Higher Maths Formula List

Exponential & Logarithmic Functions

$log_a x + log_a y = log_a(xy)$

$log_a x - log_a y = log_a(\frac{x}{y})$

$log_a x^n = n\, log_a x$

$log_a a = 1$

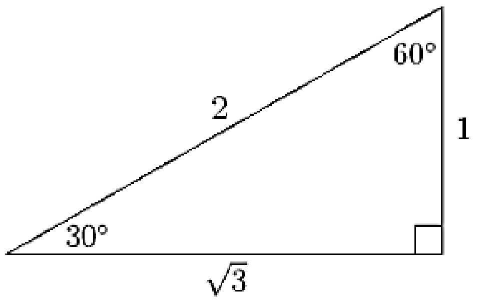
$log_a 1 = 0$

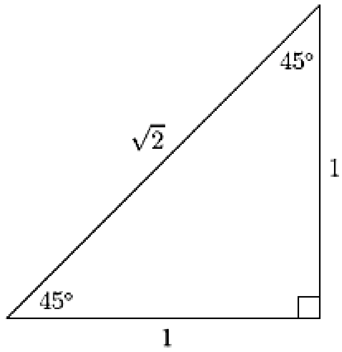
Trigonometry

Trigonometric exact values for common angles in degrees

Angle	0	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$No\ value$

The values in the above table are generated from these triangles





Degrees	30°	45°	60°	90°	120°	150°	180°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π

Degrees	180°	210°	240°	270°	300°	330°	360°
Radians	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

The Addition Formulas

$sin\ (x + a) = sin\ x\ cos\ a + cos\ x\ sin\ a$

$sin\ (x - a) = sin\ x\ cos\ a - cos\ x\ sin\ a$

$\cos\ (x + a) = \cos x \cos a - \sin x \sin a$

$cos\ (x - a) = cos\ x\ cos\ a + sin\ x\ sin\ a$

The Double Angle Formulas

$sin\ 2A = 2sin\ A\ cos\ A$

$\cos 2A = cos^2A - sin^2A$

$\cos 2A = 2cos^2A - 1$

$cos\ 2A = 1 - 2sin^2A$

Trigonometric Identities

$sin^2x + cos^2x = 1$

$\frac{sin\ x}{cos\ x} = tan\ x$

Vectors

For the vector $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, the magnitude of $\underline{a}$, written $|\underline{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

For the vectors $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ the scalar product of $\underline{a}$ & $\underline{b}$, written $\underline{a} \cdot \underline{b}$, is given by: $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|cos\theta$ where θ is the angle between the positive direction of $\underline{a}$ & $\underline{b}$

Quadratic Functions

For the quadratic function $ax^2 + bx + c$ the discriminant is defined by $b^2 - 4ac$.

If $b^2 - 4ac < 0$ the quadratic function has no real roots

If $b^2 - 4ac = 0$ the quadratic function has one real root

If $b^2 - 4ac > 0$ the quadratic function has two real roots

Note that one real root is sometimes referred to as *equal* roots.

Differentiation

The Chain Rule

To differentiate the function $y = (ax + b)^n$ we use the chain rule.

$$\frac{dy}{dx} = na(ax + b)^{n-1}$$

Trigonometric Functions

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Rates of Change

Let $s(t)$ be a function which gives distance, d, at time t, called *displacement*.

Then $V(t) = s'(t)$ i.e. velocity is the derivative of displacement

And $A(t) = V'(t)$ i.e. acceleration is the derivative of velocity

Stationary Points

Stationary points occur where $f'(x) = 0$ or $\frac{dy}{dx} = 0$

Stationary points can be a *minimum turning point*, *maximum turning point*, or less commonly a *point of inflection*.

Integration

To integrate the function $y = (ax + b)^n$ we use the reverse chain rule.

$$\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{(n + 1) \cdot a} + c$$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax$
$\cos ax$	$\frac{1}{a}\sin ax$

For the curve $y = f(x)$ the area between the curve, the x-axis and the limits a & b is given by

$$Area = \int_a^b f(x)\,dx$$

The area between two curves, $f(x)$ & $g(x)$, is given by $\int_a^b \big(g(x) - f(x)\big)\,dx$, where $g(x)$ is the <u>upper</u> function.

The formula for the area between curves can be written as

$$Area\ between\ curves = \int_a^b (upper\ function - lower\ function)\,dx$$

Straight Lines

The equation of a straight line is given by $y = mx + c$, where m is the gradient and c is the y-intercept.

An alternative form of the equation of a straight line is $y - b = m(x - a)$, where m is the gradient and (a, b) is any point on the line.

Recurrence Relations

A general form of a recurrence relation is $u_{n+1} = au_n + b$.

The sequence generated by the recurrence relation $u_{n+1} = au_n + b$ *converges* to a *limit* if $-1 < a < 1$ otherwise the sequence *diverges* i.e. has no limit.

The limit of the sequence generated by $u_{n+1} = au_n + b$ is given by $L = \frac{b}{1 - a}$

Circles

The circle with centre (a, b) and radius r is defined by $(x - a)^2 + (y - b)^2 = r^2$

A special case is the circle centre at the origin $(0, 0)$ and radius r which has equation $x^2 + y^2 = r^2$

The general equation of a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$ is $x^2 + y^2 + 2gx + 2fy + c = 0$