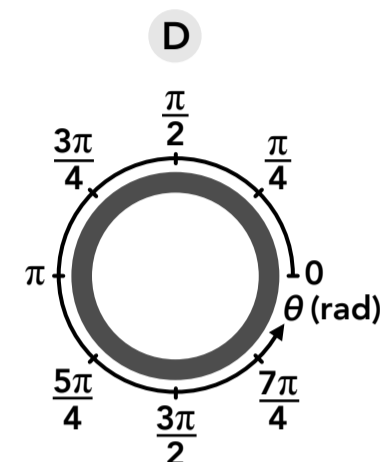
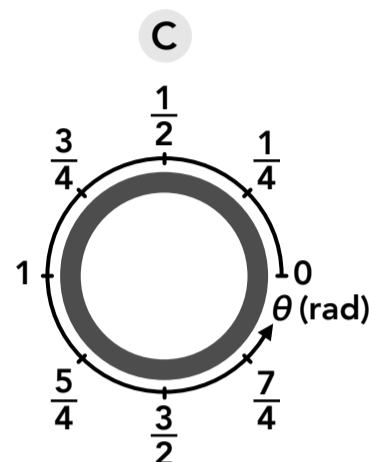
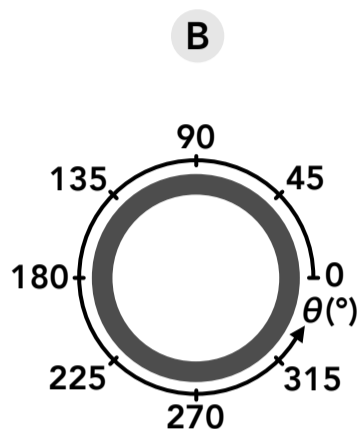
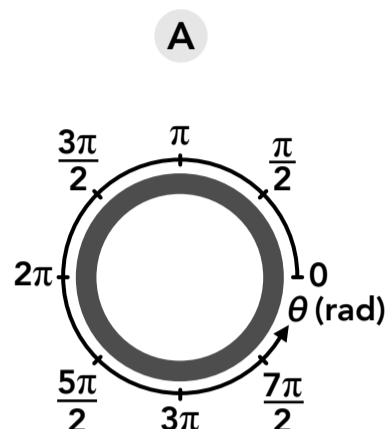


Angular position and displacement

1. Which of the following are examples of an object in rotational motion? (Select all that apply)

- A A CD spinning in a CD player (the CD)
- B The knob on a door being opened or closed (the knob)
- C A screw being tightened with a screwdriver (the screw)
- D A person riding on a Ferris wheel (the person)

2. Which of these are valid axes? (Select all that apply)



3. What is the SI unit for angular position?

- A m
- B deg
- C rad
- D rev

4. What is the SI unit for angular displacement?

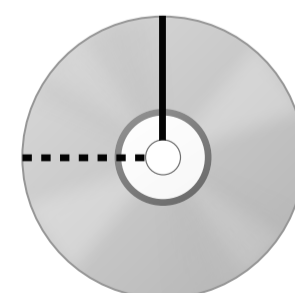
- A rev
- B rad
- C m
- D deg

5. A wrench is used to tighten a bolt clockwise 3 revolutions. The angular displacement of the bolt is

- A Positive
- B Negative

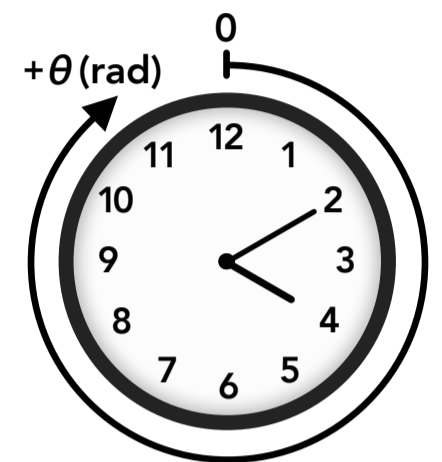
6. If a bike wheel rotates 270° over a short period, how many radians does the wheel rotate?

7. If the solid black line on the spinning CD to the right rotates 270° , how many degrees does the dotted black line rotate in the same amount of time?



8. If a door starts at an angular position of -15° and rotates clockwise 45° , what is the final angular position?

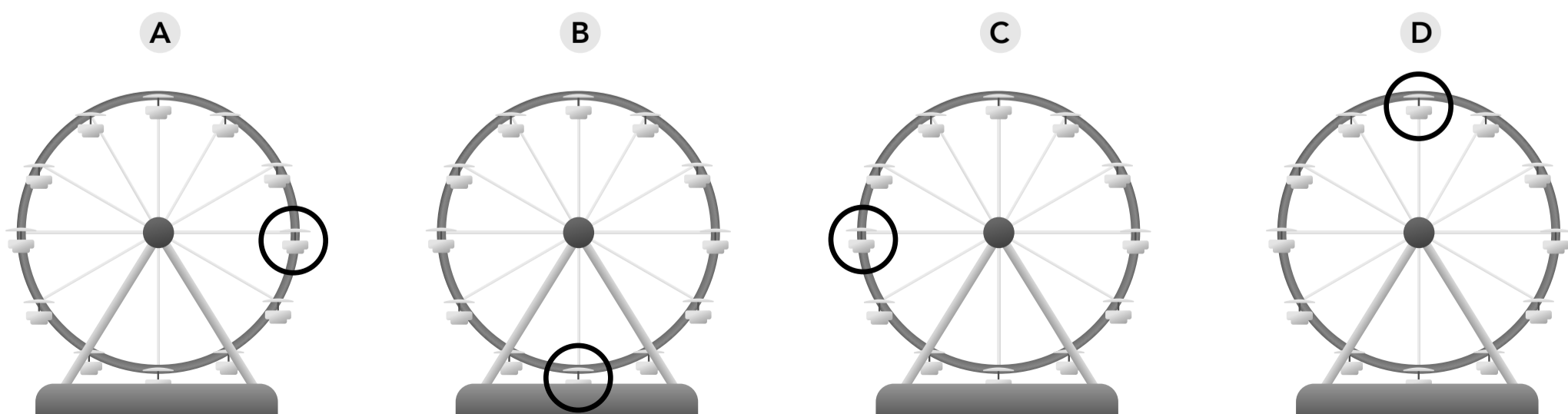
9. The drum in a washing machine starts at an angular position of $3\pi/2$ rad and rotates counterclockwise 2.5 revolutions. What is the final angular position of the drum in radians?
10. An olympic diver jumps and begins flipping through the air. During a short period of time, an observer sees the diver rotate from an angular position of 115° to a position of -80° . What was the angular displacement of the diver based on that point of view?
11. If we overlay an axis on a clock as seen to the right, what would be the angular positions of the hour hand at the following times, in radians: 3:00, 6:30, 10:45 ?



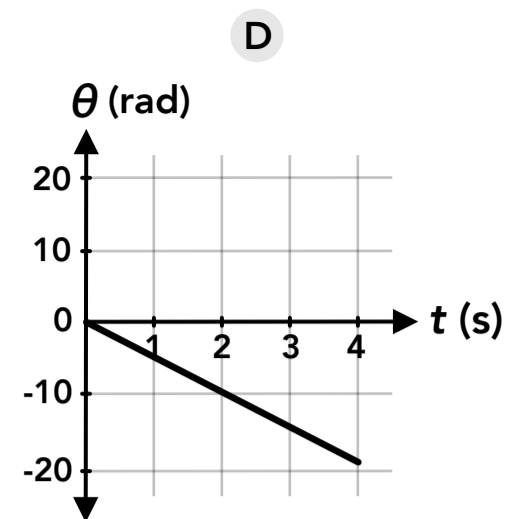
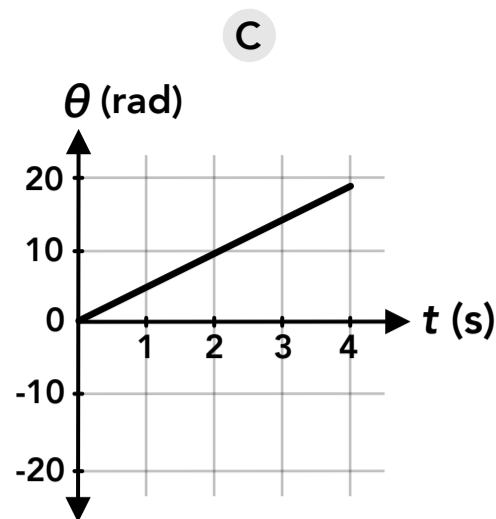
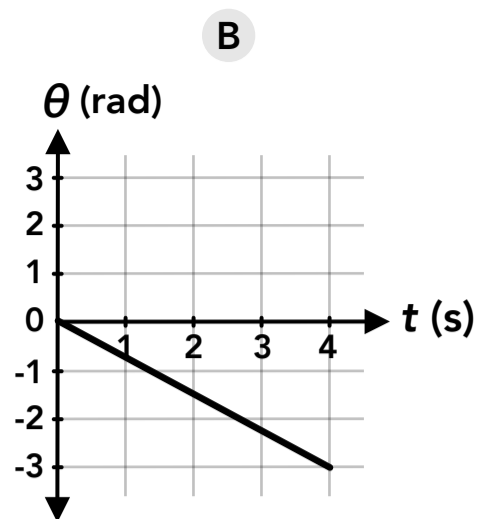
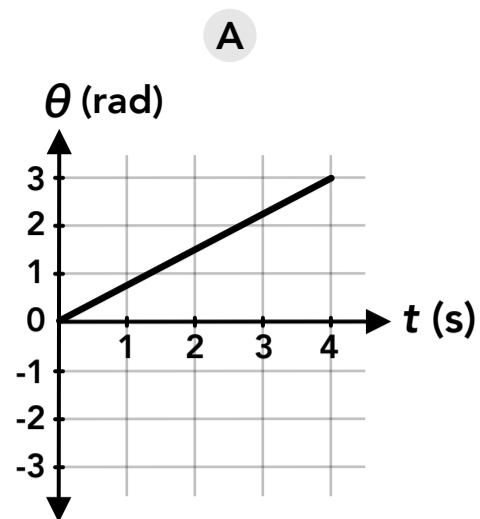
12. A carousel ride starts at an angular position of 270° and rotates clockwise 5.5π radians. What is the final angular position of the carousel in deg?

Angular speed and velocity

13. What is the SI unit for angular velocity and speed?
- A m/s
 - B rad/s
 - C deg/s
 - D rpm
14. A football is passed down the field with some spin added to it. From one perspective, if the ball is spinning counterclockwise, is the angular velocity of the ball positive or negative?
- A Positive
 - B Negative
15. You're watching your friend on a Ferris wheel which is rotating clockwise at a constant 1.5 rpm. When you look away, your friend is at the top of the Ferris wheel. When you look back 30 sec later, where is your friend?



16. A vinyl record is spinning at a constant 45 rpm clockwise. Which of these graphs could represent its motion?



17. A carousel at a local fair rotates at an angular velocity of 18 rad/s. What is the angular velocity in rpm?

18. If the Earth rotates once per day counterclockwise as seen from a point above the North Pole, what is the angular velocity of the Earth in rad/s?

19. What is the angular speed of the hour, minute, and second hands on a clock (a 12-hour clock) in rpm?

20. If a vinyl record rotates 700° in the clockwise direction in 3.5 s, what is its angular velocity in deg/s?

21. An art student is using a potter's wheel to make a vase. Over a period of 0.5 s, the wheel rotates from an angular position of $-\pi/2$ radians to a position of $3\pi/2$ radians. What is the angular velocity in rad/s?

22. If a Ferris wheel is currently at an angular position of $\pi/8$ rad and rotating at a constant angular velocity of $-\pi/60$ rad/s, how long does it take for the Ferris wheel to reach an angular position of $-4\pi/3$ rad?

23. If a wheel rotates 500π rad in 4 min, what is the average angular velocity of the wheel in deg/s?

24. What is the angular speed of the hour, minute, and second hands on a clock (a 12-hour clock) in rad/s?

25. A vinyl record is spinning at $33 \frac{1}{3}$ rpm. How many seconds does it take for the record to rotate 3π radians?

Angular acceleration

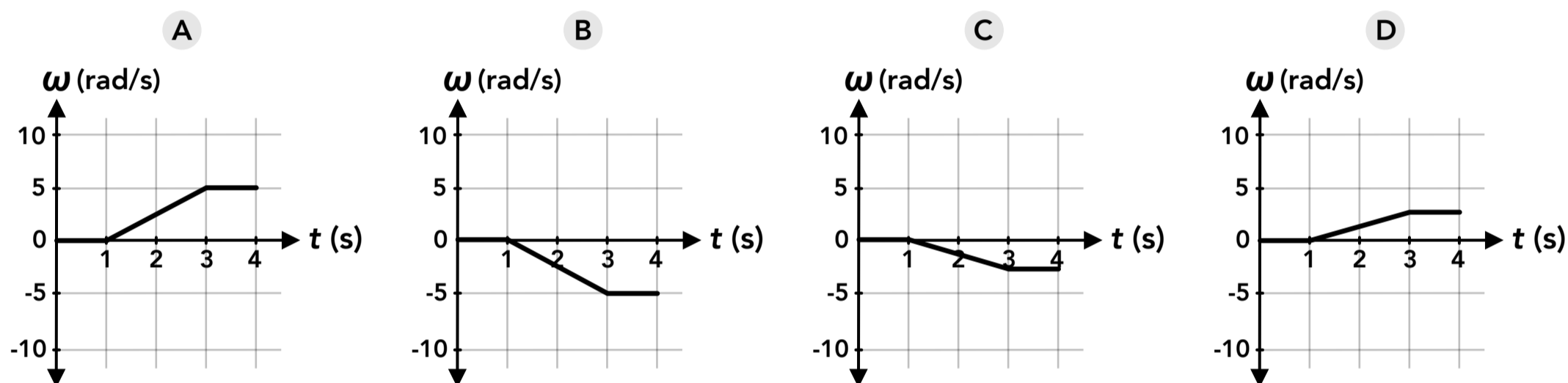
26. What is the SI unit for angular acceleration?

- A m/s^2
- B rev/min^2
- C deg/s^2
- D rad/s^2

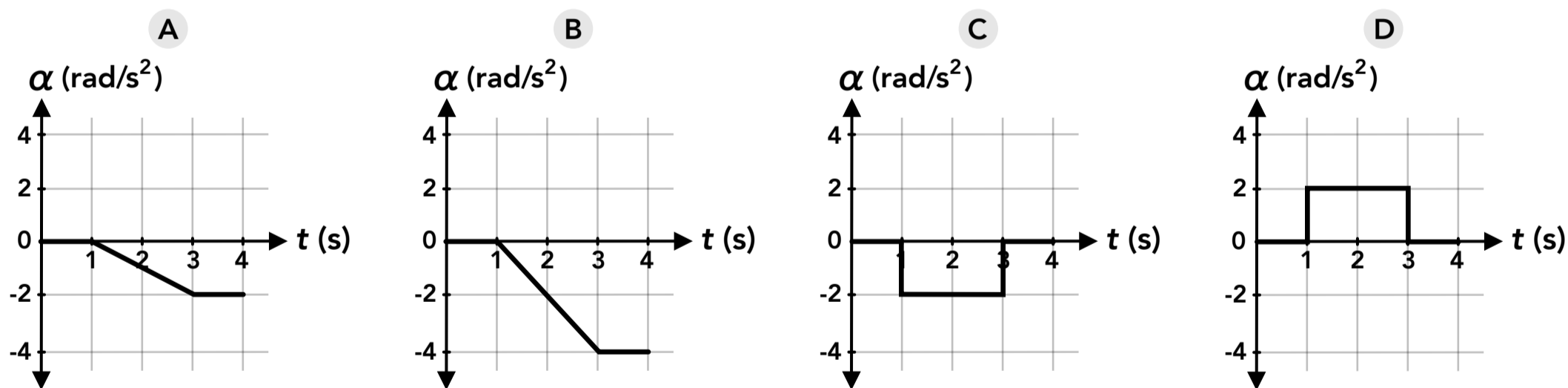
27. If an object is rotating and experiences an angular acceleration, its angular speed must be increasing.

- A True
- B False

28. A vinyl record starts from rest and accelerates clockwise at 2.5 rad/s^2 for 2 s, then maintains its speed. Which of these graphs could represent its motion?



29. A vinyl record starts from rest and then speeds up with a constant clockwise acceleration for 2 s, reaching a final angular velocity of -4 rad/s . Which of these graphs could represent its motion?



30. When a blender is turned on, the blades inside experience an angular acceleration of 20 rev/s^2 . What is the angular acceleration in rad/s^2 ?

31. A lab centrifuge spinning counterclockwise at 100 rad/s has its speed turned up. If it takes 8 s to reach an angular speed of 500 rad/s , what was the angular acceleration of the centrifuge?

32. A cyclist is stopped on a hill. The moment they release the brakes, the bike accelerates down the hill and the wheels experience an angular acceleration of 1.5 rad/s^2 . After 4 s, what is the angular speed of the wheels in rad/s ?
33. You're in the mood to listen to some music. You pick an album, and when you turn on your record player the record (starting from rest) takes 2 s to speed up to an angular velocity of 45 rpm clockwise. What was the angular acceleration of the record during that period in rad/s^2 ?
34. After a few seconds of listening to the record you start to wonder why the singer's voice is a higher pitch than you remember. You suddenly realize the record is supposed to be played at $33 \frac{1}{3}$ rpm, so you press the button for that speed. If it takes 0.6 s for the record to change from 45 rpm to $33 \frac{1}{3}$ rpm (both clockwise), what is the angular acceleration of the record during that time in rad/s^2 ?
35. When the first side of the record is finished playing you hit "Stop". The record that was spinning at $33 \frac{1}{3}$ rpm clockwise comes to a stop in 1.5 s. What was the angular acceleration of the record during that time in rad/s^2 ?
36. If a lab centrifuge starts from rest at an angular position of 0 deg and accelerates counterclockwise at 200 deg/s^2 , what is the final angular position after 3.5 s, in deg?
37. The operator of a carousel ride that is spinning counterclockwise decides to increase the speed. Over a period of 5 s the ride accelerates at 1 rad/s^2 and rotates 20 rad during that time. What was the initial angular velocity of the ride?
38. The axle connected to the front wheels of a car is spinning at a constant 420 rpm as the car drives forwards. The car then accelerates and the axle experiences a constant angular acceleration. After a period of 4 s the axle has turned 60 revolutions since the car began accelerating. What was the angular acceleration of the axle in rev/s^2 during that time?
39. A vinyl record starts from rest and then accelerates, reaching an angular velocity of -3.5 rad/s while rotating through an angular displacement of -10 rad . What was the angular acceleration of the record?

40. A CD is spinning at 20 rad/s and then the CD accelerates at a constant 8 rad/s^2 . From the moment the CD begins to accelerate, how many radians does the CD rotate by the time it reaches a velocity of 50 rad/s ?
41. The wheels on a bike are spinning at 6.3 rad/s . When the rider hits the brakes, the wheels experience a constant angular acceleration and rotate 25 rad as the bike and wheels come to a full stop. What was the angular acceleration of the wheels?
42. When a ceiling fan is turned on, it accelerates (from rest) at a constant 3 rad/s^2 . How long does it take the fan to rotate 10 revolutions?
43. A ceiling fan is spinning counterclockwise at a constant speed. The fan is turned off and experiences an angular acceleration of -2 rad/s^2 . During the time it takes the fan to come to a stop, the fan turns 7 revolutions. What was the initial angular speed of the fan, in rpm?
44. A wheel on a car is spinning at 840 rpm . The car accelerates, and the wheel experiences an angular acceleration for 3 s . If the wheel rotates 50 revolutions during that period of time, what is the angular velocity of the wheel after those 3 s , in rpm?

Answers - Angular position and displacement

1. Answer: A, C

A) A spinning CD is rotating around its own center so the CD is in rotational motion

B) The knob on a rotating door follows a circular path so the knob is in circular motion

C) A screw being turned by a screwdriver is rotating around its own center so the screw is in rotational motion

D) A person on a rotating Ferris wheel follows a circular path so the person is in circular motion

2. Answer: B, D

A) One circle has 2π radians, this circle has 4π radians

B) One circle has 360°

C) One circle has 2π radians, this circle has 2 radians

D) One circle has 2π radians

3. Answer: C

A) m is the SI unit for linear or tangential position

B) deg is a valid unit for angular position but is not the SI unit

C) rad is the SI unit for angular position

D) revolutions is a valid unit for angular position but is not the SI unit

4. Answer: B

A) revolutions is a valid unit for angular displacement but is not the SI unit

B) rad is the SI unit for angular displacement

C) m is the SI unit for linear or tangential displacement

D) deg is a valid unit for angular displacement but is not the SI unit

5. Answer: B

By convention, clockwise is the negative direction.

6. Answer: $3\pi/2$ rad or 4.7 rad

$$\frac{270^\circ}{360^\circ} \times \frac{2\pi \text{ rad}}{1} = 3\pi/2 \text{ rad}$$

7. Answer: 270°

All points or lines on the same, solid rotating object will rotate together the same amount.

8. Answer: -60°

$$\Delta\theta = \theta_f - \theta_i \quad (-45^\circ) = \theta_f - (-15^\circ) \quad \theta_f = -60^\circ$$

9. Answer: $13\pi/2$ rad or 20.4 rad

$$\frac{2.5 \text{ rev}}{1 \text{ rev}} \times \frac{2\pi \text{ rad}}{1} = 5\pi \text{ rad}$$

$$\Delta\theta = \theta_f - \theta_i \quad (5\pi \text{ rad}) = \theta_f - (3\pi/2 \text{ rad}) \quad \theta_f = 13\pi/2 \text{ rad}$$

10. Answer: -195°

$$\Delta\theta = \theta_f - \theta_i = (-80^\circ) - (115^\circ) = -195^\circ$$

11. Answer: 3:00 = $\pi/2$ rad, 6:30 = $13\pi/12$ rad, 10:45 = $43\pi/24$ rad

$$3:00 \quad \frac{3}{12} \times \frac{2\pi \text{ rad}}{\text{rev}} = \pi/2 \text{ rad}$$

$$6:30 \quad \left(\frac{6}{12} + \frac{30}{720} \right) \times \frac{2\pi \text{ rad}}{\text{rev}} = \frac{780\pi \text{ rad}}{720} = 13\pi/12 \text{ rad}$$

$$10:45 \quad \left(\frac{10}{12} + \frac{45}{720} \right) \times \frac{2\pi \text{ rad}}{\text{rev}} = \frac{1290\pi \text{ rad}}{720} = 43\pi/24 \text{ rad}$$

12. Answer: -720°

$$\frac{-5.5\pi \text{ rad}}{2\pi \text{ rad}} \times \frac{360^\circ}{1} = -990^\circ$$

$$\Delta\theta = \theta_f - \theta_i \quad (-990^\circ) = \theta_f - (270^\circ) \quad \theta_f = -720^\circ$$

Answers - Angular speed and velocity

13. Answer: B

A) m/s is the SI unit for linear or tangential velocity

B) rad/s is the SI unit for angular velocity and speed

C) deg/s is a valid unit for angular velocity and speed but is not the SI unit

D) rpm (rev/min) is a valid unit for angular velocity and speed but is not the SI unit

14. Answer: A

By convention, counterclockwise is the positive direction.

15. Answer: C

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (-1.5 \text{ rpm}) = \frac{\Delta\theta}{(0.5 \text{ min})} \quad \Delta\theta = -0.75 \text{ rev (3/4 rev clockwise)}$$

16. Answer: D

All of the graphs are angular position-time graphs. The slope of the angular position-time graph is the angular velocity. A slope of -45 rpm is equal to -4.7 rad/s . After 4 seconds the angular position would be -18.8 rad .

$$\frac{45 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -4.7 \text{ rad/s}$$

$$(-4.7 \text{ rad/s})(4 \text{ s}) = -18.8 \text{ rad}$$

17. Answer: 171.9 rpm

$$\frac{18 \text{ rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 171.9 \text{ rev/min (rpm)}$$

18. Answer: $7.27 \times 10^{-5} \text{ rad/s}$

$$\frac{1 \text{ rev}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 7.27 \times 10^{-5} \text{ rad/s}$$

19. Answer: hour hand = 1/720 rpm, minute hand = 1/60 rpm, second hand = 1 rpm

$$\text{hour hand: } \frac{1 \text{ rev}}{12 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} = 1/720 \text{ rev/min (rpm)}$$

$$\text{minute hand: } \frac{1 \text{ rev}}{1 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} = 1/60 \text{ rev/min (rpm)}$$

$$\text{second hand: } \frac{1 \text{ rev}}{1 \text{ min}} = 1 \text{ rev/min (rpm)}$$

20. Answer: -200 deg/s

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{-700^\circ}{3.5 \text{ s}} = -200 \text{ deg/s}$$

21. Answer: 4π rad/s or 12.6 rad/s

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(3\pi/2 \text{ rad}) - (-\pi/2 \text{ rad})}{0.5 \text{ s}} = 4\pi \text{ rad/s}$$

22. Answer: 87.5 s

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (-\pi/60 \text{ rad/s}) = \frac{(-4\pi/3 \text{ rad}) - (\pi/8 \text{ rad})}{\Delta t} \quad \Delta t = 87.5 \text{ s}$$

23. Answer: 375 deg/s

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(500\pi \text{ rad})}{(4 \text{ min})} \times \frac{360^\circ}{2\pi \text{ rad}} \times \frac{1 \text{ min}}{60 \text{ s}} = 375 \text{ deg/s}$$

24. Answer: hour hand = $\pi/21600$ rad/s, minute hand = $\pi/1800$ rad/s, second hand = $\pi/30$ rad/s

$$\text{hour hand: } \frac{2\pi \text{ rad}}{12 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/21600 \text{ rad/s}$$

$$\text{minute hand: } \frac{2\pi \text{ rad}}{1 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/1800 \text{ rad/s}$$

$$\text{second hand: } \frac{2\pi \text{ rad}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \pi/30 \text{ rad/s}$$

25. Answer: 2.7 s

$$\frac{33 \frac{1}{3} \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \times \frac{1 \text{ min}}{60 \text{ s}} = 10\pi/9 \text{ rad/s}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (10\pi/9 \text{ rad/s}) = \frac{(3\pi \text{ rad})}{\Delta t} \quad \Delta t = 2.7 \text{ s}$$

Answers - Angular acceleration

26. Answer: D

A) m/s^2 is the SI unit for linear or tangential acceleration

B) rev/min^2 is a valid unit for angular acceleration but is not the SI unit

C) deg/s^2 is a valid unit for angular acceleration but is not the SI unit

D) rad/s^2 is the SI unit for angular acceleration

27. Answer: False

Angular acceleration is the change in angular speed, but the speed may be increasing or decreasing.

28. Answer: B

All of the graphs are angular velocity-time graphs. The slope of the angular velocity-time graph is the angular acceleration, so the slope of the graph should be -2.5 rad/s^2 because clockwise is the negative direction. After 2 seconds the angular velocity would be: $(-2.5 \text{ rad/s}^2)(2 \text{ s}) = -5 \text{ rad/s}$

29. Answer: C

All of the graphs are angular acceleration-time graphs. The acceleration is constant (a flat line) and negative because clockwise is the negative direction.

30. Answer: $40\pi \text{ rad/s}^2$ or 125.7 rad/s^2

$$\frac{20 \text{ rev}}{\text{s}^2} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 40\pi \text{ rad/s}^2$$

31. Answer: 50 rad/s^2

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(500 \text{ rad/s}) - (100 \text{ rad/s})}{(8 \text{ s})} = 50 \text{ rad/s}^2$$

32. Answer: 6 rad/s

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad (1.5 \text{ rad/s}^2) = \frac{\omega_f - (0 \text{ rad/s})}{(4 \text{ s})} \quad \omega_f = 6 \text{ rad/s}$$

33. Answer: $-3\pi/4 \text{ rad/s}^2$ or -2.4 rad/s^2

$$\frac{-45 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -3\pi/2 \text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(-3\pi/2 \text{ rad/s}) - (0 \text{ rad/s})}{(2 \text{ s})} = -3\pi/4 \text{ rad/s}^2$$

34. Answer: 2.0 rad/s^2

$$\frac{-45 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -3\pi/2 \text{ rad/s}$$

$$\frac{-33 \frac{1}{3} \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -10\pi/9 \text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(-10\pi/9 \text{ rad/s}) - (-3\pi/2 \text{ rad/s})}{(0.6 \text{ s})} = 2.0 \text{ rad/s}^2$$

35. Answer: 2.3 rad/s^2

$$\frac{-33 \frac{1}{3} \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -10\pi/9 \text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(0 \text{ rad/s}) - (-10\pi/9 \text{ rad/s})}{(1.5 \text{ s})} = 2.3 \text{ rad/s}^2$$

36. Answer: 1225°

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 = (0 \text{ deg}) + (0 \text{ deg/s})(3.5 \text{ s}) + \frac{1}{2} (200 \text{ deg/s}^2)(3.5 \text{ s})^2 = 1225 \text{ deg}$$

37. Answer: 1.5 rad/s

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad (20 \text{ rad}) = \omega_i(5 \text{ s}) + \frac{1}{2}(1 \text{ rad/s}^2)(5 \text{ s})^2 \quad \omega_i = 1.5 \text{ rad/s}$$

38. Answer: 4 rev/s²

$$\frac{420 \text{ rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 7 \text{ rev/s}$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad (60 \text{ rev}) = (7 \text{ rev/s})(4 \text{ s}) + \frac{1}{2}\alpha(4 \text{ s})^2 \quad \alpha = 4 \text{ rev/s}^2$$

39. Answer: -0.6 rad/s²

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \quad (-3.5 \text{ rad/s})^2 = (0 \text{ rad/s})^2 + 2\alpha(-10 \text{ rad}) \quad \alpha = -0.6 \text{ rad/s}^2$$

40. Answer: 131.3 rad

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \quad (50 \text{ rad/s})^2 = (20 \text{ rad/s})^2 + 2(8 \text{ rad/s}^2)\Delta\theta \quad \Delta\theta = 131.3 \text{ rad}$$

41. Answer: -0.8 rad/s²

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \quad (0 \text{ rad/s})^2 = (6.3 \text{ rad/s})^2 + 2\alpha(25 \text{ rad}) \quad \alpha = -0.8 \text{ rad/s}^2$$

42. Answer: 6.5 s

$$\frac{10 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 20\pi \text{ rad}$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad (20\pi \text{ rad}) = (0 \text{ rad/s})t + \frac{1}{2}(3 \text{ rad/s}^2)t^2 \quad t = 6.5 \text{ s}$$

43. Answer: 126.6 rpm

$$\frac{7 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 14\pi \text{ rad}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \quad (0 \text{ rad/s})^2 = \omega_i^2 + 2(-2 \text{ rad/s}^2)(14\pi \text{ rad}) \quad \omega_i = 13.26 \text{ rad/s}$$

$$\frac{13.26 \text{ rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 126.6 \text{ rev/min (rpm)}$$

44. Answer: 1160.4 rpm

$$\frac{840 \text{ rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 14 \text{ rev/s}$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad (50 \text{ rev}) = (14 \text{ rev/s})(3 \text{ s}) + \frac{1}{2}\alpha(3 \text{ s})^2 \quad \alpha = 1.78 \text{ rev/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad (1.78 \text{ rev/s}^2) = \frac{\omega_f - (14 \text{ rev/s})}{(3 \text{ s})} \quad \omega_f = 19.34 \text{ rev/s}$$

$$\frac{19.34 \text{ rev}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} = 1160.4 \text{ rev/min (rpm)}$$