Sequences Mixed Exam Questions MS

Attempt these exam questions independently showing full and clear solutions. Check each answer as you go against the exam board mark scheme.

Arithmetic Sequences

1.

3(a)	(Tenth term) = $a + (10-1)\,d$	M1		
	…………… $= 1 + 9(6) = 55$	A1	2	NMS or rep. addn. B2 CAO SC if M0 award B1 for $6n-5$ OE
(b)(i)	$S_n = \frac{n}{2}[2+(n-1)6]$	M1		Formula for $\{S_n\}$ with either $a = 1$ or $d = 6$ substituted
	$\frac{n}{2}[2+6n-6] = 7400$	A1		Eqn formed with some expansion of brackets
	$3n^2 - 2n = 7400 \Rightarrow 3n^2 - 2n - 7400 = 0$	A1	3	CSO AG
(ii)	$(3n+148)(n-50) = 0$	M1		Formula/factorisation OE
	$\Rightarrow n = 50$	A1	2	NMS single ans. 50.. B2 CAO NMS 50 and –49.3(3…) B1 CAO
	Total		7	

2.

(a)	$a + 17d = 25$ or equiv. (for 1st B1), $a + 20d = 32.5$ or equiv. (for 2nd B1),	B1, B1	(2)
(b)	<u>Solving</u> (Subtract) $3d = 7.5$ so $\boldsymbol{d} = \underline{2.5}$ $a = 32.5 - 20\times 2.5$ so $\boldsymbol{a} = \underline{-17.5}$ (*)	M1 A1cso	(2)
(c)	$2750 = \frac{n}{2}\left[-35 + \frac{5}{2}(n-1)\right]$	M1A1ft	
	$\{\ 4\times 2750 = n(5n-75)\ \}$ $4\times 550 = n(n-15)$	M1	
	$\underline{n^2 - 15n = 55\times 40}$ (*)	A1cso	(4)
(d)	$n^2 - 15n - 55\times 40 = 0$ or $n^2 - 15n - 2200 = 0$	M1	
	$(n-55)(n+40) = 0$ $n = \ldots$	M1	
	$\underline{\boldsymbol{n = 55}}$ (ignore - 40)	A1	(3) **[11]**

3.

(i)	$u_5 = 8 + 4 \times 3$	M1		Attempt $a + (n - 1)d$ or equiv inc list of terms
	$= 20$ **A.G.**	A1	**2**	Obtain 20
(ii)	$u_n = 3n + 5$ ie $p = 3$, $q = 5$	B1		Obtain correct expression, poss unsimplified, eg $8 + 3(n - 1)$
		B1	**2**	Obtain correct $3n + 5$, or $p = 3$, $q = 5$ stated
(iii)	arithmetic progression	B1	**1**	Any mention of arithmetic
(iv)	$\frac{2N}{2}(16+(2N-1)3)-\frac{N}{2}(16+(N-1)3)=1256$	M1		Attempt S_N, using any correct formula (inc $\sum(3n+5)$)
		M1		Attempt S_{2N}, using any correct formula, with $2N$ consistent (inc $\sum(3n+5)$)
	$26N + 12N^2 - 13N - 3N^2 = 2512$ $9N^2 + 13N - 2512 = 0$	M1*		Attempt subtraction (correct order) and equate to 1256
	$(9N + 157)(N - 16) = 0$	M1dep*		Attempt to solve quadratic in N
	$N = 16$	A1	**5**	Obtain $N = 16$ only, from correct working
				OR: alternative method is to use $^n/_2\,(a + l) = 1256$
		M1		Attempt given difference as single summation with N terms
		M1		Attempt $a = u_{N+1}$
		M1		Attempt $l = u_{2N}$
		M1		Equate to 1256 and attempt to solve quadratic
		A1		Obtain $N = 16$ only, from correct working
			10	

Geometric Sequences

4.

Q	Solution	Marks	Total	Comments
(a)	$ar = 48; \quad ar^3 = 3$	B1		For either. OE
	$\Rightarrow 16r^2 = 1$	M1		Elimination of a OE
	$r^2 = \frac{1}{16} \Rightarrow r = -\frac{1}{4}$	A1		CSO **AG** Full valid completion. **SC** Clear explicit verification (max B2 out of 3.)
	or $r = \frac{1}{4}$	B1	4	
(b)(i)	$a = -192$	B1	1	
(ii)	$\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{4}\right)}$	M1		$\frac{a}{1-r}$ **used**
	$S_\infty = \frac{-768}{5}$ $(= -153.6)$	A1ft	2	Ft on candidate's value for a. i.e. $\frac{4}{5}a$ **SC** candidate uses $r = 0.25$, gives $a = 192$ and sum to infinity = 256. (max. B0 M1A1)
	Total		**7**	

5.

(a)	$r = 16 \div 20 = 0.8$	B1	1	OE
(b)	$\frac{a}{1-r} = \frac{20}{1-0.8}$	M1		OE Using a correct formula with $a = 20$ or $r =$ c's 0.8
	$= 100$	A1F	2	ft on c's value of r provided $\lvert r \rvert < 1$
(c)	$\{S_{20} =\} \frac{a(1-r^{20})}{1-r}$	M1		OE Using a correct formula with $n = 20$
	$= 100(1-0.8^{20}) = 98.847\{07..\}$	A1	2	Condone > 3dp
(d)	nth term $= 20\, r^{n-1} = 20(0.8)^{n-1}$ $= 20 \times 0.8^{-1} \times 0.8^n$	M1		Ft on c's r. Award even if 16^{n-1} seen
	$= 25 \times 0.8^n$	A1	2	CSO; AG
	Total		**7**	

6.

(a) $(S =)\ a + ar + ... + ar^{n-1}$ "$S =$" not required. Addition required.	B1	
$(rS =)\ ar + ar^2 + ... + ar^n$ "$rS =$" not required (M: Multiply by r)	M1	
$S(1-r) = a(1-r^n)$ $S = \dfrac{a(1-r^n)}{1-r}$ (M: Subtract and factorise) (*)	M1 A1cso	(4)
(b) $ar^{n-1} = 35000 \times 1.04^3 = 39400$ (M: Correct a and r, with $n = 3, 4$ or 5).	M1 A1	(2)
(c) $n = 20$ (Seen or implied)	B1	
$S_{20} = \dfrac{35000(1-1.04^{20})}{(1-1.04)}$	M1 A1ft	
(M1: Needs <u>any</u> r value, $a = 35000$, n =19, 20 or 21).		
(A1ft: ft from n =19 or n =21, but r must be 1.04).		
= 1 042 000	A1	(4)
		10

Inductive Sequences

7.

(a)	(i)	$u_2 = \frac{1}{2}$ $u_3 = 4$	B1 B1 FT **[2]**	State ½ State 4, following their u_2
(a)	(ii)	periodic / alternating / repeating / oscillating / cyclic	B1 **[1]**	Any correct description

8.

[1], $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	**2**	**B1** for [1], $\frac{1}{2}, \frac{1}{3}$

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9.

iA	10+20+30+40+50+60	B1	or $\frac{6}{2}(2\times10+5\times10)$ or $\frac{6}{2}(10+60)$	1
iB	correct use of AP formula with a = 10 and d = 10	M1		
	$n(5 + 5n)$ or $5n(n + 1)$ or $5(n^2 + n)$ or $(5n^2 + 5n)$	A1		
	$10n^2 + 10n - 20700 = 0$	M1	Or better	
	45 c.a.o.	A1		4
iiA	4	1		1
iiB	£2555	2	M1 for $5(1 + 2 + \ldots 2^8)$ or $5(2^9 - 1)$ o.e.	2
iiC	correct use of GP formula with a =5, r = 2	M1		
	$5(2^n - 1)$ o.e.= 2621435	DM1	"S" need not be simplified	
	2^n = 524288 www	M1		
	19 c.a.o.	A1		4

10.

(a)	(i)	$u_4 = \log_2 27 + 3\log_2 x$ $= \log_2 27 + \log_2 x^3$ $= \log_2(27x^3)$ **AG**	**M1** **M1** **A1** **[3]**	Use $u_4 = a + 3d$ Use $b \log a = \log a^b$ on $3\log_2 x$ Show $\log_2(27x^3)$ convincingly
(a)	(ii)	$27x^3 = 2^6$ $x = {}^4/_3$	B1* B1d* **[2]**	State correct equation no longer involving $\log_2 x$ Obtain ${}^4/_3$

(b)	(i)	$\frac{1}{2} < y < 2$	**M1** **A1** **[2]**	Identify at least one of ½ and 2 as end-points Obtain $\frac{1}{2} < y < 2$
(b)	(ii)	$\frac{\log_2 27}{1-\log_2 y} = 3$ $\log_2 27 = 3 - 3\log_2 y$ $\log_2 27 = 3 - \log_2 y^3$ $\log_2(27y^3) = 3$ $27y^3 = 8$ $y^3 = {}^8/_{27}$ $y = {}^2/_3$	B1 M1* M1d* A1* A1d* **[5]**	State $\frac{\log_2 27}{1-\log_2 y} = 3$ Attempt to rearrange equation to $\log_2 f(y) = k$ Use $f(y) = 2^k$ as inverse of $\log_2 f(y) = k$ Obtain correct exact equation no longer involving $\log_2 y$ Obtain ${}^2/_3$

11.

(i)		$(x + 4) - 2x = (2x - 7) - (x + 4)$ **OR** $2x + d = x + 4 \quad 2x + 2d = 2x - 7$ $2x = 15$ $x = 7.5$	M1 A1 A1 **[3]**	Attempt to eliminate d to obtain equation in x only Obtain correct equation in just x Obtain $x = 7.5$
(ii)	**(a)**	terms are 16, 12, 9 ${}^{12}/_{16} = 0.75$, ${}^{9}/_{12} = 0.75$ common ratio of 0.75 so GP $S_\infty = {}^{16}/_{1-0.75}$ $= 64$	B1 B1 M1 A1 **[4]**	List 3 terms Convincing explanation of why it is a GP Attempt use of ${}^{a}/_{1-r}$ Obtain 64

(ii)	**(b)**	${}^{(2x-7)}/_{(x+4)} = {}^{(x+4)}/_{2x}$ $4x^2 - 14x = x^2 + 8x + 16$ **OR** $2xr = x + 4 \quad 2xr^2 = 2x - 7$ $3x^2 - 22x - 16 = 0$ $(3x + 2)(x - 8) = 0$ $x = {}^{-2}/_3, x = 8$	M1* A1 M1d* A1 **[4]**	Attempt to eliminate r to obtain equation in x only Obtain $3x^2 - 22x - 16 = 0$ Attempt to solve quadratic Obtain $x = {}^{-2}/_3$