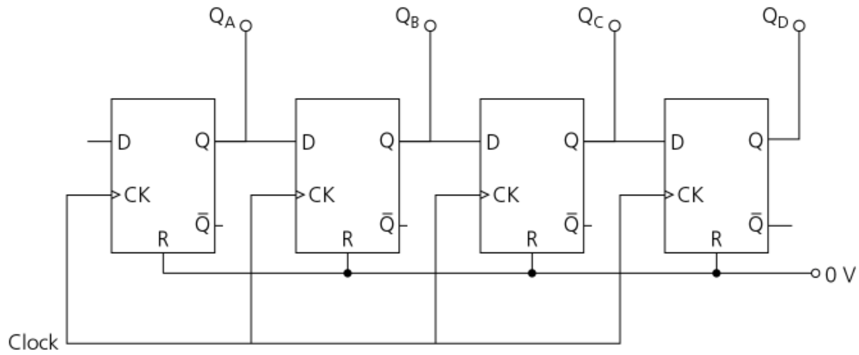
A	B	S	T	Z
0	0	1	0	1
1	0	0	0	0
0	1	0	0	0
1	1	0	1	1

3b. i.

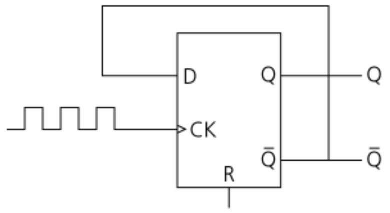
A	**B**	**Out**
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

3b. ii. A NOT gate

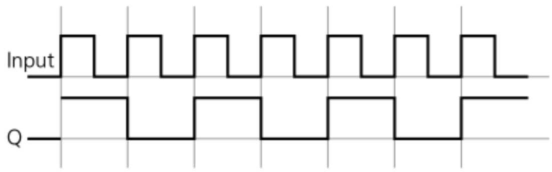
4. D = data input, CK = clock input



5a. i.



5a. ii.

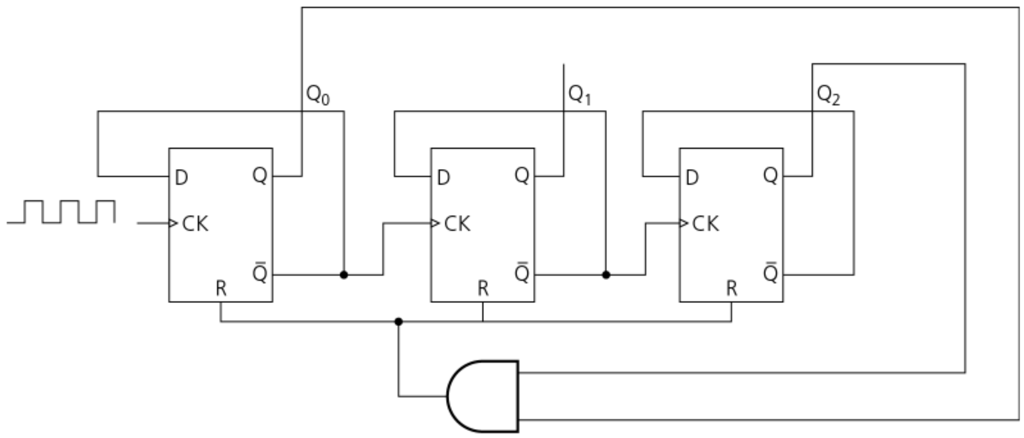


5b. i. 2.4 kHz

5b. ii.

Count	Q_2	Q_1	Q_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1

Unique state for 5 is $Q_2 = 1$ and $Q_0 = 1$ fed as input into AND gate to reset counter to zero, so circuit is as here.



6a. i. The output is a continuous output of regular ON–OFF pulses with a constant time period.

6a. ii. It can be used as a clock signal to trigger a sequential logic circuit, such as a counter.

6b. i. Duty cycle is

$$\text{duty cycle} = \frac{R_1 + R_2}{R_1 + 2R_2} \times 100\%$$

$$= \frac{1000 + 2000}{1000 + 4000} \times 100\% = 60\%$$

6b. ii. Frequency is

$$f = \frac{1.4}{(R_1 + 2R_2) \times C_1}$$

$$= \frac{1.4}{(1000 + 4000) \times 10 \times 10^{-6}} = 28 \text{ Hz}$$

6b. iii. Period is

$$T = \frac{1}{f} = \frac{1}{28} = 0.036 \text{ s or } 36 \text{ ms}$$

6b. iv. $\text{mark} = \frac{\text{period} \times \text{duty cycle}}{100\%} = \frac{\text{duty cycle}}{\text{frequency}} \times 100$

$= 0.7 \times (R_1 + R_2) \times C_1 = 0.7 \times (1000 + 2000)\ \Omega \times 10 \times 10^{-6}\text{ F} = 21 \text{ ms}$

space = period-mark - 36 ms – 21 ms = 15 ms

6b. iv. Mark-to-space ratio = $\frac{21}{15}$ = 1:4

Chapter 5: Data communication systems

ASSIGNMENT 1

A1 a.

A	B	OUT
0	0	0
0	1	1
1	0	1
1	1	0

b. **i.** 0 + 0 = 0

ii. 0 + 1 = 1

iii. 1 + 0 = 1

iv. 1 + 1 = 0

A2 **a.** ASCII code for the letter 'a' is 01100010

b.

Plain text	01100010
Random binary number (the key)	11010001
Binary sum (the encrypted data)	10110011

c.

Received encrypted data	10110011
Random binary number (the key)	11010001
Binary sum	01100010

The binary sum is the plain text recovered or de-encrypted.

d. $2^8 = 256$

A3 Use more bits to represent the letter. Transmit over a more secure guided transmission medium such as optical fibre.

PRACTICE QUESTIONS

1a. Downstream bandwidth = 1.1 MHz – 138 kHz = 962 kHz

1b. Number of downstream sub-channels = $\frac{962}{4.31}$ = 223.2 = 223 sub-channels (there must be a whole number of channels).

1c. 223 × 56 kbps = 12.488 Mbps

1d. 3 GBytes = $3 \times (1024)^3 \times 8$ bits = 3 221 225 472 × 8 = 25 769 803 776 bits.

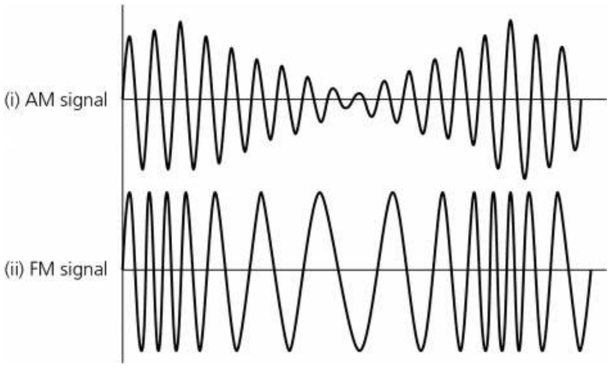
To every 8 bits of data, 3 are added = 35 433 480 192 bits.

$$\frac{35\ 433\ 480\ 192\ \text{bits}}{8\ \text{Mbps}} = \frac{35\ 433\ 480\ 192\ \text{bits}}{8\ 388\ 608\ \text{bps}} = 4224\ \text{s}$$

1e. Upload bandwidth = (138 – 26) kHz = 112 kHz. This is a smaller bandwidth than upload so there are fewer sub-channels available.

1f. People download data from the internet more than they upload data.

2a.



2b. AM bandwidth = 2 × 3 kHz = 6 kHz

2c. Practical FM bandwidth = 2(3 kHz + 5 kHz) = 16 kHz

3a. i. Ultra high frequency

3a. ii. Frequency modulation

3a. iii. The instantaneous amplitude/voltage of the information signal determines variations in the frequency of the carrier wave.

3b. i. $c = f \times \lambda;\ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m s}^{-1}}{446.09375 \times 10^6\ \text{Hz}} = 67.25\ \text{cm}$

3b. ii. Use any two adjacent channels.

For example, channel 2 – channel 1 = 446.01875 – 446.00625 = 12.5 kHz.

3b. iii. FM bandwidth = 2(3 kHz + 2.5 kHz) = 11 kHz. This is *less* than the channel spacing of 12.5 kHz so the maximum information frequency can be supported.

3b. iv. FM bandwidth = 2(3 kHz + 5 kHz) = 16 kHz. This is *mor*e than the channel spacing so there would be interference created on adjacent channels.

3c. Half-duplex means that there is only one communication channel available for transmission and reception so two-way communication between users is possible but only one at a time.

4a. With guided transmission media, information is guided along a physical path such as coaxial cables, twisted pair cables and optical fibres to its destination.

With unguided transmission media, information is sent without the use of a physical path to guide it to its destination. Examples of unguided transmission media are radio and microwave transmissions where electromagnetic waves are sent, but are not guided to their destination.

4b.

		Metal Wire	**Optical fibre**
i.	Physical properties	Can corrode; heavier/more expensive	Glass does not corrode; thinner/less expensive; but more complex to make joins
ii.	External interference	Less secure – can be tapped without breaking cable; can pick up electrical noise	More secure – cannot be tapped unless cable is broken into; immune from electrical noise and can be used in environments with electrical noise
iii.	Signal-carrying properties	Can have high attenuation of signal; lower bandwidth can carry fewer channels	Low attenuation of signal; much higher bandwidth and data capacity

5a. Amplitude modulation conveys information over a carrier wave by varying its amplitude while keeping its frequency constant. Frequency modulation conveys information over a carrier wave by varying its frequency while keeping its amplitude constant.

5b. bandwidth = 2 × 1.8 kHz = 3.6 kHz

5c. FM radio needs a large bandwidth, so if low frequency carriers were used this would limit the number of FM broadcasting channels available.

5d. FM signals have greater noise immunity as the information is not contained in the amplitude. The original signal can be completely recovered as long as all frequencies are detectable.

AQA A-Level Physics Year 2
Astrophysics Answers

Chapter 1

ASSIGNMENT 1

A1 f_o = 200 mm and f_e = 25 mm, so $M = \frac{200}{25} = 8.0$

A2 Collecting power is proportional to the square of the objective diameter so the ratio is $\frac{50^2}{10^2} = 25$.

The telescope can collect 25 times more light than the human eye.

A3 **a.** $\theta = \frac{\lambda}{D}$

i. $\frac{685 \times 10^{-9}\,\text{m}}{50 \times 10^{-3}\,\text{m}} = 1.37 \times 10^{-5}$ rad = 2.8 arcsec

ii. $\frac{550 \times 10^{-9}\,\text{m}}{50 \times 10^{-3}\,\text{m}} = 1.10 \times 10^{-5}$ rad = 2.3 arcsec

iii. $\frac{445 \times 10^{-9}\,\text{m}}{50 \times 10^{-3}\,\text{m}} = 0.89 \times 10^{-5}$ rad = 1.8 arcsec

The best resolution is achieved with blue light.

b. A telescope is diffraction-limited if it meets the Rayleigh limit of angular resolution for two closely spaced sources. The constructed telescope is unlikely to approach the diffraction limit as the practical resolution depends of the quality of the lenses (how free they are from aberrations), the 'seeing' though the turbulent atmosphere of the Earth and the sharpness of the observer's vision.

c. Using your answers to part **a**, theoretically the telescope should be able to resolve the first double at green wavelengths and the second double at blue wavelengths. In practice this will depend on the factors mentioned in the answer to part **b**.

A4 The lens telescope would be subject to chromatic and spherical aberration.

A5 **a.** Increase the objective lens diameter (but this may give more aberration).

b. Decrease the focal length of eyepiece (or increase the focal length of objective).

c. Increase the objective lens diameter (but this may give more aberration).

A6 At higher magnification a solid rigid mount will be needed to keep the image steady when you look at it in the eyepiece (because the field of view has been decreased).

ASSIGNMENT 2

A1 **a.** $\theta = \frac{\lambda}{D} = \frac{1.5 \times 10^{-2}\,\text{m}}{28\,\text{m}} = 5.3 \times 10^{-4}$ rad = 109.3 arcsec

b. $\theta = \frac{6 \times 10^{-2}}{217 \times 10^{3}\,\text{m}} = 2.8 \times 10^{-7}$ rad = 0.06 arcsec

c. At a wavelength of 6 cm the angular resolution of MERLIN is about the same as that of the Hubble telescope at optical wavelengths.

d. $\theta = \frac{\lambda}{D} = \frac{0.2\text{ m}}{6.4\times10^{6}\text{ m}} = 3.1 \times 10^{-8}$ rad = 0.006 arcsec, which is about 10 times better resolution than the Hubble.

e. The telescopes can be connected together in variety of combinations and operate simultaneously to produce much improved resolution.

A2 a. ALMA operates in the part of the electromagnetic spectrum between infrared and radio waves. Sub-millimetre wavelengths are absorbed by water vapour in the atmosphere so building the radio telescope at high altitudes where the atmosphere is thinner reduces the effects of absorption allowing sub-millimetre waves to be detected.

b. i. Single dish diameter = 12 m. 1 THz = $\frac{3.00\times10^{8}\text{ m s}^{-1}}{1\times10^{12}\text{ Hz}} = 3 \times 10^{-4}$ m

$\theta = \frac{\lambda}{D} = \frac{3.\times10^{-4}}{12} = 2.5 \times 10^{-5}$ rad = 5.1 arcsec

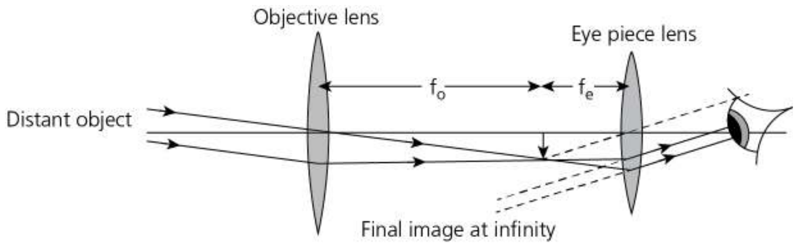
ii. Maximum baseline is 16 km, so $\theta = \frac{3\times10^{-4}}{16\times10^{3}} = 1.9 \times 10^{-8}$ rad = 0.004 arcsec

iii. 10^3

c. One of ALMA's key objectives is to investigate the births of stars in dust clouds. The sub-millimetre waves emitted by cool gas can penetrate the dust revealing the process of star formation that would be otherwise be hidden from optical telescopes.

PRACTICE QUESTIONS

1a. A refracting telescope in normal adjustment means that an object is viewed at infinity. To achieve this the focal point of the eyepiece lens and the objective lens are set to coincide and the length of the telescope is the sum of the focal lengths $f_o + f_e$ of the two lenses.



1b. Since the telescope is in normal adjustments $f_o + f_e = 1$

Magnification $M = \frac{f_o}{200} = 200$

So $f_o = 1 - f_e = 1 - \frac{f_o}{200}$

$f_o\left(1+\frac{1}{200}\right) = 1$

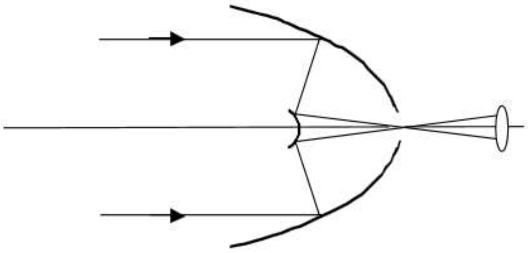
$f_o = \frac{200}{201} = 0.955 = 995$ mm

$f_e = 1000 - 995 = 5$ mm

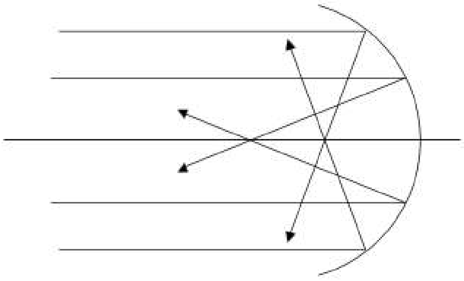
1c. $s = r \times \theta$

$$\theta = \frac{s}{r} = \frac{14.3}{50 \times 10^3\,\text{m}} = 2.7 \times 10^{-4}\ \text{rad}$$

2a.



2b. i.



2b. ii. $\theta = \frac{\lambda}{D} = \frac{630 \times 10^{-9}\,\text{m}}{0.15\,\text{m}} = 4.2 \times 10^{-6}$ rad

2b. iii. $s = r\theta$ so $\theta = \frac{4.8 \times 10^3\,\text{km}}{1.4 \times 10^9\,\text{km}} = 3.43 \times 10^{-6}$ rad. The claim is unlikely to be valid as this angular resolution is less than the minimum angular resolution calculated in part **b(ii)**.

3. Answers should include some of the following factors.

3a. Size

Telescopes are often built as large as possible in order to increase the collecting power, which is proportional to $(\text{diameter})^2$.

The diameter of the objective of telescopes is also often as large as possible in order to improve the resolving power, as the minimum angular resolution is proportional to $\frac{1}{\text{diameter}}$.

3b. Siting

Apart from visible and some parts of the radio wave section, all the other parts of the EM spectrum are significantly absorbed by the atmosphere.

To reduce the effects of absorption, IR telescopes are often placed in dry areas and/or very high up.

UV is significantly absorbed by the ozone layer, so UV telescopes are generally put into orbit.

X-ray telescopes are also put into orbit to avoid atmospheric absorption.

To avoid atmospheric distortion, visible telescopes are often placed high up.

To avoid interference from terrestrial sources, radio telescopes may be situated away from centres of population.

To avoid light pollution, visible telescopes are often placed a long way from centres of population.

4. Use $\theta = \frac{\lambda}{D}$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{8}\,\mathrm{m\,s^{-1}}}{230 \times 10^{9}\,\mathrm{Hz}} = 1.3 \times 10^{-3}\ \mathrm{m}$$

$$\text{So } \theta = \frac{1.3 \times 10^{-3}\,\mathrm{m}}{5 \times 10^{6}\,\mathrm{m}} = 2.6 \times 10^{10}\ \mathrm{rad}$$

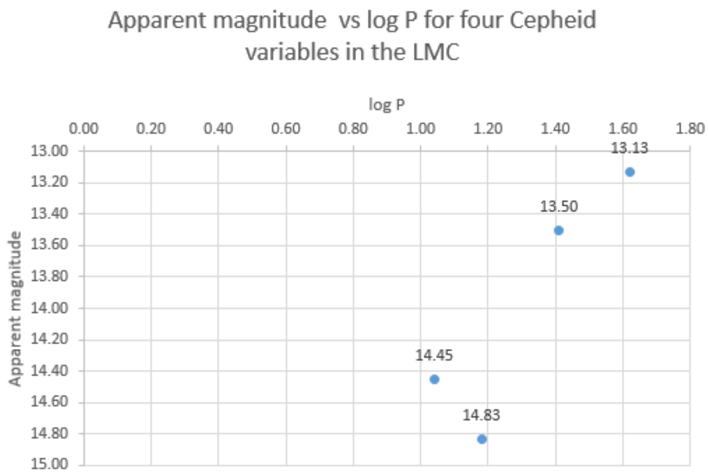
5. Chromatic aberration is due to the refractive index of the material being different for different wavelengths (and so colours). The lens is unable to bring all visible wavelengths to the same point of focus. This results in an image with fuzzy coloured edges. Chromatic aberration in a convex lens can be reduced by use of an achromatic doublet lens, which is a composite lens made of two different types of glass that bring two wavelengths (typically red and blue) into focus at the same point.

Chapter 2

ASSIGNMENT 1

A1 Students' own readings/graph

Star	m_{Max}	m_{Min}	$\frac{m_{Max} + m_{Min}}{2}$	$\frac{P}{\text{days}}$	log *P*
HV 837	12.60	13.65	13.13	42	1.62
HV 1967	13.00	14.00	13.50	26	1.41
HV 843	14.35	15.30	14.83	15	1.18
HV 2063	14.10	14.80	14.45	11	1.04



Estimated best straight line drawn through points.

Modulus gradient value = apparent magnitude/log *P* ≈ 4 log day^{-1}

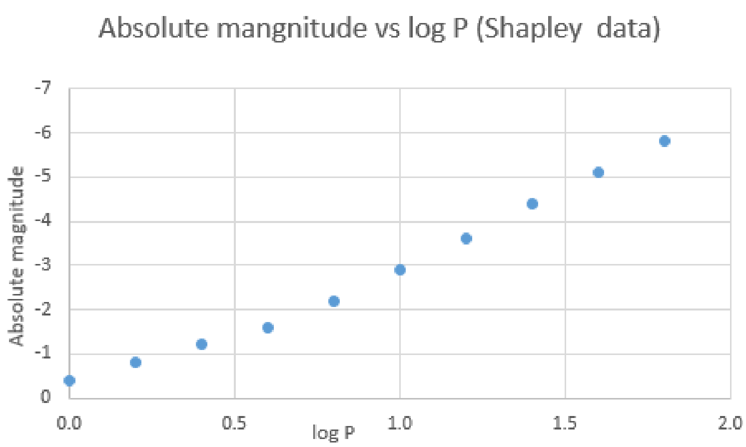
A2 Apparent magnitude is a logarithmic function, so log *T* is plotted to get a linear graph.

A3 By measuring the apparent magnitudes of more Cepheid variable stars, more points could be plotted

A4 The distance between Cepheid variables in a galaxy is much less than the distance between the galaxy and Earth.

A5 We only know the apparent magnitude – we need to know the absolute magnitude of a Cepheid variable to know its distance.

A6 Students' own graph



Estimated best straight line drawn through points.

Modulus of gradient value = absolute magnitude/log $P \approx 3$ log day^{-1}

A7 For a given Cepheid period the absolute magnitude and the apparent magnitude can be found from the graphs. Then the distance can be found using Pogson's law, $m - M = 5 \log \frac{d}{10}$.

A8 That they all have the same physical properties, giving rise to the same relationship between luminosity variation period and total luminosity.

A9 At very large distances it is not possible to resolve individual stars in galaxies.

PRACTICE QUESTIONS

1a. Use $m - M = 5 \log \frac{d}{10}$

$-1.4 - (1.4) = -2.8 = 5 \log \frac{d}{10}$

$10^{-0.56} = \frac{d}{10}$, $d = 2.75$ pc

1b. i. Use $\frac{P_{\text{SiriusA}}}{P_{\text{SiriusB}}} = \frac{\sigma A_{\text{SiriusA}} T^4_{\text{SiriusA}}}{\sigma A_{\text{SiriusB}} T^4_{\text{SiriusB}}}$

So, $\left(\frac{2400}{12}\right)^2 \left(\frac{10\,000}{25\,000}\right)^4 = 1.6 \times 10^3$

Sirius A radiates about 1000 times more power than Sirius B.

1b. ii. The difference in the apparent (or absolute) magnitudes = 9.8.

So difference in brightness = $2.51^{9.8} = 8.3 \times 10^3$, or about 8000 times.

1b. iii. The spectrum of a star is related to its temperature. Hotter stars produce a lot more power outside the visible part of the electromagnetic spectrum, whereas the apparent or absolute magnitude refers to brightness in the visible region.

2. The outer atmosphere or photosphere of the star contains hydrogen with electrons in the $n = 2$ state. Photons from lower layers pass through this outer layer. The electrons will be excited to higher energies, absorbing photons corresponding to specific wavelengths. When they de-excite they may drop though various energy levels and photons emitted will be radiated in random directions. So the characteristic Balmer pattern of spectral absorption lines appears in the star's continuous spectrum.

3a. Use $\lambda_{\text{max}} = 2.9 \times 10^{-3}$ m K/T so $T = \frac{2.9 \times 10^{-3}\,\text{m K}}{3.4 \times 10^{-7}\,\text{m}} = 8.5 \times 10^3$ K

3b. $\frac{P_{\text{D}}}{P_{\text{S}}} = \frac{\sigma A_{\text{D}} T^4_{\text{D}}}{\sigma A_{\text{S}} T^4_{\text{S}}}$

Rearranging and cancelling Boltzmann's constant

$\frac{A_{\text{D}}}{A_{\text{S}}} = \frac{P_{\text{D}} T^4_{\text{S}}}{P_{\text{S}} T^4_{\text{D}}} = 70\,000 \left(\frac{5700}{8500}\right)^4 = 1.42 \times 10^4$

So $\frac{r_{\text{D}}}{r_{\text{S}}} = \sqrt{1.48 \times 10^4} = 122$

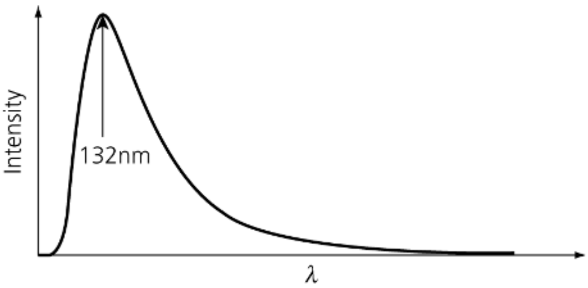
And $r_{\text{D}} = 119 \times 6.96 \times 10^8 = 8.28 \times 10^{10}$ m

4a. i. The brightness of a star as it would appear from a distance of 10 pc.

4a. ii. Betelgeuse. Bellatrix is actually a lot brighter than Betelgeuse (the absolute magnitude is a lot more negative), but only appears to be a bit brighter (the apparent magnitude is only a little smaller) so Betelgeuse must be closer.

4b. i. Use $\lambda_{max} = 2.9 \times 10^{-3}$ m K/T so $\lambda_{max} = \dfrac{2.9 \times 10^{-3}\ \text{m K}}{22\,000\ \text{K}} = 1.32 \times 10^{-7}$ m or 132 nm.

4b. ii.



4c. i. Bellatrix is a B type star

4c. ii. helium

4c. iii. Because it is much cooler and the temperature of the star is not high enough to excite electrons to the $n = 2$ state required for the Balmer lines.

5a. Similarity: both would appear the same brightness as the apparent magnitudes are the same.

Difference: Kocab would appear orange/red, Polaris yellow/white due to their spectral classes and different temperatures.

5b. Polaris is further from Earth. Both stars are same size and Polaris is hotter as it is an F type star. As $P = \sigma AT^4$ and they are the same size it would mean that Polaris has the greater power output. So Polaris must be further from Earth to appear as the same apparent magnitude as Kocab.

Chapter 3

ASSIGNMENT 1

A1 $R = \sqrt{\frac{L}{4\pi\sigma T^4}}$

A2 **a.** A G type star

b. $R = \sqrt{\frac{3.90 \times 10^{26}\,W}{4\pi \times 5.67 \times 10^{-8}\,Wm^{-2}\,K^{-4}\,(5800)^4}} = 6.96 \times 10^5$ km.

A3 **a.** Published values of absolute magnitude, temperature and radius are given here. Your values will be estimates but should be of a similar order of magnitude.

Star name	Spectral class	Type of star	Colour	Absolute magnitude M	$\frac{L}{L_{Sun}}$	$\frac{T}{K}$	$\frac{R}{R_{Sun}}$
Betelgeuse	B	Supergiant	red	–5.14	1.25×10^5	3140	1205
Aldebaran	K	Red giant	orange	–0.63	520	3190	100
Sirius A	A	Main sequence	blue	+1.42	25.4	9940	1.7
Spica	B	Main sequence	blue	–3.55	12 100	22 400	7.4
Rigel	B	Supergiant	blue-white	–7.84	1.25×10^5	12 100	79.5
Sirius B	D*	White dwarf	blue	+11.2	0.026	25 200	0.0085

*White dwarfs have a special stellar classification letter of their own labelled ‘D’.

b. Students’ own answers.

A4 No. The luminosity, temperature and radius are dictated by the mass of the star.

A5 Students’ own answers.

PRACTICE QUESTIONS

1a. X-ray telescopes need to be in orbit as X-rays are absorbed by the Earth’s atmosphere and do not reach the ground.

1b. i. A black hole is region of space-time that has such a strong gravitational field that no particle or electromagnetic radiation can escape from it and is the endpoint of stars with main sequence masses greater than 3 solar masses.

1b. ii. $R_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11}\,Nm^2\,kg^{-1} \times 7 \times 2.0 \times 10^{30}\,kg}{(3.00 \times 10^8\,m\,s^{-1})^2} = 2.1 \times 10^4$ m

2a. The absolute magnitude is apparent magnitude at a distance of 10 pc

2b. The absolute magnitude axis should have a scale from +15 to –10 and the temperature axis scale should be from 50 000 to 2500 K.

2c. i. The Sun is at position 5700 K and absolute magnitude 5.

2c. ii. W at same absolute magnitude, but further to left.

2c. iii. X at same temperature as S but greater absolute magnitude.

2c. iv. Y at same absolute magnitude or above S, on the right hand side of the diagram.

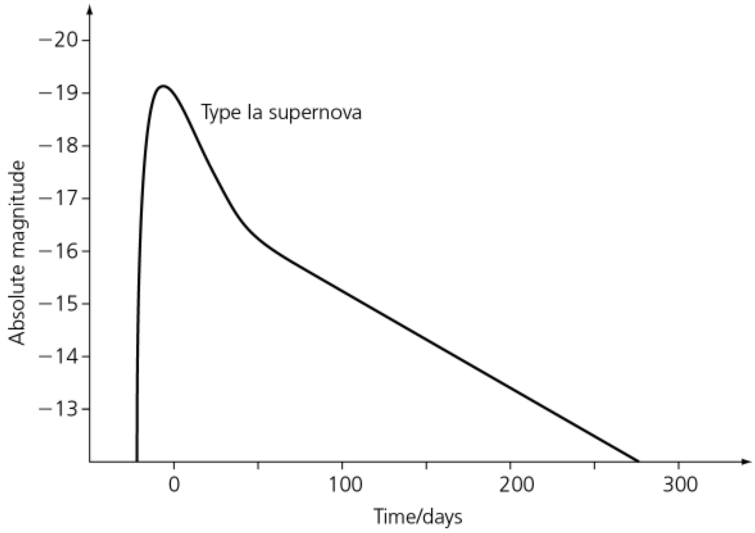
2d. W has a similar power output but is hotter. Using $P = \sigma AT^4$, A must be less so W must have a smaller diameter than the Sun.

3a. A star that suddenly and rapidly increases in absolute magnitude, because of an explosion that ejects its outer layers.

3b. A massive star (more than 1.4 times as massive as the Sun) becomes a large red giant or supergiant after its main sequence stage. When all of its nuclear fuel is used up the star collapses extremely rapidly and a supernova explosion occurs, blowing the outer parts of the star into space in an expanding gas shell. What is left of the star's core is a neutron star, an extremely compact dense object composed almost entirely of neutrons.

3c. i. All Type Ia supernovae produce the same light curves, with the same peak value of absolute magnitude. So if their apparent magnitude is measured, the distance from Earth can be calculated from $m - M = 5 \log \frac{d}{10}$.

3c. ii.



3d. $m - M = 5 \log \frac{d}{10}$

$M = -19.3$

$m - (-19.3) = 5 \log \frac{46}{10}$

$m = 5 \log 4.6 - 19.3$

so $m = -16.0$

This is a difference of 3 magnitudes from the apparent magnitude of the full Moon, which means that the supernova would be $(2.51)^3 = 16$ times brighter than the full moon. We would certainly be able to see it in daylight.

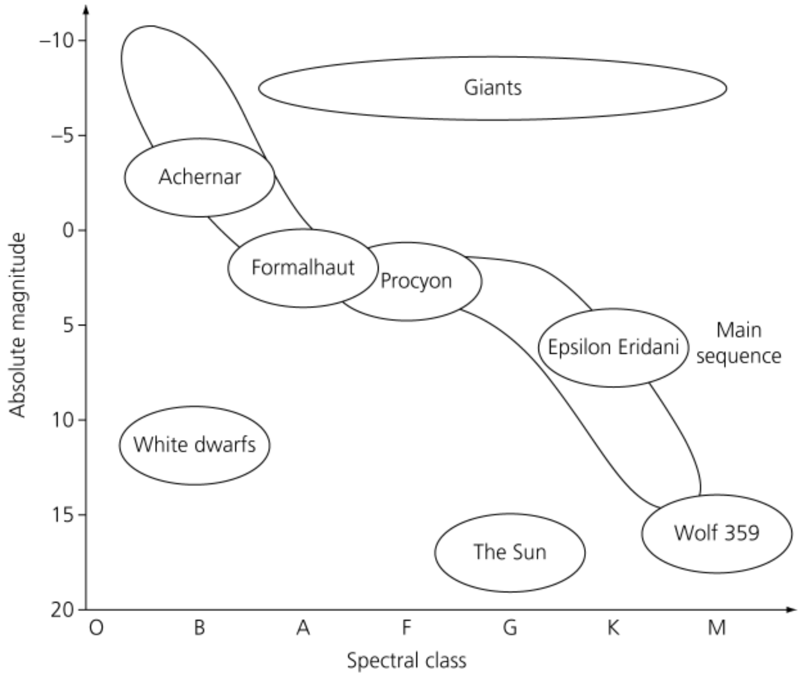
4a. i. Procyon

4a. ii. Wolf 359

4a. iii. Wolf 359

4a. iv. Achernar

4b. i. and **ii.**



4b. iii. They are all main sequence stars. This means they are in equilibrium, in the longest stage in their lives. They have stopped contracting and their energy comes from nuclear fusion of hydrogen to helium in their cores. They will remain in this stage until the hydrogen in their cores is used up.

4c. $m - M = 5 \log \frac{d}{10}$

$13.4 - 16.6 = 5 \log \frac{d}{10}$

$-0.64 = \log \frac{d}{10}$

$d = 2.3$ pc

Chapter 4

ASSIGNMENT 1

A1 Since carbon has four valence electrons it will readily form bonds with many other elements to form carbon-containing molecules. Moreover, most carbon-based molecules are very stable and will not spontaneously fall apart but also react to make further new molecules

A2 The carbon on Earth has been made in the interiors of massive stars which explode as supernovae ejecting carbon and other elements into the interstellar medium out of which is formed more stars and planets.

A3 **a.** $d_{max} = \sqrt{\dfrac{L_{star}}{4\pi\sigma T^4}} = \sqrt{\dfrac{3.90\times10^{26}\,\text{W}}{4\pi\times5.67\times10^{-8}\,\text{W m}^{-1}\text{K}^{-4}(273.2\,\text{K})^4}} = 3.13\times10^{11}\,\text{m} = 2.09\,\text{AU}$

$d_{min} = \sqrt{\dfrac{L_{star}}{4\pi\sigma T^4}} = \sqrt{\dfrac{3.90\times10^{26}\,\text{W}}{4\pi\times5.67\times10^{-8}\,\text{W m}^{-1}\text{K}^{-4}(373.2\,\text{K})^4}} = 1.67\times10^{11}\,\text{m} = 1.11\,\text{AU}$

b. The Earth is not a perfect black body. It reflects some radiation. It is not in perfect equilibrium. In addition, its atmosphere and surface features will also produce local variations in temperature.

c. At 1.0 AU, the Earth is 0.3 AU outside its minimum orbital radius and 0.5 AU inside its maximum for where water can exist in liquid form.

A4 $T = \sqrt[4]{\dfrac{L}{4\pi\sigma r^2}} = \sqrt[4]{\dfrac{0.04\times3.90\times10^{26}\,\text{W}}{4\pi\times5.67\times10^{-8}\,\text{W m}^{-2}\text{K}^{-4}(0.37\times1.5\times10^{11}\,\text{m})^2}} = 290\,\text{K}$

This is in the range where liquid water can exist.

A5 A planet in a highly elliptical orbit will experience extreme changes in temperature and move in and out of its habitable zone, which means that water may vaporise or freeze depending where it is in its orbit.

A6 As life has developed on Earth over many millions of years during the main-sequence lifetime of the Sun (about 10 billion years), stars with longer main-sequence lifetimes may have a better chance of developing life.

PRACTICE QUESTIONS

1a. The red-shift is when light or other electromagnetic radiation from an object is increased in wavelength or shifted to the red end of the spectrum due the receding motion of the object relative to the observer.

1b. (From Data sheet) $H = 67.3\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

$v = H \times d = 67.3\ \text{km s}^{-1}\ \text{Mpc}^{-1} \times 25\ \text{Mpc} = 1.7\times10^3\ \text{m s}^{-1}$

2a. $z = \dfrac{v}{c}$, therefore $v = c \times z = 3\times10^8\ \text{m s}^{-1} \times 0.025 = 7500\ \text{km s}^{-1}$

(From Data sheet) 1 ly = $\dfrac{1}{3.26}$ pc

So in pc, $d = \dfrac{340\times10^6}{3.26} = 104\times10^6$ pc or 104 Mpc

$H = \dfrac{v}{d} = \dfrac{7500\ \text{km s}^{-1}}{104.2\ \text{Mpc}} = 72\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

2b. $H = 72\times10^{-3}\ \text{m s}^{-1}\ \text{pc}^{-1}$

Using 1 pc = 3.09×10^{16} m, $H = 2.33\times10^{-18}\ \text{s}^{-1}$

Age of universe = $\dfrac{1}{H_0} = \dfrac{1}{2.33\times10^{-18}\ \text{s}^{-1}} = 4.3\times10^{17}$ s or about 13.6 billion years.

3a. $z = \dfrac{v}{c} = \dfrac{900\times10^3\ \text{m s}^{-1}}{3\times10^8\ \text{m s}^{-1}} = 0.003$

3b. (From Data sheet) $H = 65$ km s^{-1} Mpc^{-1}

$$d = \frac{v}{H} = \frac{900\ \text{km s}^{-1}}{65\ \text{km s}^{-1}\ \text{Mpc}^{-1}} = 13.8\ \text{Mpc}$$

4. Answer should include the fact that type 1a supernovae are standard candles – their peak absolute magnitude is known and so when the apparent magnitude measured the distance can be determined by $m - M = 5 \log \frac{d}{10}$. They are extremely bright so can be observed in distant galaxies. Observations of type 1a supernovae in distant galaxies showed that they were further from us than was expected, assuming a constant rate of expansion of the universe (i.e. Hubble's law). This evidence showed that the universe is expanding at faster rate than when the supernovae occurred. This was a controversial conclusion because cosmologists had been generally of the opinion that the expansion of the universe was slowing down, because of the effects of gravity. There was no theory at the time for what could cause an acceleration.

5a. Quantum efficiency = % of photons hitting the CCD which are detected and produce a signal.

5b. i. $v = \frac{\lambda}{D} = \frac{750 \times 10^{-9}}{0.6} = 1.25 \times 10^{-6}$ rad

5b. ii. (From Data sheet) 1 AU = 1.50×10^{11} m, 1 ly = 9.46×10^{15} m

$$s = r \times \theta = \frac{s}{r} = \frac{5 \times 1.50 \times 10^{11}\ \text{m}}{10.5 \times 9.46 \times 10^{15}\ \text{m}} = 7.55 \times 10^{-6}\ \text{rad}$$

5b. iii. Answer to part **b. i.** is the theoretical resolving limit. In practice it will be worse than this due to Earth's atmosphere. Also, the planet will be too dim to see against the star. The brightness of the star is much too great for TRAPPIST to observe the planet directly and the planet is much smaller than the star. It can only measure the small dimming of the light of the star as the exoplanet transits it.

6a. The universe has expanded from an extremely hot, extremely dense state. The expansion began about 1 billion years ago.

6b. Observations of the red shifts of galaxies which show that they are moving out from a single common point.

Observations of the cosmological microwave background radiation. This follows a black body radiation curve corresponding to a temperature of 2.7 K, and can be interpreted as leftover 'heat' of the Big Bang.

Observations that hydrogen and helium are found in the universe in the ratio 3:1, which is as predicted by the very early period of fusion in the Big Bang theory.

AQA A-Level Physics Year 2
Engineering physics Answers

Chapter 1

ASSIGNMENT 1

A1 **a.** (11.44 + 10.94 + 11.21) ÷ 3 = 11.20 s

b. Range = 11.44 – 10.94 = 0.50 s

So the uncertainty is ±0.25 s

As a percentage of the mean dropping time, the uncertainty is 0.25 ÷ 11.20 × 100% = 2.2%

c. The precision of the timer is 0.01 s, and 0.01 ÷ 11.20 × 100% = 0.09%

So the uncertainty in reaction time is more dominant. It is also random, which is why there is a spread of times.

A2 **a.** The acceleration will not be 9.81 m s^{-2} as the mass is not in free fall, due to the upward tension in the string.

b. Average velocity $= \frac{h}{t} = \frac{100\text{ cm}}{11.20\text{ s}}$ = 8.93 cm s^{-1} (to 3 s.f.), which is 0.0893 m s^{-1}.

Since the acceleration is constant, the final velocity will be double the average value, or 0.179 m s^{-1}.

A3 **a.** Loss of gravitational potential energy, $\Delta E_p = mg\Delta h$ = 0.7 kg × 9.81 m s^{-2} × 1.00 m = 6.87 J

Gain in linear kinetic energy, $\Delta E_k = \frac{1}{2}mv^2$ = 0.5 × 0.7 kg × (0.179 m s^{-1})^2 = 0.0112 J

b. The rotational kinetic energy of the flywheel is the difference between the E_p lost and the E_k gained by the mass: 6.87 – 0.0112 = 6.86 J (to 3 s.f.)

A4 **a.** The final angular velocity of the flywheel is the same as the final angular velocity of the shaft, which is

$$\omega = \frac{v}{r_{shaft}} = \frac{0.179\text{ m s}^{-1}}{12.5\times10^{-3}\text{ m}} = 14.3\text{ rad s}^{-1}$$

b. rotational $E_k = \frac{1}{2}I\omega^2$, so $I = \frac{2E_k}{\omega^2} = \frac{2\times6.86}{14.3^2} = 0.067\text{ kg m}^2$

c. The percentage uncertainty in:

- t is 2.2% (from **A1b**)
- r_{shaft} is 0.1 ÷ 25 × 100% = 0.4%
- h is 0.1 ÷ 100 × 100% = 0.1%

So the uncertainty in $v = \frac{h}{t}$ is 0.1% + 2.2% = 2.3%

and hence the uncertainty in $\omega = \frac{v}{r_{shaft}}$ is 2.3% + 0.4% = 2.7%

Therefore the uncertainty in ω^2 is 2 × 2.7% = 5.4%

To calculate the uncertainty in the rotational E_k value:

as the linear E_k acquired is so much smaller than the rotational E_k, we can take the uncertainty in the rotational E_k to be equal to the uncertainty in ΔE_p.

% uncertainty in ΔE_p = % uncertainty in m + % uncertainty in g + % uncertainty in h

Assuming zero uncertainty for the first two terms on the right-hand side, we can say that

% uncertainty in E_k = % uncertainty in ΔE_p = % uncertainty in h = 0.1%

The total percentage uncertainty in $I = \frac{2E_k}{\omega^2}$ is therefore 0.1% + 5.4% = 5.5%

The absolute uncertainty in the value of I is 0.067 × 5.5% = 0.004 to 3 decimal places, so

I = (0.067 ± 0.004) kg m^2

Experimentally, the value of I ranges from 0.063 to 0.071 kg m^2.

A5 The theoretical moment of inertia, assuming the flywheel is shaped like a ring, is

$$I = mr^2_{\text{flywheel}} = 7.26 \times 0.1^2 = 0.0726 \text{ kg m}^2$$

A6 **a.** The flywheel is not actually a ring but rather a composite disc-and-ring shape, so the formula used in **A5** will give an answer that is higher than the experimentally determined one.

Also, we have not allowed for work done against frictional torque, nor the moment of inertia of the axle.

b. The axle is a solid cylinder, so

$$I = \frac{1}{2}mr^2_{\text{axle}} = \frac{1}{2} \times 1.13 \times (12.5 \times 10^{-3})^2 = 8.83 \times 10^{-5} \text{ kg m}^2$$

Compared with the moment of inertia calculated in **A4b**, as a percentage this is 0.000 088 3 ÷ 0.067 × 100% ≈ 0.1%, so we are justified in ignoring it.

ASSIGNMENT 2

A1 The moment of inertia for a disc is $\frac{1}{2}mr^2$, so I = 0.5 × (1.5 × 907 kg) × (0.80 m)^2 = 435.4 kg m^2

A2 Rotational kinetic energy is calculated from $E_k = \frac{1}{2}I\omega^2$

We know I = 435.4 kg m^2 (from **A1**).

To find ω, we need to convert 3000 rpm to rad s^{-1}: ω = (3000 ÷ 60) × 2λ = 314.2 rad s^{-1}

So $E_k = \frac{1}{2} \times 435.4 \times 314.2^2 = 2.15 \times 10^7$ J, or 21.5 MJ

A3 2000 rpm = (2000 ÷ 60) × 2λ rad s^{-1} = 209.4 rad s^{-1}

So the kinetic energy at 2000 rpm is $\frac{1}{2} \times 435.4 \times 209.4^2 = 9.55 \times 10^6$ J

Energy transferred from the flywheel = $2.15 \times 10^7 - 9.55 \times 10^6 = 1.20 \times 10^7$ J, or 12 MJ

A4 As starting up would have taken too long if the flywheel were allowed to run down fully, it was kept spinning overnight at a lower rate (2850 rpm).

A5 Angular momentum is a vector quantity, and gyroscopes will try to maintain a specific direction. In this case, there will be an enormous gyroscopic effect because of the mass of the flywheel and its fast rotation. Consequently, a large torque will be required to change the direction of the flywheel and hence the bus. As there was no power-assistance with the steering, the driver would have needed a great deal of strength just to drive the bus normally.

(An interesting possibility is to use two counter-spinning flywheels. Each flywheel will produce a gyroscopic effect, but by using two that spin in opposite directions, there will be a 'balancing out', or cancelling, of the two gyroscopic effects – the angular momentum vectors will cancel out.)

A6 Bearings were not made particularly well in those days, so there was usually a great deal of friction to contend with. The massive flywheel by itself (1.5 tons) would already be causing problems and, with it rotating, there would have been an excessive amount of frictional torque in addition. When the bus turned a corner, the gyroscopic effect would have exerted a further torque, so the bearings could not be expected to have a great life expectancy.

A7 The bearings quickly wore down and needed constant monitoring. If they were to break when the bus was travelling with a full load of passengers, the flywheel would be freed and able to move inside the bus. It would possess a large amount of kinetic energy, so the passengers would need a great deal of protection; otherwise, there could be fatal consequences. The hydrogen gas from the flywheel chamber could also cause problems.

A8 The bus itself does not produce carbon emissions, but it does need electrical charging to get the flywheel going.

Lighter materials could be used for bus construction, thereby improving the efficiency.

Better materials for the flywheel could allow a faster spin, thereby storing more energy.

Magnetic bearings would also improve efficiency because there would be less friction. This would extend the life of the bearings between refits. (If the flywheel were to be spun in a vacuum, then this would also keep friction to a minimum, thereby taking pressure off the bearings.)

PRACTICE QUESTIONS

1a. $\alpha = 5\,\text{rad s}^{-2}$ and $t = 8\,\text{min} = 480\,\text{s}$

(remember to convert to seconds)

Using the equation

$$\omega = \omega_0 + \alpha t$$

$$\omega = 0 + (5)(480) = 2400\ \text{rad s}^{-1}$$

which converted to rev min^{-1} gives

$$\omega = 2400 \times \frac{60}{2\pi} = 22\,900\ \text{rev min}^{-1}$$

1b. Using

$$E_K = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 1.2 \times (2400)^2$$

$$= 3.5 \times 10^6\,\text{J} = 3.5\,\text{MJ}$$

1c. Using

$$\text{power} = \frac{\text{work done}}{\text{time taken}} = \frac{E_k}{t}$$

and rearranging gives

$$t = \frac{E_k}{\text{power}} = \frac{3.5 \times 10^6\,\text{J}}{15\,\text{W}}$$

$$= 2.3 \times 10^5\,\text{s}\ (\approx 64\,\text{h})$$

1d. Using

$$P = T_{average}\,\omega_{average}$$

and rearranging gives

$$T_{average} = \frac{P}{\omega_{average}} = \frac{15\,\text{W}}{\frac{2400}{2}\,\text{rad s}^{-1}}$$

$$= 1.3 \times 10^{-2}\ \text{Nm}$$

2a. Angular momentum is

$$L = I\omega = 0.84 \times \left(\frac{2400 \times 2\pi}{60}\right)$$

$$L = 0.84 \text{ kg m}^2 \times 251 \text{ rad s}^{-1}$$

$$= 210 \text{ N ms} \left(\text{or kg m}^2 \text{ s}^{-1}\right)$$

2b. $E_k = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 0.84 \times (251)^2$

$$= 2.6 \times 10^4 \text{ J} = 2.6 \text{ kJ}$$

2c. Angular momentum is conserved, i.e. is constant.

Total moment of inertia increases, so ω decreases, hence speed of motor falls.

2d. $I = I_{motor} + I_{flywheel}$

$$I = 0.84 + 1.4 = 2.20 \text{ kg m}^2$$

Angular momentum is conserved, hence

$$L = I \omega_{new}$$

$$\omega_{new} = \frac{L}{I} = \frac{210 \text{ N ms}}{2.20 \text{ kg m}^2}$$

$$= 95 \text{ rad s}^{-1}$$

2e. Angular impulse = change in angular momentum

Angular momentum of the flywheel before engagement

$$= I_{flywheel} \times \omega_0 = 0$$

Angular momentum of the flywheel after engagement

$$= I_{flywheel} \times \omega_{new} = 1.4 \times 95 = 130 \text{ N ms}$$

Angular impulse = 130 N ms

3a. The following points should be included:

The energy stored in a flywheel depends on the moment of inertia and ω^2.

The moment of inertia is defined as Σmr^2, so the shape is important and the mass needs to be distributed as far as possible from the axis of rotation.

By using a high density material, the flywheel can be made massive for a relatively small size.

The material's tensile strength will limit the maximum speed of rotation and hence the maximum energy that can be stored.

Friction will need to be reduced, so the bearings could be magnetic. All other surfaces need to be as smooth as possible.

Balance is essential and any gyroscopic effects need to be allowed for.

3b. i. The KE (rotation) of a flywheel is $\frac{1}{2} I\omega^2$, so the loss of KE of the flywheel

$$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2 = \frac{1}{2} I(\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} \times 0.036 \times (6400^2 - 3100^2) = 5.6 \times 10^5 \text{ J (2 s.f.)}$$

This assumes no energy loss in the transfer from the flywheel to the car.

3b. ii. Using power = $\frac{\text{energy}}{\text{time}}$,

power = $\frac{5.6 \times 10^5}{6.6}$ = 85 000 W

3b. iii. Torque = moment of inertia × angular acceleration ($T = I\alpha$)

$\alpha = \frac{(\omega_2 - \omega_1)}{t} = \frac{(3100 - 6400)}{6.6}$ = –500 rad s^{-2} (deceleration)

$T = I\alpha$ = 0.036 × (–500) = –18 N m, so the decelerating torque = 18 Nm.

3b. iv. Angular displacement as the flywheel can be calculated in a variety of ways. For example:

$W = T \times \theta$ (work = torque × angle), so $\theta = \frac{W}{T}$

$\frac{560\ 000}{18}$ = 31 000 rad. Since there are 2π rad in 1 revolution,

$\frac{31\ 000}{2\pi}$ = 4900 rev

4a. i. The turntable makes 8.3 revolutions, which is 8.3 × 2π = 52 rad.

Use $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ as we need to calculate α. The final angular velocity is zero.

0 = 6.42 + 2 × α × 52, gives us α = –0.39 rad s^{-2}

4a. ii. $T = I\alpha$, so $T = 8.2 \times 10^{-3} \times 0.39$ = 0.0032 Nm (3.2×10^{-3})

4b. i. The turntable is kept at an angular velocity of 0.78 rad s^{-1}.

The friction torque is 0.0032 Nm, so we use $P = T \times \omega$.

Power = 0.0032 × 0.78 = 0.0025 W

So, for 270 s, the work done = P × time = 0.68 J

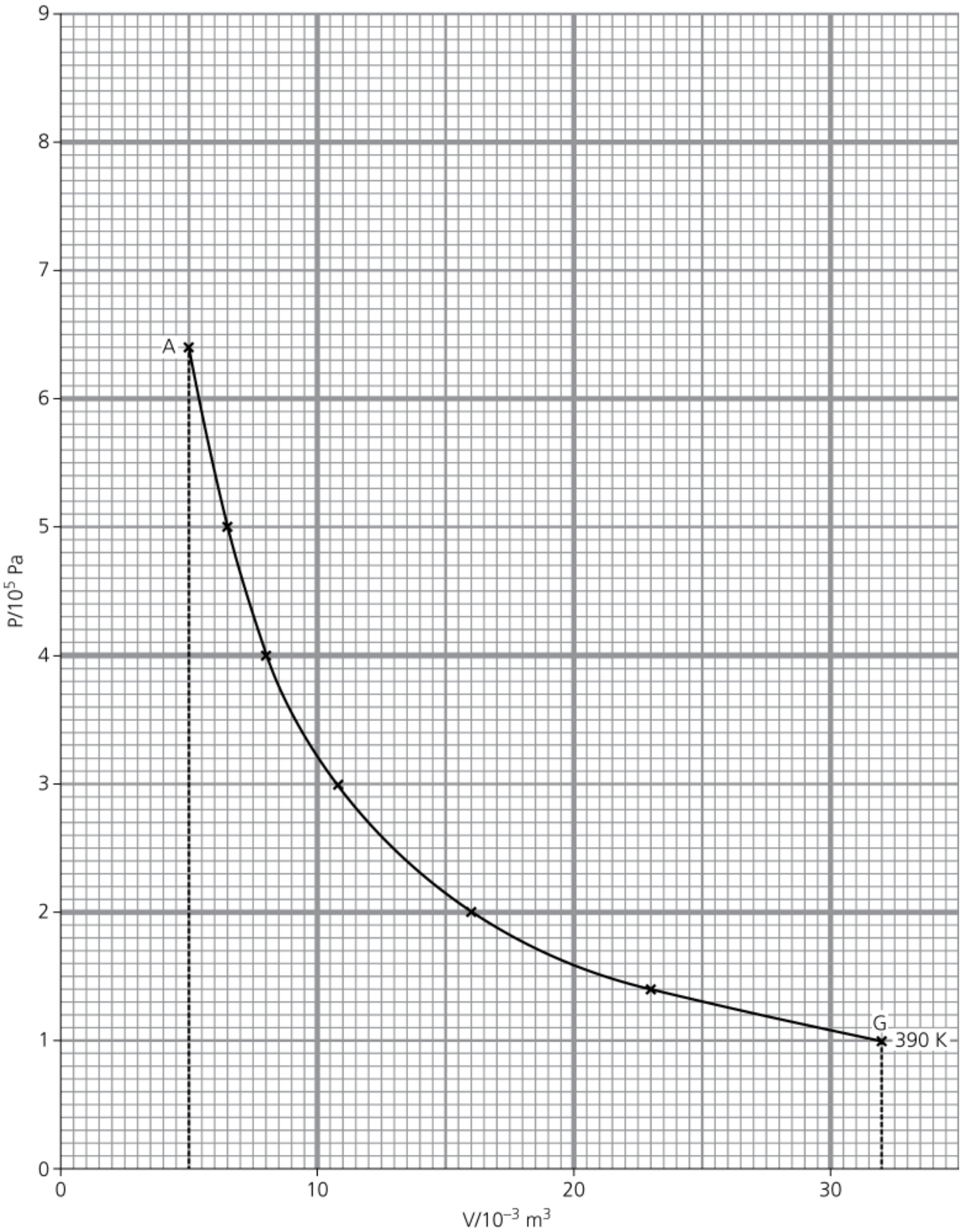
4b. ii. Using the ratio given in the question:

$\frac{\text{energy supplied to the oven}}{\text{work done to drive the turntable}}$, we get

$\frac{900 \times 270}{0.68} = 3.6 \times 10^5$, which is of the order of 10^5.

Chapter 2

ASSIGNMENT 1



A1 a, b.

c. Answers will vary. For example:

- Smallest squares are 2 mm × 2 mm = 4 mm^2, and each represents (0.5×10^{-3} m^3) × (0.1×10^5 J) = 5 J
- Medium squares are 1 cm × 1 cm = 1 cm^2, and each represents (2.5×10^{-3} m^3) × (0.5×10^5 J) = 125 J
- Largest squares are 2 cm × 2 cm = 4 cm^2, and each represents (5×10^{-3} m^3) × (1×10^5 J) = 500 J

d. The size of square you choose should be the largest one that would give you an estimate with an *acceptable* uncertainty (see part **f** below). Depending on the axis scales you have used, you can probably choose the medium (cm) squares: the largest squares would give an answer that is too rough; the smallest (mm) squares, although more accurate, would take too long to count and add up. The emphasis here is on *estimating.* There are occasions in which smaller squares may need to be taken into account, especially when the graph lines are right in the middle of the cm squares.

Using the cm squares in this case would give about 48 cm^2.

e. Since 1 cm^2 represents 125 J, an estimate of the total work done is 125 J × 48 = 6000 J.

f. Assuming that the estimation could be 1 cm^2 out in either direction, the uncertainty will be (48 ± 1) cm^2, which as a percentage is 1 ÷ 48 × 100% = 2.1% ≈ 2%.

So the work done is 6000 ± 120 J to 3 s.f, or about 6000 ± 100 J to 2 s.f.

2% is a reasonable uncertainty in this case.

g. **i.** A → G is an isothermal expansion and work will be done *by* the gas.

ii. G → A is an isothermal compression and work will be done *on* the gas.

h. $pV = nRT$, so $T = \frac{pV}{nR}$

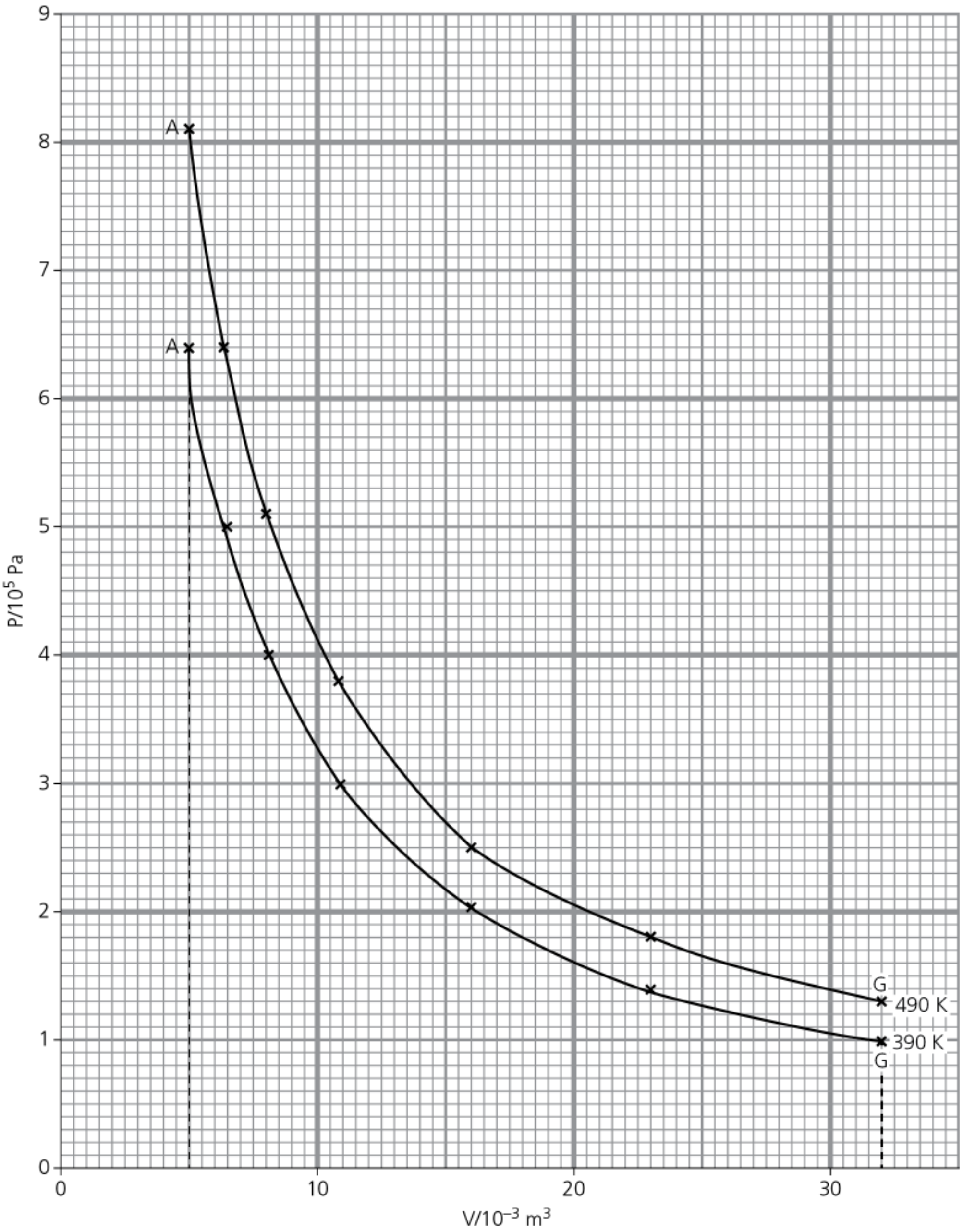
Choosing one pair of points (such as A) and using n = 1 mol, we get

$$T = \frac{6.4 \times 10^5 \times 5.0 \times 10^{-3}}{1 \times 8.31} = 385\text{ K} \approx 390\text{ K (2 s.f.)}$$

A2 The new temperature is 490 K. The pressures are calculated from $p = \frac{nRT}{V} = \frac{1 \times 8.31 \times 490}{V} = \frac{4072}{V}$

The values are tabulated as follows:

	$\frac{p}{10^5\ \text{Pa}}$	$\frac{V}{10^{-3}\ \text{m}^3}$
A	8.1	5.0
B	6.4	6.4
C	5.1	8.0
D	3.8	10.7
E	2.5	16.0
F	1.8	23.0
G	1.3	32.0



A3

There are about 61 cm squares below the curve.

So an estimate of the total work done is 125 J × 61 = 7625 J (about 7630 J to 3 s.f. or 7600 J to 2 s.f.)

Assuming the same uncertainty in area estimation as in **A1f**, we get the uncertainty (61 ± 1) cm^2, which as a percentage is 1 ÷ 61 × 100% = 1.6% ≈ 2%.

So the work done is 7630 ± 150 J to 3 s.f, or about 7600 ± 200 J to 2 s.f.

A4 No. The 490 K curve is a vertically stretched copy of the 390 K curve; they have the same shape but will never cross.

ASSIGNMENT 2

(This assignment involves research, group discussion and presentation; students' answers will vary.)

PRACTICE QUESTIONS

1a. i. For an adiabatic process, $p_1V_1^{\gamma} = p_2V_2^{\gamma}$

$1.0 \times 10^5 \times (2.1 \times 10^{-5})^{1.4} = p_2 \times (1.2 \times 10^{-5})^{1.4}$

The new pressure $= 2.2 \times 10^5$ Pa.

1a. ii. For an adiabatic process, $T_1V_1^{(\gamma-1)} = T_2V_2^{(\gamma-1)}$

$290 \times (2.1 \times 10^{-5})^{0.4} = T_2 \times (1.2 \times 10^{-5})^{0.4}$

The maximum temperature reached is 360 K to 2 s.f.

This can also be worked out by using $pV = nRT$.

1b. The first law of thermodynamics is $Q = \Delta U + W$

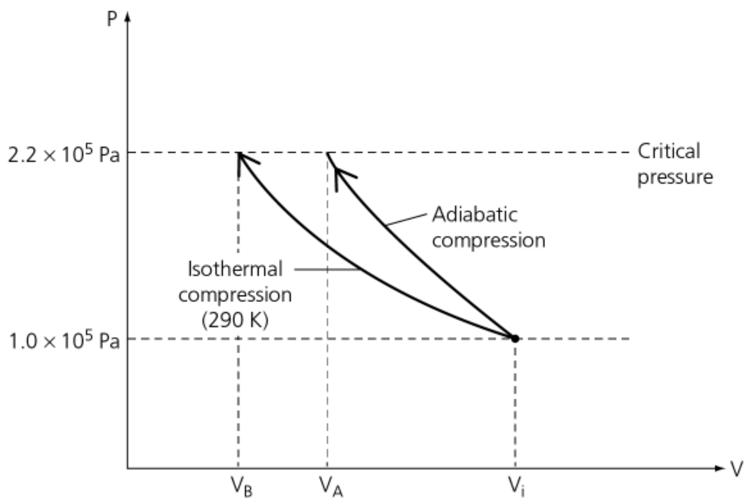
So, $\Delta U = Q - W$

The work has been done on the gas, so $W = -1.4$ J

As it is adiabatic, $Q = 0$.

$\Delta U = Q - W = 0 - (-1.4) = + 1.4$ J

1c. To fire the cork, the same pressure is needed (2.2×10^5 Pa).



In the sketch graph, V_i is the initial pressure. V_A is the final volume when the adiabatic curve is followed up to the critical pressure. If the handle is pushed in slowly, the compression can be viewed as nearly isothermal, which is a shallower curve. To reach the required pressure, the compression has to reduce the volume to V_B. The decrease in volume is much greater in the isothermal scenario (compared to the adiabatic), so the volume of air left in the barrel will be *less/the piston is pushed in further.*

2a. The work done equals the area under the graph from 3.6×10^5 Pa to 1.0×10^5 Pa.

Since one vertical line is almost at the half-way mark inside a centimetre square, we will need to count up smaller squares instead of square centimetres, which would have rounding problems.

There are about 150 of these (again, only counting those that are more than half-full).

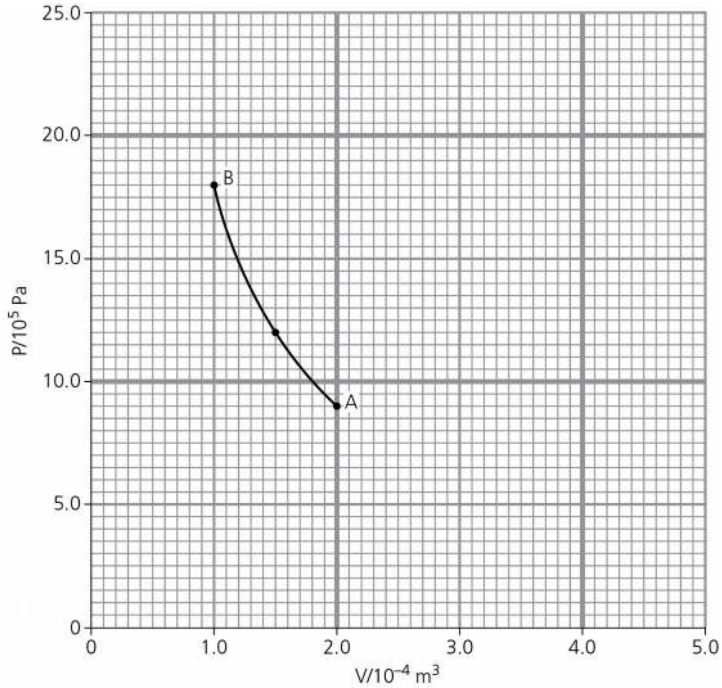
1 small square corresponds to $0.1 \times 10^5 \times 0.1 \times 10^{-3} = 1$ J

Total work done $= 150 \times 1 = 150$ J

2b. An isothermal line is shallower than an adiabatic, so the area under the curve will be greater. This means more work done by the gas, which means the rocket would rise higher.

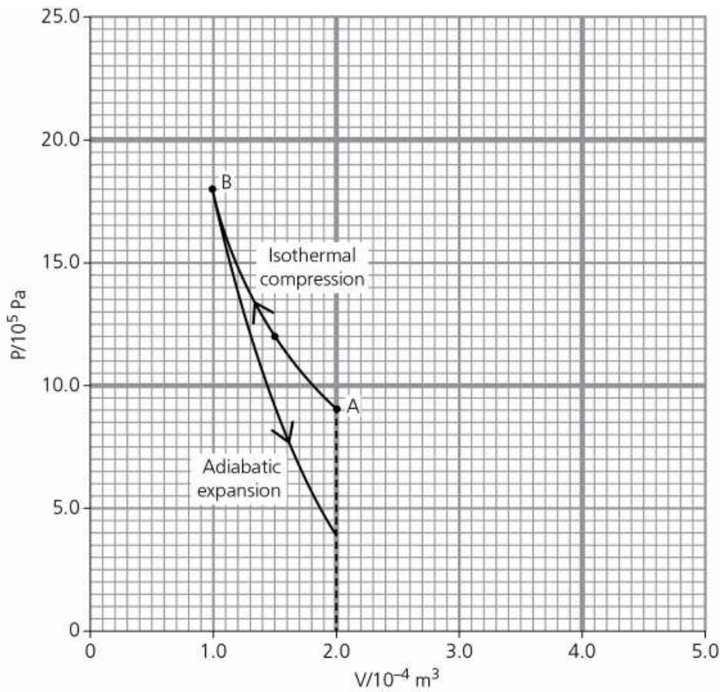
3a. The volume now drops to 1×10^{-4} m^3. As pV = constant for an isothermal, the new pressure is doubled: 18.0×10^5 Pa.

The new point is marked as B. A further point in between is added to help sketch the curve. Choosing V as 1.5×10^{-4} gives us a pressure of 12.0×10^5 Pa.



3b. i. A change where there is no heat (Q) into or out of the system.

3b. ii. The adiabatic curve is steeper than the isothermal, but will end on the same volume (indicated by the dotted line).



3b. iii. The temperature falls and work is done by the gas as it expands.

3c. Using $pV = nRT$, we get $9.0 \times 10^5 \times 2.0 \times 10^{-4} = n \times 8.31 \times (1100 + 273)$

$$n = \frac{180}{11400} = 0.0158\ (1.58 \times 10^{-2}) \text{ moles}$$

Chapter 3

ASSIGNMENT 1

A1 $2^{10} = 1024$, so the likelihood of getting a string of 10 heads is 1 in 1024.

A2 $2^{100} \approx 1.3 \times 10^{30}$, so there is a 1 in 1.3×10^{30} chance of getting all heads in 100 coin tosses. This is extremely unlikely to happen!

A3 Using $\Delta S = \frac{\Delta Q}{T}$ for the hot reservoir and the cold reservoir in each case:

a. Loss of entropy from the hot reservoir $= \frac{-300}{500} = -0.6\ \mathrm{JK^{-1}}$

Gain of entropy in the cold reservoir $= \frac{+300}{200} = +1.5\ \mathrm{JK^{-1}}$

Overall change in entropy $= +1.5 - 0.6 = +0.9\ \mathrm{JK^{-1}}$

b. Loss of entropy from the hot reservoir $= \frac{-1050}{350} = -3.0\ \mathrm{JK^{-1}}$

Gain of entropy in the cold reservoir $= \frac{+1050}{300} = +3.5\ \mathrm{JK^{-1}}$

Overall change in entropy $= +3.5 - 3.0 = +0.5\ \mathrm{JK^{-1}}$

c. Loss of entropy from the hot reservoir $= \frac{-1000}{100+273} = -2.7\ \mathrm{JK^{-1}}$

Gain of entropy in the cold reservoir $= \frac{+1000}{90+273} = +2.8\ \mathrm{JK^{-1}}$

Overall change in entropy $= +2.8 - 2.7 = +0.1\ \mathrm{JK^{-1}}$

There is always an increase in entropy, and the bigger the difference between the temperatures of the two reservoirs, the greater the overall increase in entropy.

A4 **a.** $\varepsilon_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{200}{500} = 0.6 = 60\%$

b. An efficiency of 0.6 means that the work done is $W = 0.6 \times 1000\ \mathrm{J} = 600\ \mathrm{J}$.

As $Q_H = 1000\ \mathrm{J}$, $Q_C = (1000 - 600)\ \mathrm{J} = 400\ \mathrm{J}$. So:

Loss of entropy from the hot reservoir $= \frac{-1000}{500} = -2.0\ \mathrm{JK^{-1}}$

Gain of entropy in the cold reservoir $= \frac{+400}{200} = +2.0\ \mathrm{JK^{-1}}$

Overall change in entropy $= +2.0 - 2.0 = 0\ \mathrm{JK^{-1}}$ (*no change* in entropy)

c. If $W = 500\ \mathrm{J}$, then $Q_C = (1000 - 500)\ \mathrm{J} = 500\ \mathrm{J}$. So in this case:

Loss of entropy from the hot reservoir $= \frac{-1000}{500} = -2.0\ \mathrm{JK^{-1}}$

Gain of entropy in the cold reservoir $= \frac{+500}{200} = +2.5\ \mathrm{JK^{-1}}$

Overall change in entropy $= +2.5 - 2.0 = +0.5\ \mathrm{JK^{-1}}$ (entropy has *increased*)

d. If W = 700 J, then Q_C = (1000 - 700) J = 300 J. So in this case:

Loss of entropy from the hot reservoir = $\frac{-1000}{500} = -2.0\ JK^{-1}$

Gain of entropy in the cold reservoir = $\frac{+300}{200} = +1.5\ JK^{-1}$

Overall change in entropy = +1.5 – 2.0 = -0.5 JK^{-1} (entropy has *decreased*)

e. The theoretical efficiency, as calculated from $\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$, is the best efficiency it is ever possible to achieve. The (theoretically) *most efficient* engine corresponds to *no change in entropy* (part **b**). For real engines there will always be an overall increase in entropy (as in part **c**). The situation in part **d** is *not possible* as entropy can never decrease.

A5 a. Photosynthesis is the process by which energy is used to create structures in plants. There is a movement of disorder (the energy from the Sun) towards order (structures in living organisms), which implies that entropy is decreasing as the result of the processes of life. The same applies to carnivores and up the food chain.

(James Lovelock was asked by NASA in 1964 what they should be looking for on Mars to decide whether life was present. His reply was that they should look for a reduction or reversal of entropy.)

b. The statement 'entropy must increase' refers to a closed system. Although life may be reducing entropy, the systems in which it occurs will have an increase in entropy, so the net effect will be an overall increase in entropy. An animal is effectively a heat engine: consuming food is similar to burning fuel (combustion occurs), which is accompanied by the release of heat energy into the environment, raising the entropy in the surroundings.

A6 If entropy is always increasing, the entropy at the Big Bang must have been lower (some cosmologists think it may have been zero!), and this suggests that the initial conditions were extremely fine-tuned. It also means that the beginning of the Universe had only a single state. We do not have a good enough understanding of high-energy physics to explain this (yet).

A7 If the Universe is expanding at a faster rate, this implies a faster spreading-out of matter. You can therefore argue that heat death (averaging out) would proceed more rapidly. But there is a counter-view, with some physicists thinking that dark energy may increase with time; this would eventually rip apart all matter and atoms and may lead to the end of the Universe as a singularity (rather like within a black hole).

ASSIGNMENT 2

A1 For a water-sourced heat pump the heat transfer rate is higher than from the ground, as the circulation of water provides constant energy replacement. The return temperature to the heat pump is therefore higher than in the GSHP version, so the COP is improved. Also, no digging of trenches is required.

(As an alternative, the water can be diverted straight into the heat pump, avoiding the use of pipes in the water itself. However, permission would be needed for an abstraction licence and there may also be concerns over wildlife issues.)

A2 Although air-sourced heat pumps are cheaper and easier to install, they cannot take advantage of the heat stored in the ground, which is a 'constant temperature reservoir'. Air temperature can fluctuate dramatically over the year, and so will the efficiency of the pump.

(For example, in winter, when heat is really needed, some energy may have to be used to blow the air across the heat exchangers; also, ice may have formed on the heat exchangers. Both of these issues would drive down the efficiency.)

A3 By reversing the direction of operation of a heat pump, the house can be cooled (for example, by cold pipes under the floor) and the heat energy returned to the ground and stored. This stored energy can then be used again when the weather gets colder. Standard air-conditioning units extract heat from a building, but it gets vented into the atmosphere and is therefore lost to the system.

A4 Environmental benefits of heat pumps are, effectively, a significant reduction in the consumption of fossil fuels (thereby reducing our 'carbon footprint') as the whole economy moves towards 'low carbon' through the reduction of the burning of fossil fuels. The usage of heat pumps means that surplus energy from the summer is saved and recycled for use in the colder months. The pumps still require a source of electrical energy, but this could be from solar or wind. And, even if the mains supply is used, heat pumps can still provide about 4 J of heating from 1 J of input, and so will still be cheaper.

(Currently, almost half the energy consumed in the UK is in the form of heat.)

A5 At first glance, the theoretical values for the COPs are far better than those quoted by manufacturers of heat pumps. The difference stems from the same reason that efficiencies of real heat engines are so much lower than those predicted from theory. The two reservoir temperatures will not stay fixed, and there will be inefficiencies in the operating cycle of the pump; energy is bound to be lost at various points in the system.

(Remember that maximising the heat insulation of the house would be an essential requirement of the whole system.)

A6 (Answers will vary.)

PRACTICE QUESTIONS

1a. Compression (or decrease in volume) with no heat transfer from the gas.

1b. The first law of thermodynamics states

$Q = \Delta U + W$

where ΔU is the increase in internal energy.

At constant volume $W = 0$

Hence $Q = \Delta U = 850$ J

1c. C → D is an adiabatic expansion (1)

D → A is a reduction in pressure (cooling) at constant volume

1d. At point A the air in the cylinder has a pressure of 0.1×10^6 Pa and a volume of 0.5×10^{-3} m^3. The temperature is 300 K.

Using the gas equation, $pV = nRT$, and rearranging gives

$$n = \frac{pV}{RT} = \frac{0.1\times10^6 \times 0.5\times10^{-3}}{8.31\times300}$$

$$= 0.020 \text{ mol}$$

1e. The work output is the area enclosed by the loop.

Count the number of small squares.

Each small square is equivalent to the scaling factor

$$(0.01\times10^{-3})\times(0.1\times10^6) = 1 \text{ J}$$

Total number of squares is ~350 , so work output = 350 J (±30 J)

1f. No sharp corners in real cycles because valves take time to open and close.

Expansion and compression strokes are not truly adiabatic in real cycles, as heat losses occur.

Real cycles require both induction and exhaust strokes.

The maximum temperature is never realised because of imperfect combustion.

In real engines, the pistons are always moving so that the heating is not at a constant volume.

2a. For a heat pump, the COP is given by the ratio of heat delivered at the hot reservoir (Q_H) to the work done,

or $COP_{hp} = \frac{Q_H}{W}$

2b. i. The efficiency of the heat engine (E) can be calculated from either the energies or the temperatures:

$$\varepsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

So the percentage rejected as heat is $\frac{290}{1600}$ = 18%. The efficiency of the heat engine is therefore 82%.

This represents the 80 kW.

The input power is $\frac{100}{82} \times 80 = 98$ kW

Using input power = calorific value × flow rate we get 98 000 = 49 000 000 J × flow rate

So, flow rate of propane = 0.002 kg s^{-1}

2b. ii. By definition, $COP_{hp} = \frac{Q_H}{W}$

If the electrical input to the pump is 16 kW, then the heat delivered = 16 × 2.6 = 42 kW.

In this setup, the heat rejected from the heat engine is also being used to heat the buildings.

For the heat engine, we know Q_1 is 18%, which is 98 × 18% = 18 kW.

The total heat input into the building = 42 kW + 18 kW = 60 kW.

2b. iii. A heat pump will deliver more heat energy than drawn from the mains.

3a. The indicated power is derived from the area enclosed in one cycle in the p–V plot.

There are 470 mm^2. The scaling factor = $(0.1 \times 10^6) \times (0.01 \times 10^{-3}) = 1$ J

The work done per cycle = 470 × 1 = 470 J

4100 rev min^{-1} = $\frac{4100}{60}$ = (about) 68 rotations of the engine per second or 34 cycles per second.

Using P_{ind} = area of $p-V$ loop (J) × number of cycles per second (s^{-1}) × number of cylinders

The indicated power = 470 × 34 × 4 = 64 kW

3b. The overall efficiency of an engine = $\frac{\text{output (brake) power}}{\text{input power}}$

The output (brake) power = 55.0 kW.

The input power is calculated from P_{input} = calorific value of fuel(J kg^{-1}) × fuel flow rate(kg s^{-1})

In this case, the units are MJ $litre^{-1}$ × litre s^{-1} = MJ s^{-1}

P_{input} = 38.6 MJ per litre × $\frac{0.376}{100}$ (the amount of fuel used in 100 seconds)

= 145 kW (3 s.f.)

Overall efficiency = $\frac{55}{145}$ = 38%

4a. Adiabatic change means no heat transfer to/from the surroundings

The compression stroke occurs very quickly, so there will be no time for heat transfer.

So the change can be viewed as adiabatic.

4b. i. As the change is adiabatic, $p_1V_1^{\gamma} = p_2V_2^{\gamma}$

$(1.0 \times 10^5) \times (4.5 \times 10^{-4})^{1.4} = (6.2 \times 10^6) \times (V_2^{1.4})$

$V_2 = 2.4 \times 10^{-5}$ m^3

4b. ii. Using the ideal gas equation ($pV = nRT$), we can say $p_1 \times \frac{V_1}{T_1} = p_2 \times \frac{V_2}{T_2}$

The final temperature $T_2 = \left(\frac{p_2}{p_1}\right) \times \left(\frac{V_2}{V_1}\right) \times T_1$

$$= \left(\frac{6.2 \times 10^6}{1.0 \times 10^5}\right) \times \left(\frac{2.4 \times 10^{-5}}{4.5 \times 10^{-4}}\right) \times 297 = 982 \text{ K}$$

4b. iii. To ensure that when the fuel has started to burn, the piston is at the top of the stroke. This means the pressure will be at a maximum.

AQA A-Level Physics Year 2
Medical physics Answers

Chapter 1

ASSIGNMENT 1

A1 Sensorineural hearing loss is damage to the hair cells on the basilar membrane of the cochlea in the inner ear and/or damage to the nerve cells that transmit electrical impulses along the auditory nerve to the brain.

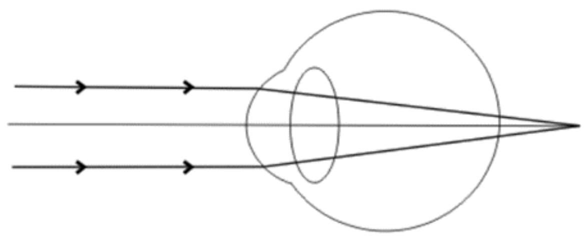
A2 The audiogram would resemble Figure 33(c) or graph B in Figure 26: lower frequencies (below about 500 Hz) are barely affected, with hearing level roughly between 0 and 20 dBA, whereas there is fall-off in sensitivity at higher frequencies, with the hearing level perhaps reaching 60–80 dBA for frequencies of 4 kHz and above.

A3 The cochlear implant divides sound into channels to mimic the response of the normal human cochlea, where groups of hair cells along the length of the basilar membrane respond to different ranges of frequencies. The channels transmit to electrodes assigned to different parts of the basilar membrane and so stimulate the nerve cells in those particular regions. The brain then perceives the signals as sounds of different frequencies.

A4 Human *speech* can be understood by perceiving sounds within the range of 200 Hz to 7500 Hz. (In fact, speech can be understood over a bandwidth of 300 Hz to 3000 Hz, so the bandwidth of the sound processor is more than adequate.)

PRACTICE QUESTIONS

1a. Hypermetropia is commonly known as long-sightedness and is when the eye refracts rays of light from a near object and form an image behind the retina.



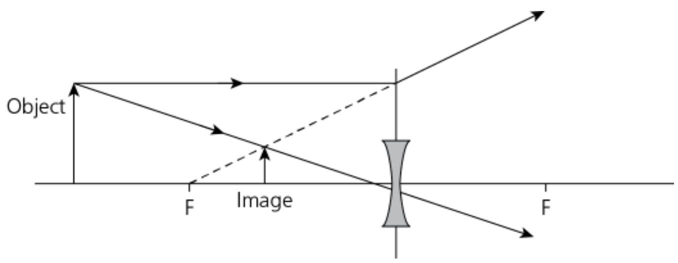
1b. Power of unaided eye = $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{1}{0.65}+\frac{1}{0.017}=60.4\text{D}$

Power of normal eye = $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{1}{0.25}+\frac{1}{0.017}=62.8\text{D}$

The power need to correct the defect of vision is 62.8D – 60.4D = +2.4D

The +ve sign indicates that this is a convex lens.

2a.



2b. Power of a lens is reciprocal of the focal length when the focal length is expressed in metres. The unit of power is dioptre.

2c. **i.** Myopia or short-sight (since diverging lens of negative focal length used).

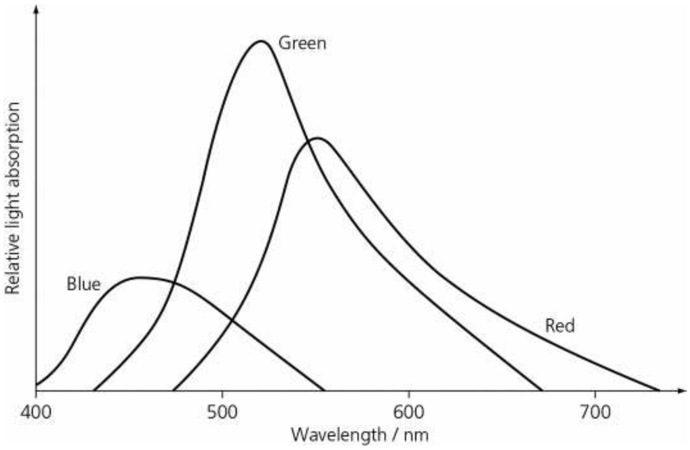
2c. ii. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ $\frac{1}{-0.56} = \frac{1}{u} + \frac{1}{-0.15}$

$u = 0.2049$ m $= 0.20$ m (to 2 s.f.)

2c. iii. The prescription needs to give the power and the axis direction of the cylindrical lens.

3a. On the retina there are only cones at fovea. As you move away from fovea there are fewer cones and more rods.

3b.



3c. The two images fall on receptors with at least one (unstimulated) receptor between them.

3d. Cones become active in bright light, whereas a rods are used in dim light. The resolution in bright light is better because size of cone smaller than size of rods.

4a. i. The intensity I of sound is the power per unit area (in W m^{-2}) at normal incidence on the ear. The intensity level is defined as 10 log $\left(\frac{I}{I_0}\right)$, where I_0 is the threshold intensity of human hearing and is measured in decibels.

4a. ii. The threshold of hearing is the minimum sound intensity at a frequency of 1 kHz that can be detected by a normal ear and is equal to 1.0×10^{-12} W m^{-2}

4b. Intensity (dB) $= 10 \log_{10}\left(\frac{I}{I_0}\right)$

$$55 = 10 \log\left(\frac{I}{1 \times 10^{-12}}\right)$$

Giving $I = 2.5 \times 10^{-7}$ W m^{-2}

4c. i. Intensity $= \frac{\text{power}}{4\pi r^2} = \frac{5.0}{4\pi(40)^2} = 2.5 \times 10^{-4}$ W m^{-2}

4c. ii. Intensity level $= 10 \log\left(\frac{I}{I_0}\right) = 10 \times \log\left(\frac{2.5 \times 10^{-4}}{1.0 \times 10^{-12}}\right) = 84.0$ dB

5a. The ossicles consist of bones of the middle ear (the malleus or hammer, the incus or anvil, and the stapes or stirrup). Together they form a lever system to produce an increase in force as the area of the oval is window smaller than that of the eardrum. Since pressure = force/area this results in an increase in the force on the oval window and amplifies the pressure changes.

5b. The A weighting filter ensures the response to frequency is similar to that of the human ear.

5c. i. $110.2 = 10 \log\left(\frac{I}{1.0 \times 10^{-12}}\right)$

$I = 10^{11.02} \times 1.0 \times 10^{-12} = 0.105$ W m^{-2}

5c. ii. Power $= I \times 4\pi r^2 = 0.105$W m$^{-2} \times 4\pi \times (5.0)^2$ m$^2 = 32.9$W

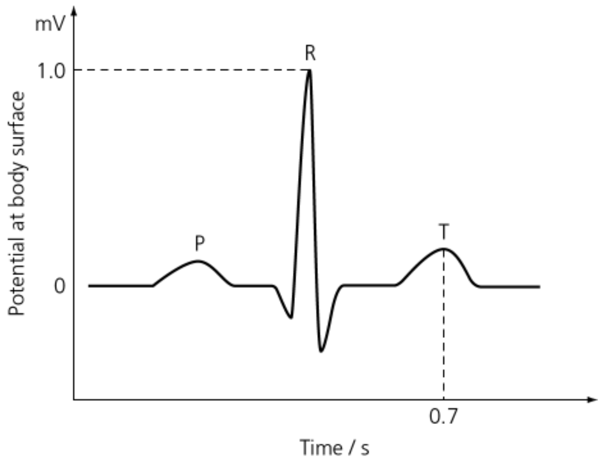
Chapter 2

PRACTICE QUESTIONS

1a. Answer should include:

- Good electrical contact between electrodes and skin by removing dead skin cells using abrasion and applying a conducting gel.
- Attach more than one electrode.
- Patient should remain relaxed and still.
- Electrodes should not react with chemicals produced by the skin.
- Electrical leads to be shielded to reduce interference.
- Signal should be amplified with a high-gain, low-noise amplifier.

1b. **i. and ii.**



1c. P – depolarisation of atria

R – depolarisation of ventricles (and repolarisation of atria)

T – repolarisation of ventricles

Chapter 3

ASSIGNMENT 1

A1 Magnetic resonance imaging is based on the magnetic properties of hydrogen nuclei (protons) in the body when subjected to a strong external magnetic field and then excited by a radio-frequency (RF) pulse. When the pulse is stopped, the protons de-excite and emit RF signals that can be processed by a computer to form an image.

A2 MR imaging provides pictures of structures inside the body and the brain; fMRI reveals areas of the brain with increased blood flow and, by inference, increased neural activity.

A3 Neurons transmit electrical signals, and energy is required for this. The energy is provided by glucose and oxygen, carried by blood. Neurons in the brain can signal nearby blood vessels to dilate in order to deliver more blood.

A4 fMRI can only tell us where blood flow to the brain has increased due to increased activity, not definitively which areas of the brain are responsible for different functions. The possibility of rehabilitating stroke patients indicates that the brain has a certain degree of ‘plasticity’, so the functions of damaged parts can be taken over by other parts, and that some neural processing functions may be distributed across the brain rather than localised to particular regions.

PRACTICE QUESTIONS

1a. An alternating potential difference (ac) applied across the crystal causes the crystal to expand and contract creating pressure waves in the crystal / plastic membrane. The frequency of the alternating pd which is fed through the coaxial cable is equal to that of the resonant frequency of crystal which is above 20 kHz. A short application of ac produces a short pulse for scanning. The use of the acoustic insulator backing material is to damp and stop vibrations of the crystal to stop it producing excessive vibrations. Dampening the vibrations will allow short pulse lengths and therefore better resolution to be achieved.

1b. i. $\frac{I_r}{I_i} = \left(\frac{1.65 \times 10^6 - 4.29 \times 10^2}{1.65 \times 10^6 + 4.29 \times 10^2}\right)^2 = 0.99896$

So $(1 - 0.99896) \times 100\% = 0.1\%$ is transmitted

1b. ii. The gel is between the probe and the skin is to exclude air. The gel should have acoustic impedance equal/close to that of the skin/soft tissue in order to ensure maximum sound energy transmission into the body and to greatly reduce reflection at body boundary.

2a. An endoscope is an imaging device for examining the inside of the body. It consists of a long thin flexible tube of optical fibres with a light source. This can be inserted into natural opening in the body for examinations or used with a cutting device to perform keyhole surgery. It has an eyepiece at one end that the surgeon can look through to view an image of the area of the body where the end of the endoscope is inserted.

2b. In an endoscope, Incoherent bundles are used for illumination purposes; they cannot transit images. Coherent bundles have optical fibres precisely lined up at both ends of the fibre so that an image can be transmitted.

2c. $\sin \theta_c = \frac{1.55}{1.60}$, $\theta = 75.6°$

3. Answers should include the following points:

Most of the human body is made up of water molecules, which consist of hydrogen and oxygen atoms. At the centre of each hydrogen atom are protons that act like tiny magnets and are very sensitive to magnetic fields.

The patient lies in a strong magnetic field which cause the protons to align themselves and precess about the direction of the field.

Short bursts of radio-frequency waves are directed at the body which knock the protons out of alignment. When the radio waves are turned off, the protons realign themselves with the magnetic field and in so doing send out radio signals which are picked up by receivers. The strength of the signals depend on the proton density and hence on the type of tissue, so different types of tissue can be distinguished.

Gradient coils in the MR scanner vary the flux density in three dimensions. This causes the RF signal frequency from different parts of the body to vary. This allows the position of the signal sources from the protons to be located.

This information is then processed by a computer to form images of cross-sections of the body.

4a. An A-scan or amplitude scan sends out ultrasound pulses and the echoes return from boundaries between different body tissues. The output is displayed as a graph on an oscilloscope, showing how the echo amplitude changes with time. If the speed of sound in the body tissue is known, it is possible to calculate the distance to any boundaries and measure the average thickness of a bodily object.

A B-scan or brightness scan creates an image by scanning the ultrasound beam over the patient. It can be thought of as a rapid sequence of A-scans. The strength of the echo signal is used to control the brightness of a picture element, rather than the height of a trace on an oscilloscope. This is combined with information about the position of the transducer to create an image.

4b. Time between peaks on A-scan = 0.08 ms

thickness $= \frac{1}{2} \times 1300 \text{ m s}^{-1} \times 0.08 \times 10^{-3} \text{ s} = 0.05 \text{ m}$ or 5 cm

4c. The extra distance the pulse has to travel before it reflects off the far surface of the tumour means more signal attenuation therefore smaller amplitude. Also a smaller fraction of the transmitted signal may be reflected at the far boundary of the tumour.

Chapter 4

PRACTICE QUESTIONS

1a. i. $E_k = e \times V = (1.60 \times 10^{-19}\,\text{C}) \times (72.4 \times 10^3\,\text{V}) = 1.16 \times 10^{-14}\,\text{J}$

1a. ii. $\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34}\,\text{J s} \times 3.00 \times 10^8\,\text{m s}^{-1}}{1.16 \times 10^{-14}\,\text{J}} = 1.71 \times 10^{-11}\,\text{m}$

1b. A beam of X-rays is produced and the X-ray generator rotated around the patient An array of detectors arranged outside of the path opposite the generator detect the transmitted intensity through the body at different angles and over time, a cross sectional image of the body is reconstructed by a computer.

2. Electrons strike the target anode in the X-ray tube. Some directly strike the target atoms and excite the electrons in the target atoms to higher energies. The excited electrons drop down to lower energy levels and photons of fixed characteristic energy and hence wavelength are emitted.

3. Examples of advantages are:

- MR uses non-ionising radiation whereas CT uses ionising radiation which is more dangerous to living cells.
- MR can give multi-plane images from same scan. CT needs new scan for each image.
- MR gives better resolution between tissue types, better resolution picture.
- MR gives real time image CT scan needs to rotate to produce final image.

4. The intensifying screen converts X-ray photons to light photons. These expose film in the correct place due to the closeness of the screens to the film. The radiation dose to the patient is less since the exposure time is much shorter.

5a. For examining the oesophagus, stomach or gastro-intestinal tract – where there are natural openings into the body.

5b. Using a barium contrast X-ray – which can allow observation of the passage of food through the digestive system.

6a. i. Soft tissues all have similar proton numbers so the degree of absorption by X-rays is roughly the same.

6a. ii. The contrast could be increased by coating the tissues of interest with an X-ray opaque material like barium sulfate.

6b. i. The half-value thickness is the thickness of the material that will reduce the initial X-ray intensity by one half. The linear attenuation coefficient is a constant that describes the rate of energy loss by a photon beam per unit thickness of a material.

6b. ii. The low-energy X-rays do not significantly penetrate the body and contribute to the image. Therefore they give the patient an unnecessary dose and so are removed.

6b. iii. $I = I_0\,e^{-\mu x}$. So $I_0 = \frac{1}{e^{-\mu x}} = \frac{250\,\text{W m}^{-2}}{e^{-250 \times 0.03}} = 4.5 \times 10^5\,\text{W m}^{-2}$

Chapter 5

ASSIGNMENT 1

A1 A drug containing a radioactive tracer which does not have any biological effect on the function of the body. It is composed of two parts: a radionuclide and a pharmaceutical.

A2 $A = A_0 e^{-\lambda t}$

$$\lambda = \frac{\ln 2}{T_P} = \frac{\ln 2}{6 \times 60} = 0.00193\ \text{min}^{-1}$$

Total time = 65 min. Activity in body after this time = $10 \times 10^9 \times e^{-0.00193 \times 65}$ = 8.8 GBq

A3 $$\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P} = \frac{1}{1} + \frac{1}{0.25} = 5$$

so T_E = 0.2 days or 4.8 h

A4 $A = A_0 e^{-\lambda t}$

Using the effective half-life,

effective decay constant $\lambda = \frac{\ln 2}{T_E} = 3.466\ \text{h}^{-1}$

Activity in body after a week (7 days) = $10 \times 10^9 \times e^{-3.466 \times 7}$ = 0.29 Bq

(This is below the background level.)

There is a variation in the value of the biological half-life (and therefore that of the effective half-life) that depends on the state of health and metabolism of the individual patient.

A5 **a.** **i.** 5

ii. Each of these produces 5 more so number of electrons = 5 × 5 = 25

iii. 25 × 5 = 125

b. There are ten dynodes in total so $5^{10} = 9.8 \times 10^6$, i.e. an electron gain of nearly 10 million.

A6 The MPS on the left shows a darker/bluer region, showing the absence of tracer and thus a lack of blood supply to this area of the heart. The MPS on the right shows a lighter/redder-orange colour in the same region, suggesting the presence of tracer and the blood supply being restored to this area.

A7 Ultrasound scanning and ECG measurements.

ASSIGNMENT 2

A1 **a.** An electronvolt (eV) is the energy equal to the work done on an electron in accelerating it through a potential difference of one volt.

b. 250 MeV = 250×10^6 V × 1.6×10^{-19} C = 4.0×10^{-11} J

A2 High-energy X-ray radiotherapy deposits energy in all parts of the body in its transmission path, which means that healthy cells get damaged as well as cancer cells. Since healthy cells recover more quickly than cancer cells, the total prescribed radiotherapy dose is given over a number of treatment sessions at fractional dosages to allow the healthy cells to recover.

A3 The Bragg peak indicates the depth in the body where the protons give up most of their energy and ultimately stop. It is where the maximum dose is delivered and is a function of the initial proton energy. Almost no dose that would otherwise be harmful to healthy tissue is deposited past the Bragg peak.

A4 Figure A1 shows that the Bragg peak and therefore the maximum dose is confined to a relatively small spatial depth of few mm in the body. By using protons of different energies the Bragg peak can be shifted to different depth levels to encompass the full extent of the tumour.

A5 Electrons lose their energy very rapidly as they enter the body and do not deliver much dose at depth.

A6 Proton therapy facilities rely on particle accelerators, which are large and very expensive to build and operate.

PRACTICE QUESTIONS

1a. Technetium-99_m is suitable as a tracer because:

- it can be combined with a number of chemical compound to go to different parts of the body and is taken up by bone and blood cells;
- it emits photons of energy that can readily be detected by a gamma camera. Gamma cameras are not designed to detect beta particles;
- it has a reasonably short half-life of 6 hours and the radiation dose to the patient is minimal.

1b. Caesium-137 is suitable as an implant because:

- beta minus radiation is more strongly ionising than gamma so better for damaging tumour cells;
- beta minus radiation has a relatively short range in the body so minimal damage to surrounding tissue;
- it has a long half-life so delivers uniform dose over the period it is in the body.

2a. Biological half-life is the time taken for the body to process and excrete half the amount of a substance introduced into it and varies is according the state of health and metabolism of the patient. Physical half-life is the time taken for a radionuclide to reduce to one half of its initial activity. It is a fixed quantity and is a property of the nucleus. The effective half-life is the period during which the quantity of a radionuclide in a biological system is reduced by half by the processes of radioactive decay and biological excretion.

2b. The effective half-life always less than the physical half-life because the physical half-life is fixed and biological removal of the nuclide from the body is also occurring removing the nuclide more quickly overall.

3. $A = A e^{-\lambda t}$

$$1800 = 3500 \times e^{-7\lambda_E}$$

$$\lambda_E = -\frac{\ln\left(\frac{1800}{3500}\right)}{7} = 0.0950 \text{ day}^{-1}$$

$$\text{so } T_E = \frac{\ln(2)}{0.0950} = 7.296$$

$$\frac{1}{7.296} = \frac{1}{14} + \frac{1}{T_B}$$

$$\frac{1}{T_B} = 0.0657$$

$T_B = 15.2$ days

4a. PET uses the annihilation of a positron and an electron to produce two gamma rays moving in opposite directions which are detected in coincidence. The tumour will lie along a line connecting the detectors. Similar measurements are repeated at different detector angles to pinpoint the position of the tumour.

4b. The tumour is irradiated at different angles in a circles centred on the position of the tumour. This maximises the dose to the tumour but minimises it to surrounding healthy cells.

Healthy tissue recovers faster that cancerous tissue so X-ray doses are given in fractions over a period of time to allow healthy tissue to recover between radiotherapy sessions.

5. Answers should include the following points.

Patient safety:

- With CT, the ionising radiation exposure needs to be considered. With ultrasound there are no known risks.

Convenience:

- A CT scan is fast, a few minutes at most, but the patient must lie still . An ultrasound scan may take up to 15 minutes but patient movement is tolerated.
- A CT scanner is large and bulky, whereas ultrasound scanning equipment is portable.
- A CT scan may need some preparation particularly if contrast medium is needed. an ultrasound scan needs little preparation apart from the use of gel.

Diagnostic information:

- CT scan is excellent for imaging bony structures but requires contrast agent to enhance soft tissues. It can detect malignant tumours. Ultrasound is good for imaging soft tissues but cannot penetrate bone or examine brain, nor pass through air spaces/lungs.
- Both can measure size of organs. CT scans cannot detect movements in body, whereas ultrasound can.

AQA A-Level Physics Year 2
Turning points in physics Answers

Chapter 1

ASSIGNMENT 1

A1 Using the data provided in the assignment generates the data table below.

Accelerating voltage/V	Flux density *B*/tesla	B^2/tesla^2
300	1.168×10^{-3}	1.364×10^{-6}
280	1.129×10^{-3}	1.275×10^{-6}
260	1.088×10^{-3}	1.184×10^{-6}
240	1.045×10^{-3}	1.092×10^{-6}
220	1.001×10^{-3}	1.002×10^{-6}
200	0.954×10^{-3}	0.910×10^{-6}

The gradient of the graph of V versus B^2 (using the data in the above table) is 2.20×10^8 V T^{-2}

which gives $\frac{e}{m} = \frac{2}{r^2} \times \text{gradient} = \frac{2}{0.05^2} \times 2.2 \times 10^8 = 1.76 \times 10^{11}\ \text{C kg}^{-1}$

The accepted value for the charge/mass ratio of an electron expressed to 3 significant figures is 1.76×10^{11} C kg^{-1}. The percentage difference between the value obtained from the data table and the accepted value is therefore 0%. Note that if a value for the electron's charge/mass ratio is generated by the student's own experiment, the percentage difference between the student's value and the accepted value can be calculated using the following equation:

$$\text{percentage difference} = \frac{\text{accepted value} - \text{student's value}}{\text{accepted value}} \times 100$$

A2 Students' own answers.

ASSIGNMENT 2

A1 $r = \sqrt{\frac{9\eta v_1}{2g(\rho_{oil} - \rho_{air})}} = \sqrt{\frac{9 \times 18.16 \times 10^{-6} \times 6.868 \times 10^{-6}}{2 \times 9.81 \times (886.1 - 1.21)}} = 2.54 \times 10^{-7}\,\text{m}$

A2 Balancing the forces produces

$$\frac{QV}{d} + \frac{4}{3}\pi r^3 \rho_{air} g = 6\pi \eta r v_2 + \frac{4}{3}\pi r^3 \rho_{oil} g$$

Rearranging for charge Q produces

$$Q = \frac{6\pi\eta r v_2 + \frac{4}{3}\pi r^3 g\,(\rho_{oil} - \rho_{air})}{\frac{V}{d}}$$

$$Q = \frac{(6\pi \times 18.16 \times 10^{-6} \times 2.54 \times 10^{-7} \times 1.163 \times 10^{-4}) + \frac{4}{3}\pi \times (2.54 \times 10^{-7})^3 \times 9.81 \times (886.1 - 1.21)}{\frac{530}{8.00 \times 10^{-3}}} = 1.62 \times 10^{-19}\ \text{C}$$

PRACTICE QUESTIONS

1a. i. The high voltage across the tube ionises some of the gas atoms creating some positive gas ions and free electrons. The positive ions are accelerated towards the cathode and on hitting the cathode cause large numbers of electrons to be ejected. The gas therefore becomes conducting because of the presence of the positive gas ions and the large number of electrons ejected from the cathode.

1a. ii. The electrons collide with gas atoms and cause them to be excited. When the gas atoms de-excite, photons in the visible region are emitted. Also some of the electrons and gas ions recombine causing the emission of photons. The gas must be at low pressure so that the gas atoms are far enough apart to allow the electrons to be accelerated to high enough speeds to be able to cause excitation of the gas atoms.

1b. The specific charge of an ion is equal to the ratio: $\frac{\text{charge}}{\text{mass}}$. The charge on the ion is equal in magnitude to the charge on an electron, however the mass of the ion is dependent on the type of gas atoms in the tube.

2a. Increasing the cathode filament current raises the temperature of the filament. Electrons are emitted from the cathode filament by the process of thermionic emission and at higher temperatures more electrons are emitted per second.

2b. The potential difference between the anode and cathode creates an electric field which exerts a force on the electrons accelerating them from the cathode towards the anode. Increasing the anode potential increases the force exerted by the electric field on the electrons accelerating them to a higher speed.

3a. The force exerted by the magnetic field on an electron acts at 90° to its velocity and therefore no work is done on the electron so its speed is unchanged. The force due to the magnetic field is given by $F = Bev$ so since the velocity is constant the force remains constant. A constant force acting at 90° to the path is a centripetal force so the electron beam is circular.

3b. We can rearrange $Bev = \frac{mv^2}{r}$ to get $r = \frac{mv}{Be} = \frac{9.11\times10^{-31}\times2.6\times10^{7}}{10\times10^{-3}\times1.6\times10^{-19}} = 1.48\times10^{-2}\,\text{m}$

3c. Time for one orbit: $T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi\times1.48\times10^{-2}}{2.6\times10^{7}} = 3.6\times10^{-9}\,\text{s}$

4a. The electric field between the plates is uniform so the downwards electric force on an electron in the beam is constant giving the electron a vertical acceleration and therefore an increasing vertical component of velocity. There is no horizontal force on the electron once it has passed through the anode, so its horizontal component of velocity is constant. The combined effect of an increasing vertical component of velocity and a constant horizontal component of velocity results in the electrons following a parabolic path so that the beam can be seen to curve downwards at an increasing angle to the horizontal.

4b. i. The direction of the magnetic field would need to be reversed so as to create an upward magnetic force on the electrons. The flux density of the magnetic field would then need to be adjusted so that the magnetic field exerted an upward force on the electrons that was equal in magnitude to the downward force exerted by the electric field.

4b. ii. The force on an electron due to the magnetic field is given by $F = Bev$. The force on the electron due to the electric field is given by $F = e\frac{V}{d}$. When the beam is undeflected these two forces are equal in size, so $Bev = e\frac{V}{d}$, which rearranges to give: $v = \frac{V}{Bd}$.

4c. The kinetic energy of an electron in the beam is related by the accelerating voltage V_A of the electron gun by the equation: $\frac{1}{2}mv^2 = eV_A$, where *m* is the mass of the electron and *e* its charge. This equation rearranges to give $\frac{e}{m} = \frac{v^2}{2V_A} = \frac{(3.9\times10^{7})^2}{2\times4200} = 1.8\times10^{11}\,\text{Ckg}^{-1}$

5a. Initially the weight of the drop is greater than the upward viscous force exerted by the air on the drop so the drop accelerates. As the speed of the drop increases the viscous force also increases until it becomes equal in magnitude to the weight of the drop, at which point there is no resultant force on the drop and it continues to fall at a constant terminal velocity.

5b. i. If a drop is held stationary, its weight is balanced by the force exerted on it by the electric field and we can write: $mg = Q\frac{V}{d}$ where *m* is the mass of the drop, *Q* is the charge on the drop, *V* is the voltage between the two horizontal plates, and *d* is the plate separation. This equation can be rearranged to give $Q = \frac{mgd}{V} = \frac{1.92\times10^{-14}\times9.81\times20\times10^{-3}}{2350} = 1.6\times10^{-18}\,\text{C}$.

5b. ii. The drop must be positively charged if it can be held stationary given the polarity of the power supply. Since the charge on the drop is equal to 10 times the electronic charge, it can be concluded that the drop has a deficit of 10 electrons.

5c. The mass of the drop is given by $m = \frac{4}{3}\pi r^3\rho$, which can be rearranged to give an expression for the radius *r* of the drop.

Radius *r* is given by $r = \sqrt[3]{\frac{3m}{4\pi r}} = \sqrt[3]{\frac{3\times1.92\times10^{-14}}{4\pi\times800}} = 1.79\times10^{-6}\,\text{m}$

5d. At the terminal speed, when no voltage is applied across the plates, the weight of the drop is equal in magnitude to the upward viscous force due to the air which gives $6\pi\eta rv = mg$. This equation can be rearranged to give: $v = \frac{mg}{6\pi\eta r} = \frac{1.92\times10^{-14}\times9.81}{6\pi\times1.8\times10^{-5}\times1.79\times10^{-6}} = 3.1\times10^{-4}\,\text{m}\,\text{s}^{-1}$.

6. Applying the equation of motion: $s = ut + \frac{1}{2}at^2$ to the vertical motion of an electron in the beam gives the electron's vertical displacement *y* at point P as $y = \frac{1}{2}at^2$, where *t* is the time an electron takes to travel between the plates on reaching point P. The vertical acceleration a of an electron is given by $a = \frac{F}{m} = \frac{Ee}{m} = \frac{Ve}{dm}$ and the time *t* is given by $t = \frac{\text{horizontal distance}}{\text{horizontal velocity}} = \frac{x}{v_H}$.

Therefore the vertical displacement $y = \frac{1}{2}\times\frac{Ve}{dm}\times\left(\frac{x}{v_H}\right)^2$, which can be written $y = \frac{Ve}{2dm(v_H)^2}x^2$ corresponding to $y = kx^2$, where $k = \frac{Ve}{2dm(v_H)^2}$, confirming that the shape of the electron beam between the plates corresponds to a parabola.

Chapter 2

ASSIGNMENT 1

A1 Shine a torch at a wall several metres away, cover the bulb with a piece of card, then, on removing the card, observe that the light appears instantly on the wall.

A2 Light would have taken just over $5\,\mu s$ to travel the distance of one mile between Galileo and his assistant, which would have been negligible compared with their reaction times and not even measurable with the clocks available at that time.

A3 Light would have to be travelling astronomical distances to create measurable time intervals.

A4 Students' own answers.

A5 The metre is defined as the length of the path travelled by light in a vacuum during the time interval of $\frac{1}{299\,792\,458}$ of a second.

ASSIGNMENT 2

A1 The gradient of the graph of $\frac{1}{V}$ versus D^2 is equal to 0.10 V^{-1} m^{-2}. Hence $\frac{d^2 m e}{2L^2 h^2} = 0.10$

which rearranges to give the carbon atom spacing:

$$d = \sqrt{\frac{0.10 \times 2 \times L^2 \times h^2}{m e}} = \sqrt{\frac{0.10 \times 2 \times (0.18)^2 \times (6.63 \times 10^{-34})^2}{9.11 \times 10^{-31} \times 1.6 \times 10^{-19}}} = 1.4 \times 10^{-10}\,\text{m}$$

PRACTICE QUESTIONS

1a. i. Huygens' wave theory was unable to explain why light formed sharp shadows when passing everyday objects whereas water waves and sound waves were observed to diffract around objects.

It was for this failure, and of course Newton's immense reputation, based on his many successful theories, that the corpuscular theory of light was supported by most scientists for the next hundred years.

1a. ii. Newton's theory predicted that, during refraction, light corpuscles were attracted to a transparent medium and increased their speed as they passed through. Foucault's speed of light measurement through water showed a reduction in speed not an increase.

1b. Huygens' wave theory predicted that the wave fronts from the two slits would diffract, overlap and undergo superposition, producing several interference fringes. A bright fringe would be created at points where the two waves were in phase, producing constructive interference, and a dark fringe where the waves were 180° out of phase, producing destructive interference. The two waves would be in phase when their path difference was zero or equal to a whole number of wavelengths.

2a. A stationary wave pattern is formed when two waves of the same frequency, speed and amplitude travel through the same space in opposite directions. Radio waves from the transmitter travel towards the metal reflector and are reflected back towards the transmitter. The superposition of the incident and reflected radio waves produces a stationary wave pattern consisting of alternate points of constructive and destructive interference called nodes and antinodes along the line XY. The nodes and antinodes are equally spaced with the distance between two consecutive nodes being equal to $\frac{1}{2}\lambda$. The detector can then be moved along XY to locate the nodes, where the signal recorded by the detector would be zero. A distance measurement from the first to the third node is equal to one wavelength. Given that a longer distance measurement has a smaller percentage uncertainty, a measurement from the first to the fifth node, which is equal to two wavelengths, would yield a more accurate value for the wavelength of the radio waves.

2b. The value for the speed of radio waves obtained by Hertz was equal to the theoretical value for the speed of electromagnetic waves that Maxwell had predicted and also equal to the measured value for the speed of light, confirming that both radio waves and light were electromagnetic waves.

3a. Einstein explained that light is made up of a beam of photons each of energy $E = hf$ and that when light is incident on a metal surface, a surface electron absorbs all the energy of a photon. In order for the electron to leave the metal surface, it must have sufficient energy to overcome the attractive electrical force holding it there. To just escape from the surface, an electron must acquire a certain minimum energy, called the work function. If the energy transferred to an electron by a photon exceeds the work function, the electron can escape from the surface as a photoelectron with a maximum kinetic energy equal to the difference between the photon energy and the work function. However, if the energy of the absorbed photon is less than the work function, the electron does not escape. If the absorbed photon has energy equal to the work function of the metal, the electron has enough energy to escape the metal but has zero kinetic energy once it has escaped. In this latter case, the frequency of the photon is called the threshold frequency. Some of the surface electrons require more energy than the work function to escape from the metal, resulting in photoelectrons being emitted with a range of kinetic energies from zero to the maximum value.

3b. i. Threshold frequency is given by

$$f_0 = \frac{\Phi}{h}$$

$$f_0 = \frac{2.3 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.6 \times 10^{14}\ \text{Hz}$$

3b. ii. Using Einstein's photoelectric equation

$$\frac{1}{2} m v^2_{\max} = hf - \Phi = h\frac{c}{\lambda} - \Phi$$

$$\frac{1}{2} m v^2_{\max} = \left(6.63 \times 10^{-34} \times \frac{3.00 \times 10^8}{4.6 \times 10^{-7}}\right) - \left(2.3 \times 1.60 \times 10^{-19}\right)$$

$$= 6.44 \times 10^{-20}$$

Therefore

$$v = \sqrt{\frac{2(hf - \Phi)}{m}} = \sqrt{\frac{2 \times 6.44 \times 10^{-20}}{9.11 \times 10^{-31}}}$$

$$= 3.8 \times 10^5\ \text{m s}^{-1}$$

3b. iii. Using $\frac{1}{2} m v^2_{\max} = eV_s$

gives $V_s = \frac{6.44 \times 10^{-20}}{1.60 \times 10^{-19}} = 0.40\ \text{V}$

4a. i. The kinetic energy *E* of a thermal neutron is given by

$$E = \frac{1}{2} m v^2 = 0.5 \times 1.675 \times 10^{-27} \times (2.2 \times 10^3)^2 = 4.1 \times 10^{-21}\ \text{J}$$

4a. ii. The de Broglie wavelength, λ, is given by $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.675 \times 10^{-27} \times 2.2 \times 10^3} = 1.8 \times 10^{-10}\ \text{m}$

4b. Photon energy $= \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.8 \times 10^{-10}} = 1.1 \times 10^{-15}\ \text{J}$

4c. i. Neutrons and X-rays with the same wavelength would have the same resolving power; however, the X-ray photon has 105 times more energy and is therefore much more likely to cause radiation damage to the specimen.

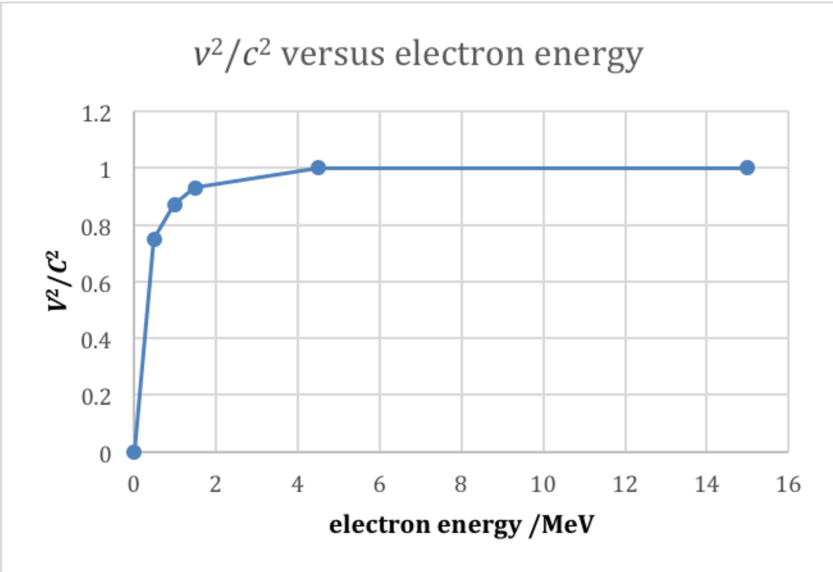
4c. ii. X-ray photons primarily interact with the electron cloud surrounding an atom, whereas neutrons are unaffected by the electron cloud and interact directly with the nuclei of the specimen atoms, enabling the identification of isotopes.

Chapter 3

ASSIGNMENT 1

A1 and A2

Accelerating voltage / MV	Number of divisions between troughs	Distance travelled at constant speed / m	Electron speed / m s^{-1}	$\frac{v^2}{c^2}$	Electron kinetic energy / MeV
0.5	3.3	8.4	2.6×10^8	0.75	0.5
1.0	3.1	8.4	2.8×10^8	0.87	1.0
1.5	2.95	8.4	2.9×10^8	0.93	1.5
4.5	2.55	7.4	3.0×10^8	1	4.5
15	2.5	7.4	3.0×10^8	1	15



A3 Total charge Q arriving at the aluminium disc during the time for 80 clicks of the coulomb meter is $Q = 80 \times 7.6 \times 10^{-8} = 6.08 \times 10^{-6}$ C

Total energy absorbed by the aluminium disc E is $E = 12.5 \times 0.8 = 10$ J

Kinetic energy of one electron $= \frac{E}{Q} \times e = \frac{10}{6.08 \times 10^{-6}} \times 1.6 \times 10^{-19} = 2.64 \times 10^{-13}$ J

The kinetic energy of one electron accelerated through 1.5 MV is 1.5 MeV, which is equal to

$1.5 \times 10^6 \times 1.6 \times 10^{-19} = 2.4 \times 10^{-13}$ J

Percentage difference between the calorimetry measurement of kinetic energy of the one electron and the value determined from the accelerating voltage is

$$\frac{2.64 \times 10^{-13} - 2.4 \times 10^{-13}}{2.64 \times 10^{-13}} \times 100 = 9\%$$

PRACTICE QUESTIONS

1a. An inertial frame of reference is non-accelerating and therefore has a constant velocity.

1b. i. Speed $= \frac{4.3 \times 9.46 \times 10^{15}}{5 \times 365 \times 24 \times 3600} = 2.58 \times 10^8 = 2.6 \times 10^8$ m s^{-1}

1b. ii. The time dilation formula, $t = t_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, can be rearranged to give

$$t_0 = t \times \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = 5 \times \left(1 - \frac{(2.58 \times 10^8)^2}{(3.00 \times 10^8)^2}\right)^{\frac{1}{2}} = 2.55 = 2.6\,\text{years}$$

2a. i. Distance $d_0 = v \times t = 1.8 \times 10^8 \times 95 \times 10^{-9} = 17.1 = 17\,\text{m}$

2a. ii. $d = d_0\sqrt{1 - \frac{v^2}{c^2}} = 17.1 \times \left(1 - \frac{(1.8 \times 10^8)^2}{(3.00 \times 10^8)^2}\right)^{\frac{1}{2}} = 13.7 = 14\,\text{m}$

2b. First determine the relativistic mass of the proton:

$$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1.673 \times 10^{-27} \times \left(1 - \frac{(1.8 \times 10^8)^2}{(3.00 \times 10^8)^2}\right)^{-\frac{1}{2}} = 2.091 \times 10^{-27}\,\text{kg}$$

$$\frac{\text{kinetic energy of the proton}}{\text{rest energy of the proton}} = \frac{(m - m_0)c^2}{m_0c^2} = \frac{2.091 \times 10^{-27} - 1.673 \times 10^{-27}}{1.673 \times 10^{-27}} = 0.25$$

3a. Light always travels at the same speed regardless of the speed of the light source or the observer.

3b. First calculate the distance between the detectors in the rest frame of the particles using the length contraction formula:

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} = 25 \times \sqrt{1 - 0.98^2} = 4.97\,\text{m}$$

Time to travel between the detectors in the rest frame of the particles:

$$= \frac{\text{distance}}{\text{time}} = \frac{4.97}{0.98 \times 3 \times 10^8}$$ = 1.69 × 10^{-8} s = 2 half-lives. So half-life = 8.5 × 10^{-9} s

4a. Total energy E = kinetic energy + rest energy = 2500 + 938 = 3438 MeV

4b. Rearranging $E = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

gives $\sqrt{1 - \frac{v^2}{c^2}} = \frac{\text{rest energy}}{E} = \frac{938}{3438} = 0.273$

$1 - \frac{v^2}{c^2} = 0.273^2$

which gives $v = 2.89 \times 10^8\ \text{m s}^{-1}$