# Exam P
## ครั้งที่ 4

# Conditional Probability

**54.** A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

**55.** The loss due to a fire in a commercial building is modeled by a random variable *X* with density function

$$f(x)=\begin{cases}0.005(20-x), & 0<x<20\\ 0, & \text{otherwise.}\end{cases}$$

Given that a fire loss exceeds 8, calculate the probability that it exceeds 16.

**56.** A group insurance policy covers the medical claims of the employees of a small company. The value, *V*, of the claims made in one year is described by $V = 100{,}000Y$

where *Y* is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

where *k* is a constant.

Calculate the conditional probability that *V* exceeds 40,000, given that *V* exceeds 10,000.

**57.** An insurance company insures a large number of homes. The insured value, *X*, of a randomly selected home is assumed to follow a distribution with density function

$$f(x)=\begin{cases}3x^{-4}, & x>1\\ 0, & \text{otherwise.}\end{cases}$$

Given that a randomly selected home is insured for at least 1.5, calculate the probability that it is insured for less than 2.

**58.** Once a fire is reported to a fire insurance company, the company makes an initial estimate, *X*, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, *Y*, to the claimant. The company has determined that *X* and *Y* have the joint density function

$$f(x,y)=\begin{cases}\dfrac{2}{x^2(x-1)}y^{-(2x-1)/(x-1)}, & x>1,\ y>1\\ 0, & \text{otherwise.}\end{cases}$$

Given that the initial claim estimated by the company is 2, calculate the probability that the final settlement amount is between 1 and 3.

**59.** A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let $X$ denote the proportion of employees who purchase the basic policy, and $Y$ the proportion of employees who purchase the supplemental policy. Let $X$ and $Y$ have the joint density function $f(x,y) = 2(x + y)$ on the region where the density is positive.

Given that 10% of the employees buy the basic policy, calculate the probability that fewer than 5% buy the supplemental policy.

**60.** A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let *X* denote the disease state (0 or 1) of a patient, and let *Y* denote the outcome of the diagnostic test. The joint probability function of *X* and *Y* is given by:

$$P[X=0, Y=0] = 0.800$$
$$P[X=1, Y=0] = 0.050$$
$$P[X=0, Y=1] = 0.025$$
$$P[X=1, Y=1] = 0.125$$

Calculate $\mathrm{Var}(Y|X=1)$

**61.** The stock prices of two companies at the end of any given year are modeled with random variables *X* and *Y* that follow a distribution with joint density function

$$f(x,y)=\begin{cases} 2x, & 0<x<1,\ x<y<x+1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the conditional variance of *Y* given that $X = x$.

**62.** An auto insurance policy will pay for damage to both the policyholder's car and the other driver's car in the event that the policyholder is responsible for an accident. The size of the payment for damage to the policyholder's car, $X$, has a marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver's car, $Y$, has conditional density of 1 for $x < y < x + 1$.

Given that the policyholder is responsible for an accident, calculate the probability that the payment for damage to the other driver's car will be greater than 0.5.

**63.** The joint probability density for $X$ and $Y$ is

$$f(x,y)=\begin{cases}2e^{-(x+2y)}, & x>0, y>0\\ 0, & \text{otherwise.}\end{cases}$$

Calculate the variance of $Y$ given that $X > 3$ and $Y > 3$.

**64.** A machine has two components and fails when both components fail. The number of years from now until the first component fails, *X*, and the number of years from now until the machine fails, *Y*, are random variables with joint density function

$$f(x,y)=\begin{cases}\frac{1}{18}e^{-(x+y)/6}, & 0<x<y \\ 0, & \text{otherwise.}\end{cases}$$

Calculate $\mathrm{Var}(Y \mid X=2)$.

# Conditional Expected & Variance

E(X) =

V(X) =

**65.** มีผู้เอาประกันอยู่ 3 ประเภท

- ประเภทที่ 1 เป็นอัตราส่วน 0.3 ของประชากรทั้งหมด มี expected loss = 10
- ประเภทที่ 2 เป็นอัตราส่วน 0.5 ของประชากรทั้งหมด มี expected loss = 8
- ประเภทที่ 3 เป็นอัตราส่วน 0.2 ของประชากรทั้งหมด มี expected loss = 7

นาย A, จงหาค่า expected loss ของนาย A

**66.** The number of workplace accidents occurring in a factory on any given day is Poisson distributed with mean $\lambda$. The parameter $\lambda$ is a random variable that is determined by the level of activity in the factory and is uniformly distributed on the interval [0,3].

Calculate the probability of one accident ,and expected number of claim on a given day.

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# Severity & frequency

$$S = X_1 + X_2 + \ldots + X_N$$

**67.** The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750. Individual losses have mean 1101 and standard deviation 70. The number of claims and the amounts of individual losses are independent.

Find expected and variance of SDIC's aggregate auto vandalism losses