Indefinite integrals

An indefinite integral is the opposite of a derivative, which is why indefinite integrals are also called *antiderivatives*.

The general rule for indefinite integration is

$$\int f(x)\ dx = F(x) + C$$

where C is the constant of integration and $F'(x) = f(x)$. In other words, the derivative of $F(x)$ is $f(x)$, which means that the integral of $f(x)$ is $F(x)$, plus the constant of integration C, which we add to account for a constant that might have disappeared when we took the derivative of $F(x)$ to get $f(x)$.

Of course, this means that you can check to see whether or not you took the integral correctly by taking the derivative of your answer. You should get back to the function you integrated originally.

Since indefinite integration happens over an open interval, taking an indefinite integral means we're finding the area under the curve in it's entire domain.

For basic power functions, we can use the integral formula,

$$\int x^a\ dx = \frac{x^{a+1}}{a+1} + C$$

Example

Evaluate the indefinite integral.

$$\int 3x^6\ dx$$

We need to remember the general rule $\int f(x)\ dx = F(x) + C$ where $F'(x) = f(x)$.

$$\int 3x^6\ dx = \frac{3x^7}{7} + C$$

Let's check our answer by taking its derivative to see if it gives us our original function, $f(x)$. Remember, the derivative of the constant C is always zero. Using power rule, we get

$$\frac{d}{dx}\left(\frac{3x^7}{7} + C\right) = \frac{21x^6}{7} + 0$$

$$\frac{d}{dx}\left(\frac{3x^7}{7} + C\right) = 3x^6$$

The derivative we just found is equal to our original function, so we know that the indefinite integral we calculated was correct.

Example

Find the antiderivative.

$$\int 2x^4 - 6x + 2\ dx$$

Using the formula $\int f(x)\ dx = F(x) + C$ where $F'(x) = f(x)$, we take the integral one term at a time and get

$$\int 2x^4 - 6x + 2\ dx = \frac{2x^5}{5} - \frac{6x^2}{2} + 2x + C$$

$$\int 2x^4 - 6x + 2 \, dx = \frac{2}{5}x^5 - 3x^2 + 2x + C$$